Coulomb Excitation of States in Th^{232} and U^{238}

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The yield of gamma rays resulting from Coulomb excitation of states in Th^{232} and U^{238} with protons of 4- to 6-Mev energy have been measured. Gamma rays of 790-, 740-, and 613-kev energy from Th^{232} and of 1.02-Mev energy from U^{238} are observed. For Th^{232}, the variation in gamma-ray yields with proton energy, the results from gamma-ray angular distributions, and the linear polarization measurements are consistent with the direct excitation of a 2+ state at 790-kev which decays by means of $E2$ radiation to the $I=0+$ and 2+ members of the ground-state rotational band. In view of the recent results from internal conversion electron measurements at Rice University, the 613-kev gamma ray is interpreted to be a transition from a 2+ β-vibrational state at 773 kev to the 4+ state at 163 kev. An interpretation of the observed gamma-ray yields is carried out taking a γ-vibrational state at 790 kev and a β-vibrational state at 773 kev. The $B(E2)_d$ of the 2+ state at 50 kev in Th^{232} is about 150 times the single-particle estimate. The $B(E2)_d$ of the vibrational states in Th^{232} and U^{238} are between 2 and 4 times the single-particle estimate. Combining Bernstein's relative internal conversion electron yields with our gamma-ray data, a value of $(8\pm2)\times10^{-2}$ for the strength parameter ρ is obtained for the $E0$ transition between the 2+ state at 773 kev and the 2+ state at 50 kev in Th^{232}.

I. INTRODUCTION

EARLY work with the Coulomb excitation process showed that states at 50[1] and 760[2] kev are excited in Th^{232}. The reduced electric quadrupole transition probability, $B(E2)$, extracted from the Coulomb excitation cross section for excitation of the 760-kev state was about 4 times larger than the Weisskopf single-particle estimate. Moore *et al.*[3] observed internal conversion electrons corresponding to a transition energy of 719 kev when Th^{232} was bombarded with 4.8-Mev protons. If these internal conversion electrons are attributed to an $E2$ transition, the $B(E2)$ deduced from their observed cross section is about 10 times larger than the result obtained from our gamma-ray data.

We have made additional studies of Coulomb excitation of states in Th^{232} with protons and α particles.[4] Gamma rays of 790, 740, and 613 kev are observed when Th^{232} is bombarded with 4.0- to 6.0-Mev protons. The composite peak in the pulse-height spectrum of the gamma radiation, which was identified as the 760-kev transition in the earlier measurements,[2] is now attributed to transitions of (790±10) kev and (740±8) kev. Both the variation in gamma-ray yields with proton energy and the results from the angular distribution of the gamma rays were consistent with the direct excitation of a 2+ state at 790 kev which decays by means of $E2$ radiation to $I=0+$ and 2+ members of the ground-state rotational band. The energy of the 613-kev transition was about 12 kev too small to correspond to the $2+\rightarrow4+$ transition. The angular distribution of the 740-kev gamma rays could, however, be fitted by $E2/M1=0.18\pm0.04$ (decay by predominantly $M1$ radiation). It seemed likely that the 2+ excitation at 790 kev in Th^{232} corresponded to a vibrational excitation of a spheroidal nucleus. In this connection, we were inclined to exclude the interpretation of decay by predominantly $M1$ radiation because $M1$ radiation is forbidden in the decay of vibrational excitations.[5]

Recently, Bernstein[6] measured the relative yields of internal conversion electrons of three transitions (795, 733, and 615 kev with an accuracy of ±10 kev) from Coulomb excitation of Th^{232}. Bernstein observed about 7 times too many internal conversion electrons in the K shell for the 733-kev transition if this transition is interpreted as $E2$ radiation. However, this excess of internal conversion electrons could nearly be accounted for if the decay is predominantly by $M1$ radiation ($\beta_1^K/\alpha_2^K=4.5$). An alternative explanation for the excess of internal conversion electrons in the 740-kev transition is that the $E0$ mode competes with the $E2$ mode in the $2+\rightarrow2+$ transition. There is an objection to this latter explanation if the 613 kev is not associated with the decay of the 790-kev state. In this case, the 2+ level at 790 kev is probably a γ-vibrational excitation and a possible $E0$ mode in the $2+\rightarrow2+$ decay is less probable.[7]

The group at Rice University have remeasured the internal conversion electrons from Coulomb excitation of states in Th^{232} with improved resolution.[8] In addition to the intense transition of 723±4 kev, a weak transition is observed at 740 kev in the internal conversion

[1] G. M. Temmer and N. P. Heydenburg, Phys. Rev. **93**, 351 (1954).
[2] P. H. Stelson and F. K. McGowan, Phys. Rev. **99**, 112 (1955).
[3] M. S. Moore, C. M. Class, F. W. Prosser, and J. P. Schiffer, Bull. Am. Phys. Soc. **1**, 88 (1956).
[4] F. K. McGowan and P. H. Stelson, Bull. Am. Phys. Soc. **2**, 207 (1957).
[5] See, e.g., the review paper by K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Revs. Modern Phys. **28**, 432 (1956).
[6] E. M. Bernstein, Bull. Am. Phys. Soc. **4**, 9 (1959) and (private communication), 1957.
[7] A. S. Reiner, Ph.D. thesis, University of Amsterdam, 1958 (unpublished).
[8] D. H. Rester, F. E. Durham, and C. M. Class, Bull. Am. Phys. Soc. **4**, 98 (1959); F. E. Durham, D. H. Rester, and C. M. Class, Bull. Am. Phys. Soc. **5**, 110 (1960); F. E. Durham, D. H. Rester, and C. M. Class, Phys. Rev. Letters **5**, 202 (1960).

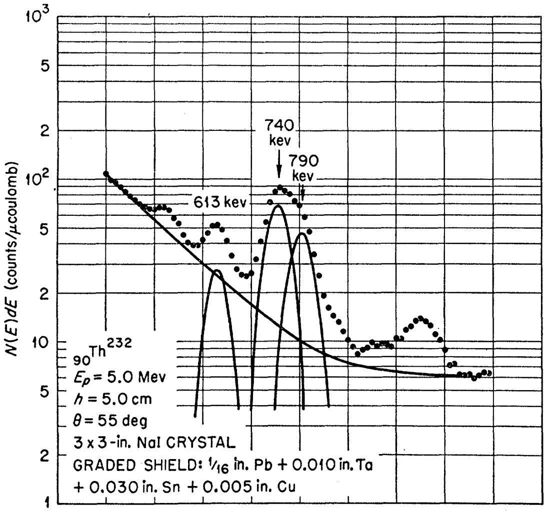

FIG. 1. Differential pulse-height spectrum of the gamma radiation for proton bombardment of Th^{232}.

electron spectrum. The intense electron transition is attributed to an $E0$ transition from a 2+ β-vibrational excitation at 773 kev to the 2+ state at 50 kev. With this evidence, the 613-kev transition, observed in the gamma-ray measurements, is probably associated with the decay of the 773-kev state and corresponds to the $2+ \rightarrow 4+$ transition.

In this paper we wish to report the results of gamma-ray measurements from Coulomb excitation of Th^{232} and U^{238}. A discussion of the results in conjunction with internal conversion electron data is given.

II. EXPERIMENTAL PROCEDURE

The projectiles used for effecting Coulomb excitation were variable-energy protons and singly- and doubly-ionized helium ions accelerated by the 5.5-Mv ORNL electrostatic generator. The target support arrangement and methods of measuring yields, angular distributions, and linear polarization of the gamma-rays have already been described.[9–11] The thorium target was prepared from a 0.003-inch foil[12] which was spotwelded onto 0.005-inch Ni. The U^{238} target was prepared from metallic disks 0.018-inch thick.[13]

A. Gamma-Ray Spectra

A typical pulse-height spectrum of the gamma radiation taken with a 3- ×3-in. NaI scintillation spectrometer is shown in Fig. 1 for 5.0-Mev protons incident on Th^{232}. The gamma rays of (613±6), (740±8), and (790±10) kev are due to excitation in Th^{232}. The peak at 760 pulse-height units is much too wide for a single gamma ray. From a knowledge of the resolution of our detector and from the spectra of the gamma rays in the angular distribution measurements, the peak has been decomposed into two peaks at 740 and 790 kev. To avoid inaccuracy resulting from slight variations in the gain of the spectrometer with changes in the counting rate, the spectrum of gamma rays was also taken simultaneously with sources of Be^7 and Zn^{65} to obtain the energy calibration. From the recent work by the group at Rice University mentioned in Sec. I, we now know that the composite peak is probably due to four gamma rays of 723, 740, 773, and 790 kev. However, in this section the analysis of the measurements is based on the excitation of a single state at 790 kev because these results were obtained between two and three years ago.[4] An analysis of the results in conjunction with the recent internal conversion electron data is deferred to Sec. III.

In addition to pulses from gamma rays resulting from Coulomb excitation, the spectrum contains pulses from (a) the local background, (b) the gamma rays from proton bremsstrahlung, and (c) the gamma rays from nuclear reactions in the light element target impurities. The peaks at 525 and 1050 pulse-height units are not associated with excitation in Th^{232} but

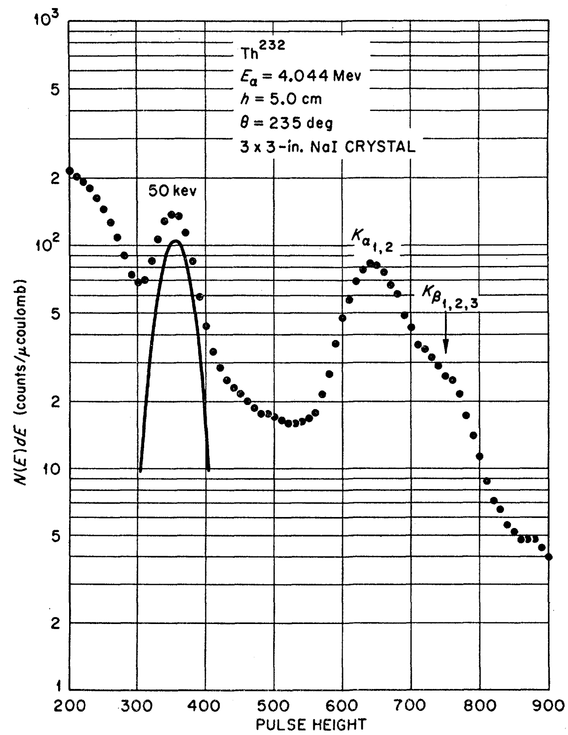

FIG. 2. Differential pulse-height spectrum of the gamma radiation for α-particle bombardment of Th^{232}.

[9] P. H. Stelson and F. K. McGowan, Phys. Rev. **110**, 489 (1958).
[10] F. K. McGowan and P. H. Stelson, Phys. Rev. **106**, 522 (1957).
[11] F. K. McGowan and P. H. Stelson, Phys. Rev. **109**, 901 (1958).
[12] The thorium foils were obtained from the Metallurgy Division of the Oak Ridge National Laboratory.
[13] The U^{238} disks were obtained from the Metallurgy Division of the Argonne National Laboratory.

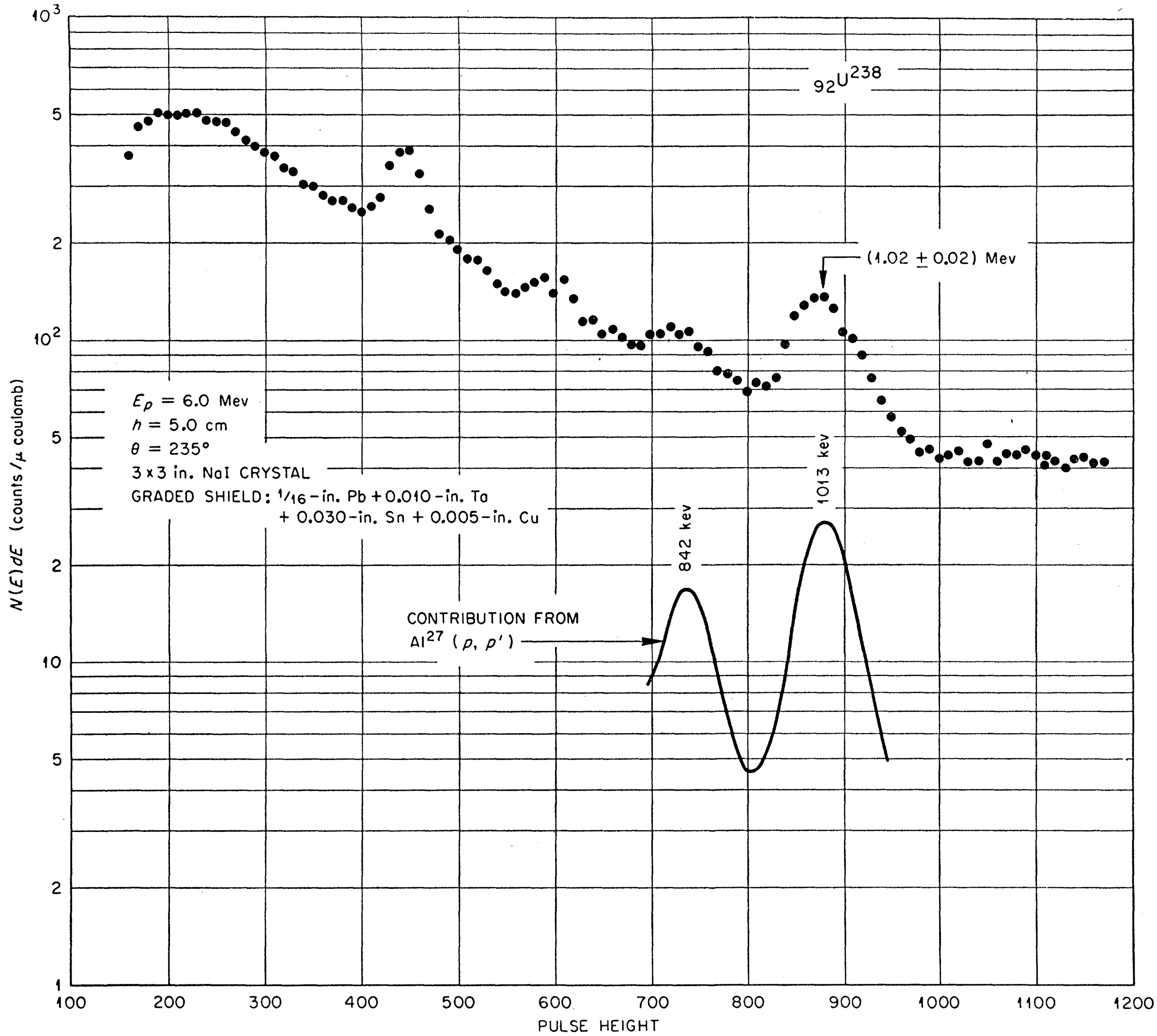

FIG. 3. Differential pulse-height spectrum of the gamma radiation for proton bombardment of U^{238}.

depend to a certain extent on the surface condition of the target.

Figure 2 is a pulse-height spectrum of the gamma radiation when Th^{232} is bombarded with 4.044-Mev He ions. The (50±3) kev gamma ray is attributed to Coulomb excitation of the 2+ state of the ground-state rotational band. The *K* x rays result from the stopping of the α particles in the target.

In Table I we have listed the gamma-ray yields from Coulomb excitation of states in Th^{232} with protons and α particles incident on a thick target.

A pulse-height spectrum of the gamma radiation taken with a 3- ×3-inch NaI scintillation spectrometer is shown in Fig. 3 for 6.0-Mev protons incident on U^{238}. The peak at (1.02±0.2) Mev is due to Coulomb excitation of a state in U^{238}. As is well known, gamma rays from nuclear reactions produced by bombardment of low-*Z* target impurities cause troublesome interference in Coulomb excitation experiments. Several targets of U^{238} from various sources were studied for Coulomb excitation of U^{238}. With exception of the sample from Argonne National Laboratory, the gamma rays from Coulomb excitation were completely obscured by low-*Z* impurities. For example, one sample contained by weight 200 ppm Si, 13 ppm Al, 60 ppm Fe, and N (amount not known), which gave rise to the following troublesome gamma rays: $Si^{28}(p,p')$ 1.78 Mev, $Al^{27}(p,p')$ 0.842, 1.013, and 2.19 Mev, $Al^{27}(p,\alpha\gamma)$ 1.37 Mev, $Fe^{56}(p,p')$ 0.845 Mev, and $N^{14}(p,p')$ 2.31 Mev. The sample of U^{238} from Argonne contained 10 ppm Si, 7 ppm Al, 12 ppm Fe, and 13 ppm N. In addition to the gamma rays shown in Fig. 3 the spectrum also contained strong lines corresponding to γ rays or 1.37, 1.78, and 2.31 Mev. Below bombarding energies of 5 Mev the gamma radiation from Coulomb excitation of U^{238} was completely obscured by the 1.013-Mev

TABLE I. Yield of gamma rays obtained from Coulomb excitation of states in Th^{232}. The evaluation of the integral is given in units of kev mg/cm².

E_p(Mev)	E_γ(kev)	Yield of γ's per μcoul	Excitations/μcoul	$\int_0^{E_i} \frac{g_2(\xi,\eta_i)E'dE}{dE/d\rho x}$	$B(E2)_{\text{exc}}$(cm⁴e²)
6.0	790	$(4.96\pm0.60)\times10^4$	14.40×10^4	9.50×10^4	2.00×10^{-49}
	740	$(6.83\pm0.82)\times10^4$			
	613	$(2.34\pm0.70)\times10^4$			
5.5	790	$(2.47\pm0.30)\times10^4$	7.76×10^4	5.36×10^4	1.91×10^{-49}
	740	$(3.89\pm0.47)\times10^4$			
	613	$(1.26\pm0.33)\times10^4$			
5.0	790	$(1.53\pm0.18)\times10^4$	4.33×10^4	2.62×10^4	2.18×10^{-49}
	740	$(2.05\pm0.25)\times10^4$			
	613	$(6.65\pm1.73)\times10^3$			
4.5	790	$(6.64\pm0.80)\times10^3$	1.85×10^4	1.09×10^4	2.25×10^{-49}
	740	$(8.56\pm1.03)\times10^3$			
	613	$(3.00\pm0.78)\times10^3$			
4.0	790	$(2.18\pm0.26)\times10^3$	5.99×10^3	3.31×10^3	2.38×10^{-49}
	740	$(2.69\pm0.32)\times10^3$			
	613	$(1.01\pm0.26)\times10^3$			
4.044[a]	50	$(1.04\pm0.12)\times10^4$	3.28×10^6	1.71×10^4	6.31×10^{-48}

[a] α particles.

gamma ray from $Al^{27}(p,p')$. In order to remove the contribution of the 1.013-Mev gamma ray from the spectrum, the yields from Al and U^{238} were measured as function of proton energy from 3.0 to 6.0 Mev. In Table II we have listed the yields of gamma rays from Coulomb excitation of U^{238}. The yields could have been studied over a wider range of proton energy if the impurity content of the U^{238} had been an order of magnitude less for Al and N. Since the excitation in U^{238} is probably analogous to that observed in Th^{232}, we have taken the excitation energy to be 1.04 ± 0.02 Mev for the 2+ state which decays by $E2$ radiation to $I=0+$ and 2+ members of the ground-state rotational band.

B. Gamma-Ray Angular Distributions

The angular distributions of the 790-, 740-, and 613-kev gamma rays from Coulomb excitation of Th^{232} were measured with respect to the incident proton beam on a thick target of Th^{232} at $E_p=5.0$ Mev. A typical pulse-height spectrum of the gamma rays taken at $\theta=90°$ and 0° is shown in Fig. 4. The fact that the 790-kev gamma-ray distribution is peaked in the forward direction suggests immediately the direct excitation of a 2+ state at 790 kev. If the nuclear excitation were electric dipole Coulomb excitation, the 790-kev gamma-ray distribution would have been peaked at 90°. The position of the composite peak in the pulse-height spectrum of the gamma rays at $\theta=90°$ is determined primarily by the 740-kev transition. In Table III we have listed the observed angular distribution coefficients ($A_\nu a_\nu G_\nu$) for the composite correlation involving the 740- and 790-kev gamma rays for several measurements at $E_p=5.0$ Mev. The errors quoted for these coefficients in Table III include the standard deviation to be expected from the finite number of counts collected and a decentering error. The coefficients for $W(\theta)_{740}$ were deduced from 0.59 $W(\theta)_{740}=W(\theta)_{\text{composite}}-0.41$ $W(\theta)_{790}$, where $W(\theta)_{790}$ is the distribution function for the transition sequence $0(E2)2(E2)0$ and 0.59 and 0.41 are the relative intensities (areas) of the full energy peaks of the 740- and 790-kev gamma rays, respectively, in the pulse-height spectrum. The thick-target particle parameters, $(a_2G_2)_t=0.89$ and $(a_4G_4)_t=0.07$, were obtained from curves in a previous paper.[11] Since the coefficient of $P_4(\cos\theta)$ is relatively small in the composite correlation, we have not used this coefficient in deducing a value for $E2/M1$ for the $2(E2+M1)2$ transition of 740 kev. The values of $\delta=(E2/M1)^{\frac{1}{2}}$ were obtained from the A_2 given in column 5 of Table III. The angular distribution of the 740-kev gamma rays can be fitted by $E2/M1=0.18\pm0.04$ (decay by predominantly $M1$ radiation) or by $E2/M1>10^2$. Since it seemed likely

TABLE II. Yield of gamma rays from Coulomb excitation of the 1.04-Mev state in U^{238}.

E_p(Mev)	Yield of γ's per μcoul	Excitations per μcoul	$\int_0^{E_i} \frac{g_2(\xi,\eta_i)E'dE}{dE/d\rho x}$	$\epsilon B(E2)_{\text{exc}}$(cm⁴e²)[a]
6.0	$(3.00\pm0.60)\times10^4$	3.03×10^4	3.62×10^4	11.8×10^{-50}
5.5	$(1.10\pm0.38)\times10^4$	1.11×10^4	1.68×10^4	9.3×10^{-50}
5.0	$(4.7\pm1.4)\times10^3$	4.75×10^3	6.37×10^3	10.5×10^{-50}

[a] ϵ is the fraction of the excitations which decay through the $2+\rightarrow0+$ and $2+\rightarrow2+$ transitions which were observed in the gamma-ray spectrum.

that the 2+ excitation at 790 kev in Th^{232} corresponded to a vibrational excitation of a spheroidal nucleus, we were inclined to exclude the interpretation of decay by predominantly $M1$ radiation for the 740-kev transition because $M1$ radiation is forbidden in the decay of vibrational excitations.[5]

In view of the recent internal conversion electron data,[8] the 613-kev transition could be interpreted as an $E2$ transition from the 2+ state at 773 kev to the 4+ state of the ground-state rotational band. To test this point we measured the angular distribution of the 613-kev gamma rays and found the distribution to be isotropic to within ±4%. For the conditions of our experiment, we should have observed an anisotropy of 13%. However, we could have missed detecting an anisotropy of this magnitude because the background associated with the gross intensity under the 613-kev full-energy peak was about 60% and we took the background to be isotropic.

Since the evidence for the assignment of the 613-kev gamma ray to the $2\rightarrow 4$ transition was somewhat in doubt, we examined other possible interpretations. Excitation of a 1− state with $K=0$ at 663 kev would account for the nearly isotropic distribution of the 613-kev gamma rays but there is no evidence for a 663-kev transition in the gamma-ray spectra of the angular distribution measurements. In addition the yield of 613-kev gamma rays increases too rapidly with the bombarding energy for the excitation of a state at 663 kev. In fact, the yield of 613-kev gamma rays agrees better with the $E2$ excitation of a state near 790 kev.

After the completion of these measurements,[4] Bernstein[6] measured the relative yields of internal conversion electrons of three transitions (795, 733, and 615 kev) from Coulomb excitation of Th^{232}. Bernstein observed about 7 times too many internal conversion electrons in the K shell for the 733±10 kev transition if this transition is interpreted as $E2$ radiation. These electrons from the 733-kev transition are probably to be identified with the internal conversion electrons from the 723±4 kev transition observed by the group at Rice University.[8] The excess of internal conversion electrons in the K shell observed by Bernstein could nearly be accounted for if the decay is predominantly by $M1$ radiation because the ratio of internal conversion coefficients β_1^K/α_2^K is 4.5.[14] In view of Bernstein's results, we have measured the linear polarization-direction correlation of the 740-kev gamma rays to remove the ambiguity in the interpretation of the gamma-ray distribution data.

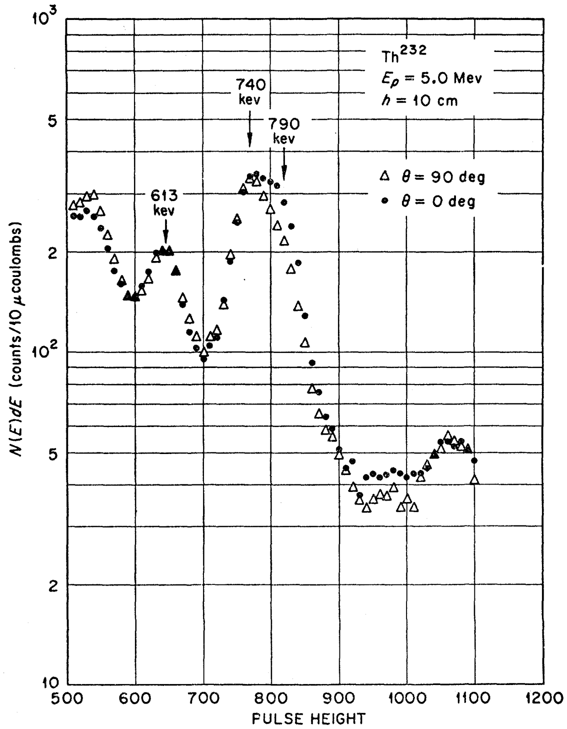

FIG. 4. Pulse-height spectrum of the gamma rays from Th^{232} taken at $\theta=0°$ and 90° with respect to the incident proton beam.

C. Polarization-Direction Correlation

Biedenharn and Rose[15] have expressed in a convenient form the polarization-direction correlation with polarization of the mixed radiation being measured.

[14] See, e.g., M. E. Rose, *Internal Conversion Coefficients* (North-Holland Publishing Company, Amsterdam, 1958) or L. A. Sliv and I. M. Band, Leningrad Physico-Technical Institute Reports, Part I, 1956; Part II, 1958 [translation: Reports 57ICC K1 and 58ICC L1, issued by Physics Department, University of Illinois, Urbana, Illinois (unpublished)].

[15] L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953).

TABLE III. Proton-gamma angular distribution coefficients observed at $E_p=5.0$ Mev for the composite correlation involving the 740- and 790-kev gamma rays. The coefficients for $W(\theta)_{740}$ were deduced from $0.59\,W(\theta)_{740}=W(\theta)_{\rm composite}-0.41\,W(\theta)_{790}$.

$W(\theta)_{\rm composite}$		$W(\theta)_{740}$				
$A_2a_2G_2$	$A_4a_4G_4$	$A_2a_2G_2$	$A_4a_4G_4$	A_2	A_4	$\delta=(E2/M1)^{\frac{1}{2}}$ [a]
0.086±0.008	−(0.029±0.010)	−0.076	0.006	−0.085	−0.08	−0.46 or −83
0.092±0.008	−(0.032±0.010)	−0.065	0.001	−0.073	−0.02	−0.44 or >100
0.121±0.010	−(0.029±0.012)	−0.015	0.006	−0.017	−0.09	−0.35 or 12
0.102±0.011	−(0.047±0.015)	−0.047	−0.023	−0.053	0.33	−0.41 or 33
0.085±0.009	−(0.042±0.012)	−0.078	−0.015	−0.087	0.21	−0.47 or −70
0.107±0.008	−(0.023±0.008)	−0.039	0.016	−0.044	−0.23	−0.39 or 23

[a] For the definition of δ, see L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953).

The correlation function has for a gamma-gamma cascade the form

$$W(\theta,\phi)=W_I+\delta^2W_{II}+2\delta W_{III},$$

where δ is $(E2/M1)^{\frac{1}{2}}$. W_I, W_{II}, and W_{III} are the polarization-direction correlation functions for pure 2^{L_I} pole-pure 2^{L_2} pole, pure 2^{L_1} pole-pure 2^{L_2+1} pole, and the interference term, respectively. ϕ is the angle between direction of polarization and the normal to the plane defined by the directions of propagation of the two gamma rays in cascade and θ is the angle between the directions of propagation of the two gamma rays. For gamma rays following Coulomb excitation, one replaces A_ν appearing in the correlation functions above by $A_\nu a_\nu$ where a_ν is the particle parameter that enters in the Coulomb excitation process, and one takes $L_1=2$ (electric quadrupole excitation).

Our polarimeter, which is based on the Compton scattering mechanism, has been described in a previous paper.[11] The polarimeter is arranged so that $\theta=90°$. We measure $N_{\|}/N_{\perp}$, the ratio of the coincidence rate for the position when the detector of the Compton scattered photons is in the plane of the two gamma rays to the coincidence rate for the perpendicular position. The ratio of the linear polarization intensities of the gamma ray is given by $P=W(90°,90°)/W(90°,0°)$ and is connected to the ratio $N_{\|}/N_{\perp}$ by the relation

$$N_{\|}/N_{\perp}=(P+R)/(PR+1),$$

where R is the sensitivity of the polarimeter. The polarimeter has been calibrated previously for gamma-ray energies of 200 to 500 kev by the use of known polarizations of gamma rays from Coulomb excitation.[11] For the measurements discussed in this paper, we have used the known polarization of the 803-kev gamma ray from Coulomb excitation of the 2+ state in Pb^{206} to calibrate the polarimeter. To assure that the axis of rotation of the detector passed through the target, the following alignment procedure was used. First, the location of the proton beam on the target was determined. Then a source of Nb^{95}(765 kev) of the same area as the beam was placed on the target at this position. The axis of rotation was adjusted until the coincidence counting rates showed that $N_{\|}/N_{\perp}$ did not differ from unity by more than 0.5%. For 5-Mev protons incident on a target of lead containing 88% Pb^{206}, the measured value of $N_{\|}/N_{\perp}$ is 0.611 ± 0.031. This value for $N_{\|}/N_{\perp}$ leads to an experimental value for R of 2.76 ± 0.49.

We observed the composite linear polarization-direction correlation of the 790- and 740-kev gamma rays from Th^{232} with 5-Mev protons. The values of $P(\theta=\pi/2)$ with weighting factors of 0.41 and 0.59 for the 790- and 740-kev transitions, respectively, are 2.49 ± 0.04 for $\delta=-(0.42\pm0.05)$, 1.22 for $\delta=10$, 1.31 for $\delta=-10$, and 1.26 for $|\delta|\geqslant10^2$. The result of the measurement is $N_{\|}/N_{\perp}=0.886\pm0.025$ and this leads to an experimental value for $P(\theta=\pi/2)=1.29\pm0.13$. This result is consistent with the assignment $\delta^2\geqslant10^2$ for the 740-kev transition.

III. REDUCED TRANSITION PROBABILITIES

The total internal conversion coefficient, α_T, must be known in order to relate the cross sections to the observed gamma-ray yields. For this purpose the calculations of Rose and of Sliv and Band have been used.[14] For the K and L shells, these calculations include the effect of the finite size of the nucleus. In the case of the M shell, the unscreened calculations by Rose have been used together with an empirical screening factor (M_{sc}/M_{unsc}) of 0.6. For the N and O shells we have used a $(N+O)/L$ ratio of 0.06. Column 6 of Table IV lists the α_T's for $E2$ transitions. To avoid confusion the reduced transition probabilities for excitation and for decay are written as $B(E2)_{exc}$ and $B(E2)_d$. Numerical values of the theoretical thick target integral,[9] which relates the cross section to the thick target yield of gamma rays, are listed in Tables I and II. In column 6 of Table IV, we have taken $B(E2)_{sp}$ to be equal to $(1/4\pi)|\frac{3}{5}R^2|^2$ where $R=1.2\times10^{-13}A^{\frac{1}{3}}$ cm.[16] The $B(E2)_{exc}$ for the 790-kev state in Table IV does not include transitions from the decay of this state by the $E0$ mode. Comparing the relative internal conversion electron yields with our gamma-ray yields for the 790- and 740-kev transitions, Bernstein obtains for the 740-kev transition a K-shell internal conversion coefficient of $(8.8\pm4)\times10^{-2}$. If this contribution to the decay is included in the yields for excitation of the 790-kev state, the $B(E2)_{exc}$ for the 790-kev state should be increased by 3.7% in Table IV.

The evidence for excitation of the β- and γ-vibrational states in Th^{232} seems fairly firm. As a result we have attempted an analysis of the gamma-ray data in conjunction with the internal conversion electron data. An analysis of the composite peak at 740 to 790 kev in the gamma-ray spectrum into 4 gamma-ray peaks at

[16] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), Chap. XII.

TABLE IV. Summary of the quantities obtained from $B(E2)_{exc}$ for Th^{232}. The values for $B(E2)$ are in units cm^4e^2. The $B(E2)_{exc}$ to the 790-kev state does not include the transitions resulting in the decay by the $E0$ mode.

E_γ(kev)	$B(E2)_{exc}\times10^{49}$	Transition	$B(E2)_d\times10^{49}$	α_T	$B(E2)_d/B(E2)_{sp}$
50 ± 3	63_{-8}^{+12}	$2\rightarrow0$	12.6	316	149
790 ± 10	2.18 ± 0.25	$2\rightarrow0$	0.44 ± 0.05	1.59×10^{-2}	5.1
740 ± 8		$2\rightarrow2$	0.82 ± 0.19	1.84×10^{-2}	9.7
613 ± 6		$2\rightarrow4$	0.72 ± 0.22	2.72×10^{-2}	8.5

TABLE V. Summary of the transition intensities for the β- and γ-vibrational excitations in Th^{232} deduced from the gamma ray and the internal conversion electron data.

E_p(Mev)	E_γ(kev)	Transitions per μcoul	Excitation energy (kev)	Excitations per μcoul	$B(E2)_{exc}\times10^{49}$ (cm^4e^2)
6.0	790	2.09×10^4	790	6.03×10^4	0.84
	740	3.94×10^4			
	773	2.95×10^4	773	8.97×10^4	1.17
	723	3.62×10^4			
	613	2.40×10^4			
5.5	790	0.93×10^4	790	3.26×10^4	0.80
	740	2.33×10^4			
	773	1.58×10^4	773	4.83×10^4	1.11
	723	1.96×10^4			
	613	1.29×10^4			
5.0	790	$7.1\ \times10^3$	790	1.94×10^4	0.98
	740	1.23×10^4			
	773	8.39×10^3	773	2.55×10^4	1.18
	723	10.33×10^3			
	613	6.83×10^3			
4.5	790	2.97×10^3	790	7.81×10^3	0.94
	740	4.84×10^3			
	773	3.78×10^3	773	1.15×10^4	1.27
	723	4.66×10^3			
	613	3.08×10^3			
4.0	790	0.93×10^3	790	2.36×10^3	0.94
	740	1.43×10^3			
	773	1.28×10^3	773	3.89×10^3	1.38
	723	1.57×10^3			
	613	1.04×10^3			

723, 740, 773, and 790 kev would be quite arbitrary with the resolution available from a scintillation spectrometer. We have instead taken an indirect approach as follows. The position of the 4+ state of the ground-state rotational band in Th^{232} is now known to be at 163 ± 2 kev from the multiple Coulomb excitation data by Stephens *et al.*[17] The 613-kev transition in our gamma-ray spectrum is identified with the $2+\rightarrow4+$ transition between the states at 773 and 163 kev. With the assumption that the relative reduced transition probabilities for the β-vibrational excitation are given correctly by the collective model,

$$B[E2, 2(\beta)\rightarrow2]/B[E2, 2(\beta)\rightarrow0]=1.43$$

and

$$B[E2, 2(\beta)\rightarrow4]/B[E2, 2(\beta)\rightarrow0]=2.57,$$

the intensities of the 773- and 723-kev transitions are deduced from a knowledge of the intensities of the 613-kev transition given in Table I. This decomposition of the transition intensities for the β- and γ-vibrational excitations in Th^{232} at 773 and 790 kev, respectively, is given in Table V. We have neglected any contribution of the $2\rightarrow4$ transition in the decay of the γ-vibrational excitation because this transition is predicted to be weak by the collective model,

$$B[E2, 2(\gamma)\rightarrow4]/B[E2, 2(\gamma)\rightarrow0]=0.07.$$

The low-lying states and transitions observed in the Coulomb excitation of Th^{232} are shown in Fig. 5.

The excess of internal conversion electrons observed from Coulomb excitation of the vibrational states in Th^{232} may be interpreted in terms of an $E0$ mode which competes with the $E2$ decay in the $2+\rightarrow2+$ transition from the β-vibrational excitation at 773 kev. The K-shell internal conversion coefficient quoted above from Bernstein's data, when reinterpreted in terms of the transition intensities in Table V, leads to a value of $(1.7\pm0.8)\times10^{-1}$ for electric-monopole conversion in the K shell. The transition probability $T(E2)$ for the 723 kev $2+\rightarrow2+$ transition is $(8.4\pm2.1)\times10^{10}$ sec^{-1}. The value of the dimensionless strength parameter,[18] ρ, which contains the $E0$ nuclear matrix elements, deduced from these data is $(8\pm2)\times10^{-2}$.

[17] F. S. Stephens, R. M. Diamond, and I. Perlman, Phys. Rev. Letters **3**, 435 (1959).

[18] E. L. Church and J. Weneser, Phys. Rev. **103**, 1035 (1956).

TABLE VI. Summary of quantities obtained from $B(E2)_{exc}$. The values for $B(E2)$ are in units cm^4e^2.

Nucleus	E_γ(kev)	$B(E2)_{exc}\times10^{50}$	Transition	$B(E2)_d\times10^{50}$	$B(E2)_d/B(E2)_{sp}$
Th^{232}	790	9.0 ± 2.3	$2\rightarrow0$	1.8 ± 0.5	2.1
	740		$2\rightarrow2$	4.6 ± 1.6	5.5
	773	12.2 ± 2.4	$2\rightarrow0$	2.4 ± 0.5	2.8
	723		$2\rightarrow2$		
	610		$2\rightarrow4$		
U^{238}	1040	16.4 ± 3.1	$2\rightarrow0$	3.3 ± 0.6	3.7

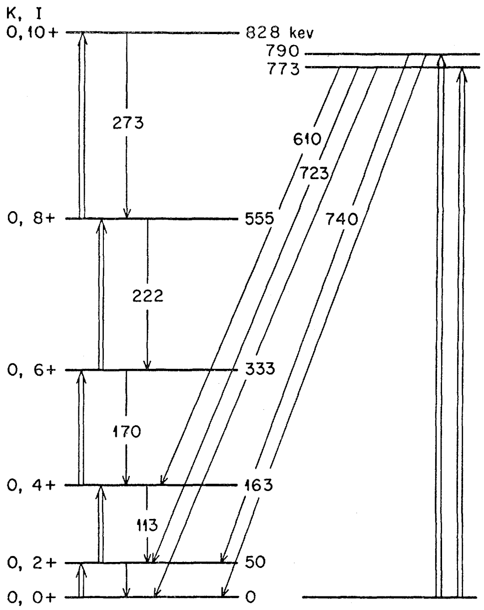

FIG. 5. Energy-level diagram of transitions observed from Coulomb excitation of Th^{232}. The rotational energy-level scheme for the ground-state band is taken from F. S. Stephens, R. M. Diamond, and I. Perlman, Phys. Rev. Letters **3**, 435 (1959).

Reiner has made a calculation of the strength parameter ρ for the case of β-vibrational excitation in strongly deformed nuclei.[7] The monopole matrix element contains a parameter $\hbar/2(B_2C_2)^{\frac{1}{2}}$, the zero-point amplitude in the harmonic approximation. A value for the zero-point amplitude was determined from the $B(E2)_{\text{exc}}$ of the vibrational state in Th^{232} given in Table IV. In addition, the nuclear deformation β_0 was taken to be equal to 0.245. The result was $\rho\approx 5\times 10^{-1}$. This result must be scaled down slightly in view of the analysis of the data from Th^{232} with the β-vibrational excitation at 773 kev. Also, the $B(E2)_{\text{exc}}$ of the 2+ rotational state of the ground band corresponds to a value of $\beta_0=0.209$. The net result is $\rho\approx 3.4\times 10^{-1}$. A direct measurement of the lifetime for the decay of the 2+ state at 50 kev in Th^{232} by Bell *et al.*[19] yields $B(E2)_{\text{exc}} = (8.50\pm 0.37)\times 10^{-48}$ $e^2\text{cm}^4$ which corresponds to $\beta_0=0.242$. The resulting ρ value is then 4.0×10^{-1}. The predicted values are considerably larger than the experimental result. Reiner has, however, pointed out a difficulty in transforming the monopole operator into collective variables. This point could leave room for some disagreement between theory and experiment for collective transitions.

Our $B(E2)_{\text{exc}}$ for the 2+ state at 50 kev in Th^{232} is about 34% smaller than the $B(E2)_{\text{exc}}$ deduced from the measured lifetime[19] for decay of the first rotational state. In the analysis of the gamma-ray yields with α-particle bombardment, we assumed that there was no oxygen in the target. If our target in the Coulomb excitation measurements had been ThO_2, we could account for this discrepancy. This explanation seems rather unlikely because we were using metallic foils and the surface was scraped before each measurement. The error in the $B(E2)_{\text{exc}}$ for the first rotational state does, however, contain a possible error due to the surface condition of the thorium target.

From the knowledge of both the position and the $B(E2)_{\text{exc}}$ for a vibrational state, one can deduce the vibrational parameters B_2 and C_2, where B_2 is the mass transport associated with the vibrational motion and C_2 represents the effective surface tension. For the β-vibrational state we have

$$E_\beta=\hbar(C_\beta/B_\beta)^{\frac{1}{2}} \quad \text{and} \quad B[E2,\, 2(\beta)\to 0] =\frac{1}{5}\left(\frac{3}{4\pi}ZeR^2\right)^2 \frac{\hbar}{2(B_\beta C_\beta)^{\frac{1}{2}}}.$$

We take $E_\beta=0.723$ Mev, $B[E2,\, 2(\beta)\to 0]=2.4\times 10^{-50}$ cm^4e^2, and $R=1.2\times 10^{-13}A^{\frac{1}{3}}$ cm. The values obtained are $C_\beta=420$ Mev and $B_\beta/B_{\text{irrot}}=22$. The value for B_{irrot} is 1.57×10^{-41} Mev sec^2 which is obtained from $B_{\text{irrot}}=(3/8\pi)AMR^2$.

For the γ-vibrational state we use[20]

$$E_\gamma=\hbar(C_\gamma/B_\gamma)^{\frac{1}{2}} \quad \text{and} \quad B[E2,\, 2(\gamma)\to 0] =\frac{(eQ_0)^2}{16\pi}\frac{\hbar}{2(B_\gamma C_\gamma)^{\frac{1}{2}}}.$$

We take $E_\gamma=0.740$ Mev, $Q_0=9.2\times 10^{-24}$ cm^2 and $B[E2,\, 2(\gamma)\to 0]=1.8\times 10^{-50}$ cm^4e^2. The Q_0 is deduced from the measured lifetime for decay of the first rotational state. The values obtained are $C_\gamma=35$ Mev and $\hbar^2/B_\gamma=0.016$ Mev. Tamura and Udagawa[20] have

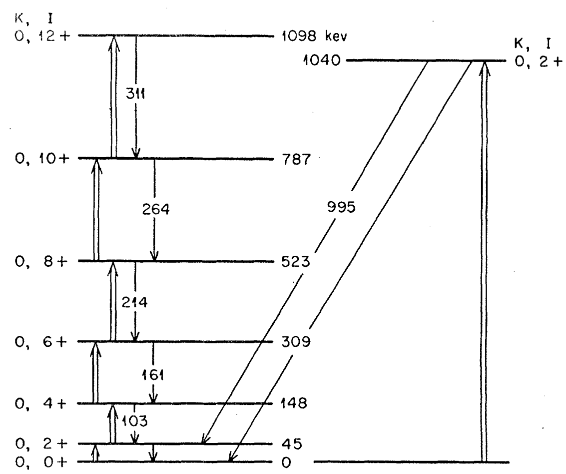

FIG. 6. Energy-level diagram of transitions observed from Coulomb excitation of U^{238}. The rotational energy-level scheme for the ground state band is taken from F. S. Stephens, R. M. Diamond, and I. Perlman, Phys. Rev. Letters **3**, 435 (1959).

[19] R. E. Bell, S. Bjornholm, and J. Severiens, Bull. Am. Phys. Soc. **5**, 338 (1960).

[20] T. Tamura and T. Udagawa, Nuclear Phys. **16**, 460 (1960).

suggested that $\hbar^2/B_\gamma \cong 3\hbar^2/\mathcal{I}$, where $\mathcal{I}$ is the moment of inertia. For Th^{232} we have $3\hbar^2/\mathcal{I}=0.050$ Mev which is 3 times larger than $\hbar^2/B_\gamma$.[21]

We have taken the excitation in U^{238} to be a β vibration because internal conversion electrons have been observed from the $E0$ mode in the $2+\rightarrow 2+$ transition.[22] The $2+\rightarrow 4+$ transition of 892 kev could have escaped detection in our gamma-ray measurements because such a transition would have been obscured by the 842-kev transition from the Al^{27} impurity.[23] The values obtained for the vibrational parameters are $C_\beta=451$ Mev and $B_\beta/B_{\text{irrot}}=11.9$. The low-lying states and transitions observed in the Coulomb excitation of U^{238} are shown in Fig. 6.

The energies of the states in the ground-state rotational band of Th^{232} have been fitted to the expression $E_I=AI(I+1)+BI^2(I+1)^2$, where the second term is a correction attributed to the rotation-vibration interaction.[17] The value obtained for B is -1.2×10^{-2} kev. An estimate of B can also be extracted from the known locations of the β- and γ-vibrational states by the use of the expression $B=-\frac{1}{2}\epsilon^3[3/E_\beta{}^2+1/E_\gamma{}^2]$, where $\epsilon=\hbar^2/\mathcal{I}$. The value obtained is -1.7×10^{-2} kev. This value is about 42% larger than the required value.

According to the strong coupling collective model the branching ratio, $B(E2, 2\rightarrow 2)/B(E2, 2\rightarrow 0)$, for the decay of a 2+ vibrational state to the 2+ and 0+ levels of the ground-state rotational band is 1.43.[5] Observed value for the decay of the γ-vibrational state at 790 kev is 2.6 ± 1.1.

As an alternative description, the 790-kev state in Th^{232} is identified with the second 2+ state of the rotational band of an axially asymmetric nucleus.[24] The energy ratio, $E_2(2'+)/E_1(2+)=15.8$, corresponds to a value of $\gamma=10°$. According to Davydov and Filippov, the ratio of the reduced transition probabilities $B(E2, 2'\rightarrow 0)/B(E2, 2\rightarrow 0)$ for the decay of the second and first 2+ states to the ground state is 2.88×10^{-2}. The experimental value is $(1.4\pm0.4)\times10^{-2}$.

The above discussion of the determination of the values of $B(E2)_{\text{exc}}$ assumed that the observed γ-ray yields resulted solely from Coulomb excitation. The cross section, σ_{comp}, for formation of the compound nucleus from estimates based on barrier penetrabilities[16] is one to two orders of magnitude smaller than the cross section for Coulomb excitation of Th^{232}. Estimates of $[\sigma(p,p')]_{\text{comp}}$ for medium weight nuclei[9] indicated that $[\sigma(p,p')]_{\text{comp}}$ is much less than σ_{comp}. These estimates suggest that compound nucleus inelastic scattering would not introduce appreciable error in the determination of Coulomb excitation cross sections.

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[21] *Note added in proof.* B. L. Birbrair, L. K. Peker, and L. A. Sliv, Soviet Phys.–JETP **9**, 566 (1959) and J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 803 (1959) have obtained an expression for $B[E2, 2(\gamma)\rightarrow 0]$ based on a Hamiltonian for γ vibrations which is not the same as that used by Tamura and Udagawa. The reduced transition probability from the $2+$ γ vibrational state to the ground state is $B(E2, 2(\gamma)\rightarrow 0]=(\frac{3}{4}\pi Z_e R^2)^2(\beta_0{}^2/5)[\hbar/(B_\gamma C_\gamma)^{\frac{1}{2}}]$ and the energy of the first γ vibrational level is $E_\gamma(2+)=\hbar\omega_\gamma+\hbar/\mathcal{I}$. In this case the values obtained are $C_\gamma=72$ Mev and $\hbar^2/B_\gamma=0.0083$ Mev for Th^{232}.

[22] B. Elbek, F. S. Stephens, and R. M. Diamond, Proceedings of Second Conference on Reactions between Complex Nuclei, Gatlinburg, Tennessee, May, 1960 (unpublished).

[23] We have included the contribution from the decay mode by the $2+\rightarrow 4+$ transition to the intensity of the $2+$ β-vibrational excitation on the assumption that the relative reduced transition probabilities for a β-vibrational excitation are given correctly by the collective model.

[24] A. S. Davydov and G. F. Filippov, Nuclear Phys. **8**, 237 (1958).