Excitation Function of the Reaction $Be^9(Li^7,Li^8)Be^8$

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(Received June 29, 1960; revised manuscript received August 31, 1960)

The cross section and thick-target yield as a function of energy are derived for the reaction $Be^9(Li^7,Li^8)Be^8$ over the laboratory energy range 1.1 to 4.0 Mev. A model is assumed in which the Li^7 picks up the final, loosely-bound neutron of Be^9 only at radial positions considerably outside the nuclear surface, direct reactions being prevented by the Coulomb repulsion.

THE simple Butler theory of surface reactions[1] can be modified quite accurately to take into account Coulomb effects for those reactions in which the direct reaction mechanism is still the chief factor determining the angular distribution. The modified theory is handicapped, however, by the large amount of numerical integration involved. Moreover, in reactions in which Coulomb effects tend to predominate over the direct reaction mechanism even the modified theory is inadequate in its description of the reaction. In addition, reactions involving projectiles more massive than H^3, He^3, or He^4 have generally not been attempted with this theory. These general comments are borne out with the $Be^9(Li^7,Li^8)Be^8$ reaction using the data of Norbeck *et al.*[2] for the laboratory energy range $E_L=2.0$ to 4.0 Mev. Even at the higher energy the bombarding energy is only 0.7 of the height of the Coulomb barrier. The peaks in the angular distribution occur at such large angles (120° for 2.0 Mev and 90° for 4.0 Mev) that the functions of the Butler theory are unable to drop sufficiently after the peaks to even approximate the data.

A second model may be proposed for the reaction. Classically, at least, the Li^7 can never penetrate to the nuclear surface because of the large Coulomb repulsion. However, the fifth and final neutron of Be^9 is so loosely bound that one expects it to spend considerable time exterior to the nuclear surface. Consequently, there is the possibility that the Li^7 may meet and capture this neutron even at considerable distances from the nucleus. The major scattering effects will be due to Coulomb deflection of the incident Li^7 and of the receding Li^8, with a small amount of deflection due to the momentum transfer of the reaction itself. This model has been independently suggested by Allison[3] from a detailed investigation of the angular distribution of the Be^9, Li^7 reaction. With this model the expression for the excitation function (production cross section versus bombarding energy) will be derived and compared with experiment.

From shell model considerations, the radial wave function of the final reutron in Be^9, when it is exterior to the nuclear surface of radius r_0, is proportional to the spherical Hankel function of the first kind and of order one:

$$\psi(r)=Ah_1^{(1)}(i\beta r)=Ai\left[\frac{1}{\beta r}+\frac{1}{\beta^2r^2}\right]e^{-\beta r}, \tag{1}$$

where $\beta=[2\mu/\hbar^2]^{\frac{1}{2}}$ and $r>r_0$. If one takes as a model for Be^9 a (Be^8+n) configuration, then the binding energy of the last neutron, $E_b=1.67$ Mev and $\beta=2.66\times10^{12}$ cm^{-1}. If one assumes, instead, a $(2\alpha+n)$ configuration, then $E_b=0.73$ Mev and β is somewhat ill-defined due to the three-body nature of the problem but is approximately equal to 1.77×10^{12} cm^{-1}. Allison's calculations[3] on the angular distributions demonstrate agreement with Eq. (1) with an experimental value of $\beta=(2.1\pm0.5)\times10^{12}$ cm^{-1} for radial distances between 1.5 and 3.0×10^{-12} cm.

As a Li^7 passes through the outer neutron cloud of a Be^9 nucleus, following a hyperbolic path due to Coulomb repulsion, the probability that a neutron will be captured is described by

$$P_c(r_p)=\int_{-\infty}^{\infty}|\psi(r)|^2\sigma_c dl, \tag{2}$$

where σ_c is a sticking probability for a neutron onto Li^7, assumed constant. Here dl is an infinitesimal section of the hyperbolic trajectory characterized by the classical perinuclear separation, r_p, which depends only upon the incident energy and the angle of scattering.

Then the cross section for production of Li^8 is given by

$$\sigma(E_L)=\int_{r_m}^{\infty}P_c(r_p)2\pi r_p dr_p, \tag{3}$$

where the integral extends outward from the classical distance of closest approach:

$$r_m=zZe^2/E_i.$$

Here E_i is the energy available for the reaction in the center-of-mass system.

Equation (2) can be evaluated numerically; however, following the suggestion of Allison,[3] most of the contribution to the integral occurs in the vicinity of $r=r_p$ over a path length equal to the perinuclear separation because of the exponential nature of the integrand. It

[1] S. T. Butler, Phys. Rev. **106**, 272 (1957).
[2] E. Norbeck, J. M. Blair, L. Pinsonneault, and R. J. Gerbracht, Phys. Rev. **116**, 1560 (1960).
[3] S. K. Allison, Phys. Rev. **119**, 1975 (1960).

is therefore a satisfactory approximation to replace the integral by

$$P_c(r_p)=|\psi(r_p)|^2\sigma_c r_p.$$

Equation (3) then reduces to a simple integral which can be evaluated as

$$\sigma(E_i)=\frac{\pi\sigma_c A^2}{\beta^3}\left(1+\frac{2E_i}{\beta zZe^2}\right)\exp\left(-\frac{2\beta zZe^2}{E_i}\right), \quad (4)$$

$$\sigma(E_L)=\text{constant}(1+0.310E_L)\exp(-12.90/E_L),$$

when Allison's experimental value of $\beta=2.1\times10^{12}$ cm^{-1} and the center-of-mass to laboratory energy conversion are used.

Curve A of Fig. 1 exhibits the behavior of Eq. (4) when normalized at the 2.0-Mev point to the thin target data of Norbeck *et al.*[2] covering the laboratory energies from 1.7 to 3.8 Mev. Also included in the figure is the thin target data of Norbeck and Littlejohn[4] over the energy range 1.1 to 2.0 Mev and normalized to the previous data in the region of overlap.

The yield expected from a very thick target will be given by

$$Y=NT\int_0^R \sigma(E)dx=NT\int_{E_L}^0 \sigma(E)\frac{dx}{dE}dE, \quad (5)$$

where N is the number of incident Li^7 ions, T is the number of Be^9 targets per unit volume, and R is the range of the Li^7 ion of energy E_L. Now $\sigma(E)$ may be taken from Eq. (4) with the modification that the energy is allowed to vary from E_L to zero as the Li^7 ion slows down in the target. Calculations of Allison[5] for the stopping power of Li^7 in argon indicate an energy dependence,

$$dE/dx=-kE^{0.53\pm0.05}, \quad (6)$$

over the energy range 0 to 2 Mev. This dependence should change very little for Li^7 in Be^9 instead of in argon. For lack of additional information, Eq. (6) will also be assumed to hold over the entire energy range from 0 to 4 Mev and the power will be taken as 0.50 for ease of calculation. Then the expression for the yield is

$$Y=\text{constant}\int_0^{E_L}(1+0.310E)e^{-12.90/E}E^{-0.50}dE. \quad (7)$$

This equation is readily integrated to give:

$$Y=N\left\{(185E_L^{\frac{3}{2}}-18.0E_L^{\frac{1}{2}})\times e^{-12.90/E_L}+10.0\int_{(12.90/E_L)^{\frac{1}{2}}}^{\infty}e^{-t^2}dt\right\}, \quad (8)$$

[4] E. Norbeck and C. S. Littlejohn, Phys. Rev. **108**, 754 (1957).
[5] S. K. Allison (private communication).

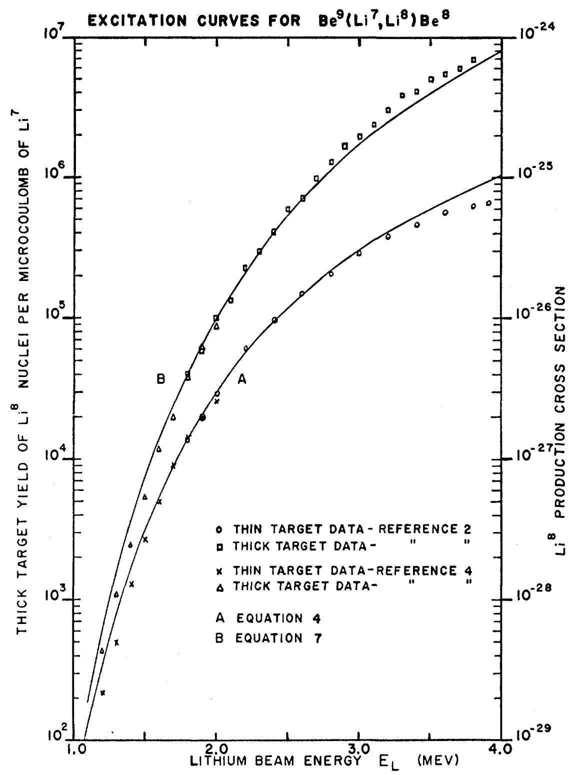

FIG. 1. The derived excitation functions for the reaction $Be^9(Li^7,Li^8)Be^8$ normalized at 2.0 Mev to the experimental data of reference 2. Also shown is the data of reference 4 normalized to the data of reference 2 in the region of overlap. Curve A: Cross section versus energy. Curve B: Thick-target yield versus energy.

where N is an unspecified constant. Equation (8) is plotted as curve B of Fig. 1 when normalized at the 2.0 Mev point of the thick-target data of Norbeck.[2,4]

The fit of Eq. (4) to the experimental points indicates a value of $\beta=(2.10\pm0.15)\times10^{12}$ cm^{-1}. If one takes the Be^9 models at all seriously, this value of β suggests that the (Be^8+n) configuration exists about 37% of the time.

The small discrepancy between the thin-target data and Eq. (4) in the high-energy region may be due to competition from the usual surface reactions and from compound nucleus formation. The somewhat larger discrepancy between the thick-target data and Eq. (8) in the high-energy region is probably due to a variation in the energy dependence of the stopping power from that assumed in Eq. (6).

ACKNOWLEDGMENTS

We wish to thank Professor S. K. Allison for a preprint of his paper and both Professor Allison and Dr. E. Norbeck for discussions of the model proposed.