Diffraction Dissociation of Beam Particles*

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A phenomenon is predicted in which a high-energy particle beam undergoing diffraction scattering from a nucleus will acquire components corresponding to various products of the virtual dissociations of the incident particle, as $p \rightarrow \Lambda + K^+$ or $\pi^- \rightarrow \bar{p} + n$. These diffraction-produced systems would have a characteristic extremely narrow distribution in transverse momentum, and would have all the same quantum numbers as the initial particle; i.e., the same spin, isotopic spin, and parity. The process is related to that discussed in the preceding paper, and has the same effective energy threshold.

THE phenomenon of diffraction scattering from nuclei is well known and well understood. We wish to point out here that a similar phenomenon should exist also, in which the diffracted or "shadow-scattered" wave acquires a component corresponding to dissociation products of the incident particle. The phenomenon is associated only with high energies of the incident particle.[1]

First we must establish that this is energetically possible. Suppose we have an incident particle A (rest mass M, momentum P) and consider the dissociation $A \rightarrow B+C$. Let the energy of $B+C$ in the rest frame of $B+C$ be M^*. We wish to consider a reaction in which the nucleus is left intact, and in its ground state. The nucleus will take up momentum $\mathbf{q}$ and essentially no energy. The requirement of energy and momentum conservation is then, for small transverse momenta,

$$q_{||} = (M^{*2} - M^2)/2p, \tag{1}$$

where $q_{||}$ is the component of $\mathbf{q}$ in the beam direction. $q_{||}$ may be very much less then $m_\pi / A^{\frac{1}{3}}$, thus justifying the assumption that the nucleus can hang together. We have then a threshold

$$P_{\text{th}} = \frac{M^{*2} - M^2}{2m_\pi} A^{\frac{1}{3}}. \tag{2}$$

We note that this is the same as the threshold for electromagnetic production by Coulomb field of the same nucleus.[2] The reaction,

$$A + (\text{nucleus}) \rightarrow B + C + (\text{nucleus in ground state}), \tag{3}$$

is thus energetically possible if the beam energy is high enough.

The next question is whether the reaction actually happens. We do not know how to calculate its rate, in general, as the strong interactions are complicated, and as this is a many-body problem.

What we will do instead is to present a physical argument which shows how such reactions would be brought about, and makes apparent some interesting properties they would have.

First we point out that the sort of phenomenon that we are discussing is really quite familiar in systems where energy degeneracies exist. The best example has to do with optics. First of all, we have the phenomenon of diffraction scattering by an opaque disk. Suppose, however, the disk is a piece of polaroid. The light whose plane of polarization is at right angles to the preferred axis is completely absorbed and the light whose plane of polarization is parallel to the axis is passed without attenuation. If unpolarized light were incident on such a disk, then a diffraction pattern would result in which the scattered light wave had its plane of polarization perpendicular to the axis of the polaroid.

Thus by absorbing a particular component of the wave, a scattering of this component of the wave results.

If the incident light were plane polarized in say the X direction and the polaroid axis was at 45° to the X axis, then the diffraction scattered wave would have in it polarization components both along the X and the Y axes.

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[1] An extensive literature exists on this subject also. The best summary is that of E. L. Feinberg and I. Ia. Pomerancuk, Suppl. Nuovo cimento **III**, 652 (1956), in which it is noted that diffraction dissociation has probably been observed in the case of the deuteron [G. P. Milburn, W. Birnbaum, W. E. Crandall, and D. S. Schechter, Phys. Rev. **95**, 1268 (1954)].

One class of calculations have to do with the diffraction disintegration of light nuclei: (a) R. J. Glauber, Phys. Rev. **99**, 1515 (1955); (b) E. L. Feinberg, J. Exptl. Theoret. Phys. (U.S.S.R.) **29**, 115 (1955) [translation: Soviet Phys.-JETP **2**, 58 (1956)]. (c) A. I. Akhiezer and A. G. Sitenko, Phys. Rev. **106**, 1236 (1957). (d) A. I. Akhiezer and A. G. Sitenko, Doklady Akad. Nauk S.S.S.R. **107**, 385 (1956) [translation: Soviet Phys. Doklady **1**, 180 (1956)]; J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 794 (1957) [translation: Soviet Phys.-JETP **5**, 652 (1957)]. (e) A. G. Sitenko and Ia. A. Berezhnoi, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 1289 (1958) [translation: Soviet Phys.-JETP **8**, 899 (1958)]. (f) J. S. Blair, Nuclear Phys. **6**, 348 (1958). (g) G. P. Milburn, W. Birnbaum, W. E. Crandall, and D. Schechter, Phys. Rev. **95**, 1268 (1954).

Another class of calculation has to do with electromagnetic radiation during diffraction scattering or electromagnetic interaction plus diffraction scattering: (a) J. A. Vdovin, thesis, Moscow, 1955 (unpublished); (b) E. M. Rabinovich, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 1563 (1957) [translation: Soviet Phys.-JETP **5**, 1272 (1957)].

[2] M. L. Good and W. D. Walker, preceding paper [Phys. Rev. **120**, 1855 (1960)].

The polarization component along the Y axis represents a component of the elastically diffracted wave which was not present in the incident wave, and may be regarded as the production of a new state by diffraction.

The various $\theta_1-\theta_2$ regeneration phenomena can also be understood from this point of view. The wave function of the θ_2 is $|\theta_2\rangle=(1/\sqrt{2})(|\theta\rangle-|\bar{\theta}\rangle)$. In this case a nucleus plays the part of the analyzer. The outgoing wave has an amplitude

$$(1/\sqrt{2})(\eta_+|\theta\rangle-\eta_-|\bar{\theta}\rangle).$$

When this is reresolved into $|\theta_1\rangle$ and $|\theta_2\rangle$ components, one finds an outgoing $|\theta_1\rangle$ wave of amplitude $\frac{1}{2}(\eta_+-\eta_-)=\frac{1}{2}(A_+-A_-)$, where A_+ and A_- are the scattering amplitudes in the positive and negative strangeness states. Thus even in the case of a purely absorptive process (η_+, η_- real and less than 1) the $|\theta_1\rangle$ component may be "diffracted into existence." This process has recently been observed.[3] Since the mass difference between $|\theta_1\rangle$ and $|\theta_2\rangle$ is small, the amount of momentum absorbed by the nucleus is very small.

We now consider our nuclear reaction (3). First let us inquire whether the state $B+C$ of mass (or proper energy) M^* may be regarded as degenerate with the incident particle state, A. If the difference in frequency between $B+C$ and A, times the time of passage through the nucleus, is small compared to unity, then they are for all paractical purposes degenerate. The condition for this is

$$(\gamma^*M^*-\gamma M)A^{\frac{1}{3}}/m_\pi \ll 1.$$

If we take $\gamma^*\beta^*M^*=p=\gamma\beta M$, this becomes $p\gg p_{\text{th}}(M^*)$, with P_{th} given by (2).

Thus for $p\gg P_{\text{th}}(M^*)$, the state of mass M^* may be regarded as degenerate with the state of mass M, and we may expect phenomena involving M^* to exist similar to those we have just discussed for polarized light and for the θ_1–θ_2 system.

Let us now consider the incident particle to be a nucleon, for definiteness. It is a "dressed" or real nucleon, $|\tilde{N}\rangle$, in contradistinction to the "bare" nucleon, $|N\rangle$. Now we may expand any state in terms of any complete set of states. For example, we could expand $|\tilde{N}\rangle$ in terms of the states of "bare" particles:

$$|\tilde{N}\rangle=\sum_i a_{Ni}|B_i\rangle$$

$|B_i\rangle=|N\rangle$, $|N\pi\rangle$, $|N2\pi\rangle$, $\cdots$, $|\Lambda K\rangle$, $\cdots$, where the $|B_i\rangle$ are all the one, two, or more particle states (of "bare" nucleons and "bare" pions) with the same quantum numbers as the nucleon, i.e., the same charge, strangeness$=0$, intrinsic angular momentum$=J=\frac{1}{2}$, etc.[4]

There is another complete set also, composed of all the "dressed" particles. We can for instance, expand any of the B_i in terms of these:

$$|B_i\rangle=\sum_j a_{ij}^*|D_j\rangle, \tag{4}$$

where the $|D_j\rangle$ are all "dressed" states of the same quantum numbers as $|B_i\rangle$ and therefore as $|\tilde{N}\rangle$. The "dressed" set is a somewhat preferred one, as these are the eigenstates of the free particle Hamiltonian.

We now consider the case where $p\gg P_{\text{th}}(M^*)$ for the final state M^* of interest. We are then justified in neglecting mass differences in discussing the behavior of the nucleon wave as it penetrates the nucleus. We may therefore expand the state $|\tilde{N}\rangle$ into an appropriate complete set, and expect that the different terms in the expansion will be attenuated separately in passing through the nucleus. The set we want is clearly neither the bare-particle set $|B_i\rangle$ nor the dressed-particle set $|D_j\rangle$ but some third set, comprised of just those linear combinations of bare particle states which are the eigenstates inside nuclear matter. Call this set $|C_i\rangle$. The $|C_i\rangle$ have the property that each is attenuated with a simple exponential dependence in traversing the nucleus.

The formulation of the problem is now simple. The incident wave is

$$|I\rangle=e^{ikz}|\tilde{N}\rangle=e^{ikz}\sum_i c_{Ni}|C_i\rangle.$$

After traversing the nucleus, the transmitted wave is[5]

$$|T\rangle=\sum c_{Ni}\eta_i|C_i\rangle,$$

where

$$|\eta_i|\leqslant|.$$

The scattered wave is the difference between $|I\rangle$ and $|T\rangle$:

$$|S\rangle=|I\rangle-|T\rangle.$$

[This is evaluated "just behind the nucleus" ($z=0$) and hence shows no coordinate dependence.]

The wave $|S\rangle$ then propagates out with a diffraction angular distribution. But $|S\rangle$ is now in general no longer a pure nucleon state; rather the projections $\langle D_j|S\rangle$ represent the amplitude in $|S\rangle$ of the various two-or-more-particle states $|D_j\rangle$ of real particles of the same quantum numbers as $|\tilde{N}\rangle$. $|S\rangle$ may be written as

$$\begin{aligned}|S\rangle&=\sum_i c_{Ni}(1-\eta_i)|C_i\rangle\\&=(1-\bar{\eta})|\tilde{N}\rangle+\sum_i(\bar{\eta}-\eta_i)c_{Ni}|C_i\rangle,\end{aligned}$$

where the first term is the scattered nucleon wave and the second represents the diffraction produced particles, i.e., not states involving only single nucleons. The amplitude for the diffraction produced particles is

[3] F. Muller *et al.*, Phys. Rev. Letters 4, 418 (1960).

[4] The summation over i includes an integration over continuous variables, where called for. For instance, the bare states $|N\pi\rangle$ may be described by their (unperturbed) c.m. energy M_0^*. The summation includes a term $\int dM^*\rho(M_0^*)a(M_0^*)|N\pi(M_0^*)\rangle$, where ρ is an appropriate density of states. Also presumably the values of the coefficients depend on the momentum. The means of writing such an expansion in a covariant fashion is unknown to the authors.

[5] There are of course many events in which scatterings or nuclear disruptions occur. These events are not the ones we are talking about. Each of the $c_{Ni}\eta_i|C_i\rangle$ wavelets is the *transmitted* wave, as measured in those events in which no nuclear excitation takes place.

proportional to the *difference* between the absorption, $\bar{\eta}$, for the nucleon and some other state.

Only if the $|C_i\rangle$ are all attenuated in the same amount, $\bar{\eta}$ is the scattered wave $|S\rangle$ a pure $|\tilde{N}\rangle$ wave. In other words, the diffraction-scattered wave has in it dissociation products of the incident particle.

This way of looking at it makes it seem likely that reactions of this general type are not rare. If, however, $\eta_i=0$ for all i (complete absorption), then $|T\rangle=0$, and $|S\rangle=|I\rangle$, so that the scattered wave is entirely composed of real nucleons. The elastic cross section is then πR^2, the familiar case of the black sphere. Our effect is therefore one of semitransparent nuclei, and so light nuclei would seem to give the highest yield.

The sort of argument made for the diffraction reactions would seem to apply roughly up to M^* such that $(M^{*2}-M^2)/2p=m_\pi/A^{\frac{1}{3}}$, above which the "regenerated" components $|D_j\rangle$ would oscillate with respect to the incident wave on the way through the nucleus. The increments to $|D_j\rangle$ will have the phase of the incident wave, and so above this limiting value of M^*, successive increments to $|D_j\rangle$ would be out of phase with each other, and no appreciable $|D_j\rangle$ amplitude would develop.

We now discuss other properties of the reaction. The above treatment makes the following points obvious:

(1) The outgoing wave $|S\rangle$ and its real-particle projections $\langle D_j|S\rangle$ have the angular dependence characteristic of diffraction scattering, since they are produced by differential absorption of the incident beam. What is meant by this is the following: one measures the momentum of each outgoing particle, and constructs, for each event, the vector sum of these momenta. The distribution in angle of this vector with respect to the incident beam should be that of a diffraction scattering. Since at high energy the diffraction pattern is very narrow, this represents a distinctive feature of the reaction, and could be used to identify it.

(2) The outgoing wave of other than incident particles will consist of two- (or more)-body systems having the same quantum numbers as the incident particle, i.e., the same charge, strangeness, nucleon number, isotopic spin, intrinsic angular momentum, and parity. Thus if $N\rightarrow N+\pi$ is observed in this way, the outgoing $N+\pi$ system will be in a $T=\frac{1}{2}$, $J=\frac{1}{2}$ state, of $+$ parity, i.e., a $P_{\frac{1}{2}}$ state. The implications of these rules are numerous. To give some examples, $\pi^+\rightarrow K^++\bar{K}^0$ if observed in this way, would establish different parity for the K^+ and K^0. With a polarized proton beam, $p\rightarrow\Lambda+K^+$ would yield Λ's of polarization ($\pm$) the proton polarization for ($\pm$) parity of the K^+ (for those events in which the plane containing the Λ and K^+ lies perpendicular to the proton polarization).

We should add a word of caution concerning possible confusion arising in the interpretation of experiments. Consider the process mentioned above, $\pi^+\rightarrow K^++\bar{K}^0$. The virtual process $\pi^+\rightarrow K^++\bar{K}^0+\pi^0$ would occur if K^+ and $\bar{K}^0$ have the same parity. One might imagine a Chew-Low or Barshay type process in which the virtual π^0 is swallowed by the nucleus, with a small momentum transfer. This however would change the parity of the nucleus and not leave it in its ground state. To be able to discriminate against such a possibility, one should choose a nucleus in the experiment with no low-lying opposite-parity states (He^4,C^{12}). The possibility of leaving the parent nucleus in an excited state must always be reckoned with. If we look at the data on p-nucleus scattering of Gerstein, Niederer, and Strauch,[6] we estimate that the contamination of such processes might be 5–10% of the elastic diffraction process. The experimental resolution is very important in such considerations.

Similarly, it does not violate any selection rules to have the orbital angular momentum of the beam particle relative to the nucleus change by one unit, the nucleus remain in its ground state, and the parity and spin of the state of mass M^* differ from that of the incident particle. We feel that such processes would be rare, for the following reason:

The orbital angular momentum change may be written as

$$\Delta l\leqslant q_{||}R,$$

$$q_{||}R\approx\left(\frac{M^{*2}-M^2}{2p}\right)\frac{A^{\frac{1}{3}}}{m_\pi}=\frac{p_{\text{th}}(M^*)}{p}.$$

If

$$p\gg p_{\text{th}}(M^*),$$

then

$$\Delta l\ll 1.$$

If the pertinent R is the radius of a nucleon, Δl is even smaller.

The amplitude for particles created in this way would be coherent with, and could interfere with, the amplitude for the same states created by the intercession of the Coulomb field.[2] The Coulomb effect would be small in the light nuclei which are useful for the diffraction process, however.

In order to make an estimate of the cross section for such a process we have assumed a specific model for the meson. The basic thing used is that the amplitude for the conversion process is always proportional to the difference between the scattering amplitudes in the "bare" state and the "bare" state of mass M^*. One can then construct imaginary potentials to describe the shadow scattering for "bare" states of mass M and M^*. The parameters appearing in the potential will be the opacity of the nucleus, the size of the nucleus, the momentum and the momentum transfer. The particular form used in our calculations was $V_i=B_i/(q^2+q_0^2)$, where $q_0=m_\pi/A^{\frac{1}{3}}$, $\mathbf{q}=\mathbf{P}_1-\mathbf{P}_2$.

[6] G. Gerstein, J. Niederer, and K. Strauch, Phys. Rev. **108**, 427 (1957).

The conversion process then goes on in this imaginary potential $(V_M - V_{M^*})$. Again the conversion comes as a result of a small longitudinal kick in the pseudopotential field. We assume that π mesons are coupled to bosons of mass M^* which represents a 3-meson intermediate state through a term in the Hamiltonian of the form,

$$\int \lambda^2(M^*)\phi_{M^*}{}^*\phi_\pi dM^*,$$

$$\phi_{M^*}{}^* = \phi_\pi{}^3 \quad (\text{i.e., } M^* \rightarrow 3\pi).$$

Using this interaction, we compute the following cross section for π+nucleus$\rightarrow M^*$+nucleus.

$$\frac{d\sigma}{dM^*} = \frac{\lambda^4(p^2+M^{*2})^{\frac{1}{2}}}{4(M^{*2}-M^2)^2}\left|1-\left(\frac{\sigma_{M^*}}{\sigma_\pi}\right)^{\frac{1}{2}}\right|^2 \frac{q_0{}^2\rho(M^*)\sigma_\pi}{(q_{||}{}^2+q_0{}^2)(2\pi)^9},$$

where σ_π = the cross section for diffraction scattering of a "bare" π meson by the nucleus (assuming $\sigma_\pi > \sigma_{M^*}$), σ_{M^*} = the cross section for diffraction scattering of the "bare" state of mass M^*, $q_{||} = (M^{*2}-M^2)/2p$, and $\rho(M^*)$ = covariant density of states in the M^* center-of-mass system (i.e., between M^* and M^*+dM^*).

In order to get an estimate of a cross section, one must make an assumption at this point. We assume that the cross section for "bare" and "dressed" states are approximately the same. One sees that the process is fairly likely until $q_{||} \geq q_0$. Again this means that the process is likely only as long as the intermediate state of mass M^* can live a distance the order of the radius of the nucleus, $A^{\frac{1}{3}}/m_\pi$. When $q_{||} \gg m_\pi/A^{\frac{1}{3}}$, then so much momentum must be transferred that the collision point is localized well inside the nucleus and consequently will very likely disrupt the nucleus.

Note added in proof. R. F. Sawyer has pointed out an exception to our argument concerning quantum numbers. Nuclei are not in eigenstates of G conjugation, since charge conjugation (which produces anti-nuclei) is a part of the G operation. This means that the G quantum number of the beam particle need not be conserved in a diffraction production process.

In a similar way, in the diffraction production of θ_1's from a beam of θ_2's, the PC quantum number of the beam particle does change (from -1 to $+1$), as a consequence of the fact that the nucleus is not in an eigenstate of PC.

Then the diffraction production $\pi \rightarrow n\pi$ is allowed regardless of whether n is even or odd (i.e., regardless of G conjugation), with the single exception that $\pi \rightarrow 2\pi$ is forbidden by angular momentum and parity considerations.

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We feel that these ideas are worthy of more detailed theoretical treatment, as well as search for experimental verification. We again acknowledge our indebtedness to the many Russian authors who started these considerations some years ago. We acknowledge numerous conversations with our colleagues here at Wisconsin, in particular Dr. Fry, Dr. Ebel, Dr. Holladay, Dr. Lewis, Dr. Sachs, Dr. Sakita, and Dr. Sawyer.

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Particle Creation in Electron-Electron Collisions

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Pair production in high-energy electron-electron collisions is studied with special attention given to pion pair production. A method of calculation is formulated which yields results with reasonable directness in the relativistic limit. The orders of magnitude of counting rates for various experimental settings are ascertained. A complete result is obtained for the case in which two pions emerge with equal energies and opposite momenta.

I. INTRODUCTION

EXPERIMENTS in which oppositely directed beams of electrons clash and interact over long periods of time are now in preparation.[1] These beams will permit measurements of electron-electron (Møller) scattering at center-of-mass energies of 500 Mev or more. Cross sections for pion and muon pair production are of the order of $(\alpha/\pi)^2$ relative to the Møller cross section, though they may be greatly enhanced in certain cases. Such processes furnish the opportunity—albeit, a re-

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[1] G. K. O'Neill and E. J. Woods, Phys. Rev. **115**, 659 (1959); Barber, Richter, Panofsky, O'Neill, and Gittelman, High-Energy Physics Laboratory, Stanford University Report, June, 1959 (unpublished). W. K. Panofsky, Fourth Annual International Conference on High-Energy Nuclear Physics, 1959 (unpublished). Similar projects have been undertaken at MURA (Midwestern Universities Research Association).