Cross Section and Polarization in the Photodisintegration of the Deuteron*

M. L. RUSTGI, W. ZERNIK,†‡ G. BREIT, AND D. J. ANDREWS
Yale University, New Haven, Connecticut

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The differential cross section and polarization of nucleons from the $H^2(\gamma,n)H^1$ reaction is investigated arranging calculations by means of the amplitude method. Electric dipole, electric quadrupole, and magnetic dipole transitions are considered taking into account coupling between states with the same J but different L such as produced by the tensor potential. The calculations include the general case of elliptically polarized gamma rays. Numerical results are obtained for two modified versions of the Signell-Marshak two-nucleon potential. Five approximations have been used to exhibit the effects of the various multipole transitions. The calculations agree with experiment rather well even at energies for which the potential used does not represent scattering data at all perfectly. The significance of polarization measurements is discussed.

I. INTRODUCTION

THE total cross section and angular distribution for the photodisintegration of the deuteron have been known for some time to show reasonably satisfactory agreement with theoretical calculation up to gamma-ray energies of about 10 Mev.[1] Until recently theoretical work[2] on the differential cross section for gamma-ray energies between 20 and 150 Mev failed to account for the observed angular distribution,[3] particularly the large isotropic component. More recently, however, it has become evident both as a result of the work reported here[4] and that carried out by de Swart and Marshak[5] and Nicholson and Brown[6] that a careful calculation on the basis of semiphenomenological two-nucleon potentials[7,8] can give results in reasonable agreement with experiment at least for the lower energies in this range. The first calculation on the polarization of the outgoing nucleon in the photodisintegration of the deuteron was published by Rosentsveig[9] using the zero-range approximation. Approximate formulas for the polarization have been later on derived by Czyż and Sawicki[10] and Kawaguchi[11] in terms of phase shifts and matrix elements. Numerical results on the basis of the Signell-Marshak potential,[7] using $E1-E1$, $E1-E2$, $E1-M1$ spin-flip, and $E2-M1$ spin-flip interference terms taking the tensor coupling of the final states exactly into account have been computed by de Swart, Czyż, and Sawicki.[12]

An improved theoretical treatment of the $H^2(\gamma,n)H^1$ reaction in the medium energy range is of interest primarily for two categories of reasons. The first category is concerned with properties of the wave function of the p-n system.

The wavelength of relative motion of the outgoing particles is comparable to the range of nuclear forces, e.g., for 20-Mev gamma rays the wavelength divided by 2π is $\lambda\cong 1.55\times 10^{-13}$ cm and for 175-Mev gamma rays, $\lambda\cong 0.49\times 10^{-13}$ cm. As a result, the matrix elements for the reaction depend on the shape of the assumed nuclear potential rather than just on the depth and range as is the case at lower energies. The short wavelength also leads to considerable cancellation in the matrix elements for transitions to low angular momentum states so that the relative importance of higher angular momenta is enhanced. The effect of this is that the tensor part of the potential plays an important

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‡ Now at Argonne National Laboratory, Lemont, Illinois.

[1] H. A. Bethe and R. E. Peierls, Proc. Roy. Soc. (London) **A148**, 146 (1935); G. Breit and E. U. Condon, Phys. Rev. **49**, 904 (1936); G. Breit, E. U. Condon, and J. R. Stehn, Phys. Rev. **51**, 56 (1937). Final-state interaction is taken into account in the last two papers. H. A. Bethe and C. Longmire, Phys. Rev. **77**, 647 (1950); E. E. Salpeter, Phys. Rev. **82**, 60 (1951), and private communication; J. M. Blatt and V. F. Weisskopf, *Theoretial Nuclear Physics* (John Wiley & Sons, Inc., New York, 1956), Chap. XII and Appendix B. N. Austern and E. Rost, [Phys. Rev. **117**, 1506 (1960)] find disagreement between theory and experiment for n-p capture cross section at thermal energies.

[2] W. S. Rarita and J. Schwinger, Phys. Rev. **59**, 556 (1941); L. I. Schiff, Phys. Rev. **78**, 733 (1950); J. F. Marshall and E. Guth, Phys. Rev. **78**, 738 (1950); J. Bernstein, Phys. Rev. **106**, 791 (1957); N. Austern, Phys. Rev. **108**, 973 (1957).

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[5] J. J. de Swart and R. E. Marshak, Phys. Rev. **111**, 272 (1958); Physica **25**, 1001 (1959).

[6] A. F. Nicholson and G. E. Brown, Proc. Phys. Soc. (London) **73**, 221 (1959). These authors employed the Gammel-Thaler potential instead of the Signell-Marshak potential used in most of the other work referred to here.

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[8] C. R. Fischer, K. D. Pyratt, Jr., M. H. Hull, Jr., and G. Breit, Bull. Am. Phys. Soc. **3**, 183 (1958). See also Phys. Rev. Letters **2**, 264 (1959).

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[11] M. Kawaguchi, Phys. Rev. **111**, 1314 (1958).

[12] J. J. de Swart, W. Czyż, and J. Sawicki, Phys. Rev. Letters **2**, 51 (1959).

role not only through the admixture of $^3D_1$ to $^3S_1$ in the ground state but also through that of $^3F_2$ to $^3P_2$ in the continuum states. The spin-orbit potential has significant effects as well especially in the triplet-odd states. The enhancement of matrix elements for transitions to high angular momentum states leads to the $E2$ and $M1$ transitions to such states producing non-negligible effects. It was, therefore, necessary to include in the calculation all $E1$, $M1$, and $E2$ transitions that are possible for the static electromagnetic interaction. In order to simplify the calculations, effects of exchange currents and of retardation effects have not been included.

The second category of reasons for the calculation is its bearing on the interaction of nucleons with the electromagnetic field. The cross-section data below 10 Mev seem to indicate that the static $E1$ interaction works quite well in this region, but they give virtually no information about the magnetic dipole interaction except at energies just above threshold. In the medium energy range the situation is considerably more complicated and one of the purposes of this work is to extablish the degree to which different multipole transitions are responsible for the observable effects.

In addition to retardation which is believed to be unimportant as a result of the work of Nicholson and Brown[6] the present report does not include the consideration of exchange current effects[13] or of the effects of higher radiation field multipoles. The potential used is also by now not the best one from the viewpoint of scattering data. The calculations of the cross section have, therefore, been arranged in a form which includes electromagnetic multipoles of arbitrarily high order in such a way as to make digital machine calculations practical. After the present work has been completed, a prepublication copy of an interesting paper by Kramer and Werntz[14] has been received. These authors find that the inclusion of $M2$ transitions produces a non-negligible increase in the isotropic term at 77 Mev. By itself such a change if applied to the calculations reported on here would not improve agreement with experimental data on angular distributions. Since the calculations described below are the most complete up to this time for $E1$, $E2$, and $M1$ transitions and since the writing of the digital machine program may take some time, it was felt that the publication of the results is advisable at this time, especially because it includes a detailed consideration of the changes produced by the inclusion of different transitions some of which have not been considered elsewhere. These are $M1(^3D_1 \rightarrow {}^3D_J)$ $(J=1, 2)$, $M1(^3S_1 \rightarrow {}^3S_1)$, and an explicit listing of all $E2$ transitions. It is also possible that the electromagnetic structure of the nucleons may produce effects comparable with those arising from the consideration of higher multipoles and that, therefore, a complete calculation of the effects of lower multipoles is likely to be useful as a point of departure for future work. Some experimental data additional to those previously compared with calculation and some revisions in older data have been included below. Formulas as well as numbers for neutron polarization are also given.

In Sec. II, the formalism used to calculate the amplitudes for the photodisintegration of the deuteron by plane-polarized gamma rays is developed. The method is similar to that used by Breit and Hull[15] in the treatment of nucleon-nucleon scattering. These amplitudes can then be combined numerically to obtain the polarization and differential cross section.

In Sec. III the effect of coupling of states of the same J but different L on the amplitudes is investigated. The required modification to Sec. II is first developed following the procedure of Breit, Ehrman, and Hull.[16] Then it is shown how the S matrix defined in Sec. II may be rather easily modified to take account of the coupling.

In Sec. IV a treatment for elliptically polarized gamma rays is given. In Sec. V, amplitudes for all $E1$, $M1$, and $E2$ transitions are written out for the outgoing protons taking the tensor coupling of the final states into account and the changes that have to be made to obtain the amplitudes for outgoing neutrons are described.

Numerical calculations were carried out with two modified versions of the "Signell-Marshak" two-nucleon potential. These modifications are due to the work of Fischer, Pyatt, Hull, and Breit.[8] The potential parameters are given in Sec. VI.

Numerical values of the differential cross section and polarization are given in Sec. VII for the two potentials using five approximations to determine the relative importance of the various transitions. In approximation A only the $E1$ transitions are considered and the coupling of the $^3P_2$ to $^3F_2$ is neglected. In approximation B only $E1$ transitions are used with full account of the tensor coupling and similarly in the succeeding approximations this coupling is fully considered. In approxima-

[13] L. D. Pearlstein and A. Klein, Phys. Rev. **118**, 193 (1960). These authors using meson theoretic techniques in certain conventions find that virtual meson effects play little role at gamma-ray energies below 100 Mev in justification of calculations carried out in this paper. They find some magnetic resonant terms which are expected to cause modifications at gamma-ray energy of the order of 250 Mev and report good agreement with experiment for the total cross section.

[14] The writers are very grateful to Dr. G. Kramer and Dr. C. Werntz for a preprint of their paper which is to be published in the Physical Review and which establishes the relative smallness of the retardation effect in a more detailed manaer than by Nicholson and Brown. Similar effects have also been considered by J. G. Brennan and R. G. Sachs (Phys. Rev. **88**, 824 (1952)]. The work of Kramer and Werntz does not include the calculation of polarization. References to earlier work by Kramer and collaborators are given in their paper. The conclusions of Nicholson and Brown and Kramer and Werntz about the smallness of the retardation effects are in contradiction with those of M. Matsumoto, Progr. Theoret. Phys. (Kyoto) **23**, 597 (1960) who finds these effects to be important at energies above $E_\gamma = 80$ Mev.

[15] G. Breit and M. H. Hull, Jr., Phys. Rev. **97**, 1047 (1955).

[16] G. Breit, J. B. Ehrman, and M. H. Hull, Jr., Phys. Rev. **97**, 1051 (1955).

tion C the effect of $M1$ transitions to singlet states is taken into account in addition while in approximation D the effect of $M1$ transitions to triplets is used as well. In approximation E there is further included the effect of $E2$ transitions to S, D, and G states. The results are discussed in Sec. VIII.

In an Appendix a brief outline is given of the transformation from specified neutron kinetic energies in the laboratory scattering system to equivalent gamma-ray energies in the laboratory photodisintegration system, this transformation having been employed incorrectly in some data reductions.

The following notation will be used throughout the work.

$\mathbf{r}_p$, $\mathbf{r}_n$ = proton and neutron coordinates in the center-of-momentum system.

$\mathbf{r}=\mathbf{r}_p-\mathbf{r}_n$ = coordinate vector of relative motion.

m_π = pion rest mass.

M = nucleon rest mass.

ψ_i, ψ_f = initial and final state wave functions for the p-n system.

μ_p, μ_n = magnetic moments of proton and neutron in nuclear magnetons.

$\boldsymbol{\sigma}_p$, $\boldsymbol{\sigma}_n$ = Pauli spin operators for proton and neutron in the usual representation.

$\boldsymbol{\kappa}$, ω = photon wave-vector and angular frequency; $\kappa/2\pi$ is the photon wave number.

$\mathcal{E}(\mathbf{r})$, $\mathcal{H}(\mathbf{r})$ = electric and magnetic field strengths.

$\mathfrak{A}$ = quantity introduced in Eqs. (4) and (4.1). In discussion following Eq. (15.4) the coefficient of $\mathfrak{A}$ in H' is $-e\mathcal{E}$ rather than $-e\mathcal{E}_x$.

$\mathbf{1}_E$ = unit vector in direction of $\mathcal{E}(\mathbf{r})$.

$\hbar\mathbf{J}$ = total angular momentum operator.

v = relative velocity of neutron and proton after the disintegration of the deuteron.

$\mathbf{k}=M\mathbf{v}/(2\hbar)$ = proton wave vector; $k/2\pi$ is the proton wave number; the same symbols represent nonrelativistically corresponding quantities for relative motion.

χ_m, $\tilde{\chi}_0$ = triplet and singlet spin functions in the primed system, with the sign defined by $\tilde{\chi}_0=(\alpha_p\beta_n-\alpha_n\beta_p)/\sqrt{2}$, where α, β are the usual Pauli spin functions.

$\hbar J$, $\hbar L$ = total and orbital angular momentum of final state, respectively.

$\hbar m$ = projection of $\hbar J$ along the z' axis.

$F_L(kr)$, $G_L(kr)$ = the regular and irregular solutions of the differential equation for $r\times$radial function for n-p system outside the region of interaction, normalized so as to be asymptotic at $r=\infty$ to $\sin(kr-\frac{1}{2}L\pi)$ and $\cos(kr-\frac{1}{2}L\pi)$, respectively.

$\mathcal{Y}_m{}^{LJ}=\sum_\mu \begin{pmatrix} L & J \\ m-\mu & m \end{pmatrix} Y_{L,m-\mu}\chi_\mu$ spin-angle function for triplet states.

$$Y_{L,m}=\frac{(-)^m}{2^L L!}\left[\frac{(2L+1)(L-m)!}{4\pi(L+m)!}\right]^{\frac{1}{2}} e^{im\varphi}\sin^m\theta\left(\frac{d}{d\cos\theta}\right)^{L+m}\times(\cos^2\theta-1)^L.$$

$\begin{pmatrix} L & J \\ m-\mu & m \end{pmatrix}$ = vector-addition coefficient for triplet states as defined by Breit and Hull.[15]

$\delta^L{}_J$ = phase shift for the triplet state described by quantum numbers L, J.

K_L = phase shift for the singlet state of angular momentum L.

$\mathfrak{F}^L{}_J(kr)\equiv F_L(kr)\cos\delta^L{}_J+G_L(kr)\sin\delta^L{}_J$.

(η_1,η_2,η_3) = combination of ground-state spin-angle functions that transform under rotations like (x,y,z).

a_i = weighting factors for the η_i.

S_{mi} = amplitudes for a triplet final state, i.e., if the initial state is $\sum_i a_i\eta_i$ the final state is proportional to $\sum_{m,i}\chi_m S_{mi}a_i$.

$\mathcal{O}_{ij}$ = transformation matrix from the primed to the unprimed coordinate system, $\mathbf{r}=\mathcal{O}\mathbf{r}'$.

s_{0i} = amplitudes for a singlet final state.

${}^1\mathfrak{F}_L(kr)\equiv F_L(kr)\cos K_L+G_L(kr)\sin K_L$.

$(S_{0i}{}^\xi, S_{1i}{}^\xi, S_{2i}{}^\xi, S_{3i}{}^\xi)$ as defined by Eqs. (9.1)–(9.4).

ϵ = deuteron binding energy taken as -2.224 Mev.

U = coupling matrix as defined by Eq. (11.3).

$u_J{}^\alpha$; $v_J{}^\alpha$; $u_J{}^\beta$, $v_J{}^\beta$ as defined by Eq. (12.1).

$\mathfrak{R}_J$ defined by Eq. (14).

$\sigma(\theta,\varphi)$ = differential cross section.

$\mathbf{P}'(\theta,\varphi)$ = polarization of the protons referred to the primed coordinate system defined in Fig. 1.

E_γ = gamma-ray energy in the laboratory system, i.e., the system in which the deuteron is initially ar rest.

σ_T = total cross section.

II. GENERAL PROCEDURE FOR OBTAINING THE AMPLITUDES

The coordinate systems shown in Fig. 1 will be used. The plane-polarized γ ray is taken as incident along the positive z axis of a Cartesian coordinate system with electric vector along the x axis. The direction of

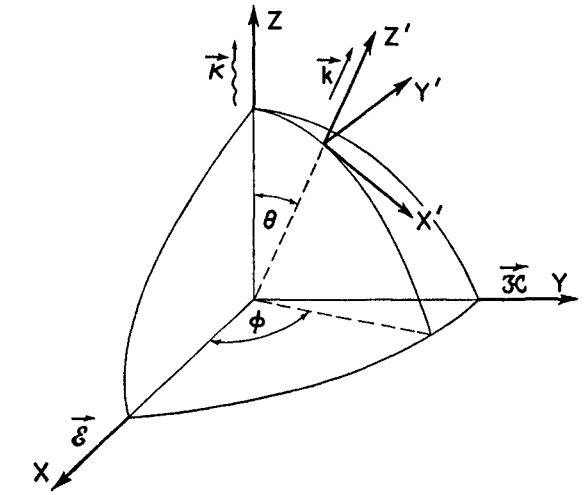

FIG. 1. Coordinate systems used. The plane polarized γ ray is incident along the positive z axis with electric vector along the x axis. The direction of the outgoing proton defines the z' axis.

the outgoing proton defines the z' axis of a second coordinate system with θ and φ being the colatitude and azimuthal angles of z' with φ referred to x. The x' and y' axes have direction cosines $(\cos\theta\cos\varphi, \cos\theta\sin\varphi, -\sin\theta)$ and $(-\sin\varphi, \cos\varphi, 0)$, respectively. All directions are in the zero-total-momentum system. Strictly speaking one should take into account the motion of the deuteron as a whole before dissociation. This is done (Appendix A) only in connection with a correction for the correlation of gamma-ray energies in the system of the deuteron and the neutron energy of scattering experiments which matters for phase shift determination. Corrections to matrix elements resulting from deuteron motion in the zero-momentum system are assumed to be negligible in the work described below. Relative amplitudes for different directions with assigned states of polarization will now be considered. The coupling of states in the continuum having the same J but different L will at first be neglected and the modification caused by the coupling will be discussed in a later section.

For triplet states, an undistorted plane wave traveling along the z' axis may be expanded as

$$e^{ikz'}\chi_m=\chi_m\sum_L i^L(2L+1)(kr)^{-1}F_L(kr)\times[4\pi/(2L+1)]^{\frac{1}{2}}Y_{L,0}(\theta',\varphi')$$
$$=(4\pi)^{\frac{1}{2}}\sum_{L,J} i^L(2L+1)^{\frac{1}{2}}(kr)^{-1}F_L(kr)\times\begin{pmatrix}L & J\\ 0 & m\end{pmatrix}\mathcal{Y}_m{}^{LJ}(\theta',\varphi',s'), \quad (1)$$

where (θ',φ',s') represent colatitude, azimuth, and spin coordinates referred to the primed coordinate system. In the presence of a nuclear potential, the radial function is distorted near the origin. Outside the range of the potential, however, the radial function is still a solution of the force-free Schrödinger equation and is, therefore, expressible as a sum of the regular and irregular functions. Furthermore, it is well known that in order to be able to identify the amplitudes with the matrix elements, the asymptotic solution must have the form of an undistorted plane wave plus an ingoing wave modification.[17] The final-state wave function formed in this way will be denoted by $\psi^{(-)}$. One has, therefore, to modify Eq. (1) by the replacement

$$F_L(kr)\rightarrow\exp(-i\delta^L{}_J)\mathfrak{F}^L{}_J(kr). \quad (1.1)$$

It will be convenient to describe the initial state in terms of combinations of spin-angle functions that transform like a vector,[15] e.g., for the ${}^3S_1$ part,

$$\eta_1=(2)^{-\frac{1}{2}}(\chi_{-1}-\chi_1)Y_{00}, \quad (2.1)$$
$$\eta_2=i(2)^{-\frac{1}{2}}(\chi_{-1}+\chi_1)Y_{00}, \quad (2.2)$$
$$\eta_3=\chi_0 Y_{00}, \quad (2.3)$$

and similarly for the ${}^3D_1$ part. The quantities (η_1,η_2,η_3) transform under rotations like (x,y,z). Using the above equations the matrix elements from an initial state ψ_i to a final state $\psi_f{}^{(-)}$ may be expressed as

$$(\psi_f{}^{(-)})|H'|\psi_i)=(4\pi)^{\frac{1}{2}}\sum_{L,J}(-i)^L(2L+1)^{\frac{1}{2}}\begin{pmatrix}L & J\\ 0 & m\end{pmatrix}\exp(i\delta^L{}_J)\times(\mathcal{Y}_m{}^{LJ}(\theta',\varphi',s')(kr)^{-1}\mathfrak{F}^L{}_J(kr)|H'|\mathfrak{g}_i). \quad (3)$$

Here $\mathfrak{g}_i$ are the ground-state wave functions that transform like η_i. If the ground state is approximated by the ${}^3S_1$ function the $\mathfrak{g}_i$ are of the form $u(r)\eta_i/r$. The collision is thus described by saying that if the initial state is $\sum_i a_i\mathfrak{g}_i$, the final state is proportional to

$$-e\mathcal{E}_x\textstyle\sum_{m,i}\chi_m S_{mi}a_i, \quad (3.1)$$

where χ_m is the spin function in the primed coordinate system, and the quantity S_{mi} is given by

$$S_{mi}=\sum_{L,J}(-i)^L(2L+1)^{\frac{1}{2}}\begin{pmatrix}L & J\\ 0 & m\end{pmatrix}\exp(i\delta^L{}_J)\times(\mathcal{Y}_m{}^{LJ}(')r^{-1}\mathfrak{F}^L{}_J(kr)|\mathfrak{A}|\mathfrak{g}_i). \quad (3.2)$$

In (3.2) $\mathcal{Y}_m{}^{LJ}$ is expressed in the primed coordinate system while $\mathfrak{A}$ and $\mathfrak{g}_i$ are expressed in the unprimed system. However, the transformation of the latter two quantities is very simple since the $\mathfrak{g}_i$ transform like components of a vector as already noted, while $\mathfrak{A}$ transforms like a component of a vector for $E1$ and $M1$ transitions and like an element of a second order tensor for $E2$ transitions.

The interaction Hamiltonian operator H' was taken to have the usual static form, i.e.,

$$H'=-e\mathcal{E}_x\mathfrak{A}, \quad (4)$$

$$\mathfrak{A}=\tfrac{1}{2}(\mathbf{1}_E\cdot\mathbf{r})+(i/8)(\boldsymbol{\kappa}\cdot\mathbf{r})(\mathbf{1}_E\cdot\mathbf{r})+(\hbar/2Mc)[\tfrac{1}{2}(\mu_p-\mu_n)(\boldsymbol{\sigma}_p-\boldsymbol{\sigma}_n)+\tfrac{1}{2}(\mu_p+\mu_n-\tfrac{1}{2})(\boldsymbol{\sigma}_p+\boldsymbol{\sigma}_n)+\tfrac{1}{2}\mathbf{J}]\cdot\mathbf{1}_H \quad (4.1)$$

for $E1$, $E2$, $M1$ to singlets and $M1$ to triplets transitions, respectively. The term in $(\mathbf{J}\cdot\mathbf{1}_H)$ of this operator reproduces the ground-state function and causes therefore no transitions. This form is readily obtained by expanding $\exp(i\boldsymbol{\kappa}\cdot\mathbf{r})\cong 1+i(\boldsymbol{\kappa}\cdot\mathbf{r})$ in the expression for the electric intensity and employing $-e(\mathcal{E}\cdot\mathbf{r})-(e\hbar/2Mc)(\mu_p\boldsymbol{\sigma}_p+\mu_n\boldsymbol{\sigma}_n)$ for H'. When looked at from this viewpoint the $E2$ part of H' is caused by taking into account a retardation correction. The classification into retarded and nonretarded terms used in the

[17] A. Sommerfeld, *Atombau und Spektrallinien* (Friedrich Vieweg und Sohn, Brauschweig, 1939), Vol. 2, pp. 457 and 502; G. Breit and H. A. Bethe, Phys. Rev. **93**, 888 (1954). An account of earlier literature will be found in the latter reference which shows that the necessity of using the ingoing wave modification is a general property of the matrix method of calculating transition probabilities employing modified plane waves and is applicable therefore to the combined system consisting of photon and matter waves even though the initial matter state is bound.

introduction was meant, however, in the sense of Appendix B in the book of Blatt and Weisskopf.[1] In the classification of the latter the term used is the static part of the quadrupole interaction.

Introducing the transformation matrix $\mathcal{O}_{ij}$ by

$$\mathbf{r}=\mathcal{O}\mathbf{r}', \tag{5}$$

one has

$$\eta_i=\sum_j \mathcal{O}_{ij}\eta'_j, \tag{5.1}$$

where the convention

$$(x,y,z)=(x_1,x_2,x_3)$$

has been used. The components of $\boldsymbol{\sigma}$ transform similarly. In the S_{mi} the whole dependence on orientation now appears in the $\mathcal{O}_{ij}$ and there are, besides, numerical coefficients such as

$$(\mathcal{Y}_m{}^{LJ}r^{-1}\mathfrak{F}^L{}_J(kr)\,|\,x_\kappa\,|\,\mathfrak{g}_j), \tag{5.2}$$

which arises as a result of substituting unprimed quantities in terms of primed ones in (3.2). The coefficients appear then first with all integrals and sums being over primed quantities which justifies dropping all primes as in the last expression. Thus, for example, for the $E1$ part one has explicitly

$$(S_{mi})_{E1}=\sum_{L,J}(-i)^L(2L+1)^{\frac{1}{2}}\begin{pmatrix}L & J\\ 0 & m\end{pmatrix}\exp(i\delta^L{}_J)\times\sum_{i,j}(\mathcal{Y}_m{}^{LJ}(')r^{-1}\mathfrak{F}^L{}_J(kr)\,|\,(-\tfrac{1}{2}x'_k)\,|\,\mathfrak{g}_j('))\times\mathcal{O}_{1k}\mathcal{O}_{ij}. \tag{5.2'}$$

The matrix elements playing the role of numerical coefficients such as (5.2) are indeed seen to be the same independently of the relative direction of the $(\mathfrak{E},\mathfrak{K})$ i.e., (x,y,z) and the (x',y',z') axes. The equation just written is for the special case of $\mathfrak{E}$ along x. Equation (5.2′) and similar relations for other multipoles are also useful in considerations of effects of the polarization of incident gamma rays.

For transitions to singlet states one has instead of S_{mi} a quantity s_{0i} defined by

$$s_{0i}=\sum_L(-i)^L(2L+1)^{\frac{1}{2}}\exp(iK_L)\times(\tilde{\chi}_0(s')Y_{L,0}(\theta',\varphi')r^{-1}\,{}^1\mathfrak{F}_L(kr)\,|\,\mathfrak{A}\,|\,\mathfrak{g}_i), \tag{5.3}$$

so that the final amplitude in the primed system arising from one initial state is

$$\Psi_i=\sum_m(\chi_m S_{mi}+\tilde{\chi}_0 s_{0i})a_i. \tag{5.4}$$

The matrix $\mathcal{O}$ is easily shown to be

$$\mathcal{O}=\begin{bmatrix}\cos\theta\cos\varphi & -\sin\varphi & \sin\theta\cos\varphi\\ \cos\theta\sin\varphi & \cos\varphi & \sin\theta\sin\varphi\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}. \tag{5.5}$$

Expanding the triplet and singlet spin functions in terms of single-particle functions and employing the sign of $\tilde{\chi}_0$ defined in the list of notation, then using Eq. (5.4) and the well-known properties of these functions one obtains

$$\sigma_{px}\Psi_i=[a_i/(2)^{\frac{1}{2}}][S_1S_{1i}(\chi_0-\tilde{\chi}_0)+S_{0i}(\chi_1+\chi_{-1})+S_{-1i}(\chi_0+\tilde{\chi}_0)+s_{0i}(\chi_{-1}-\chi_1)], \tag{6.1}$$

$$\sigma_{py}\Psi_i=[ia_i/(2)^{\frac{1}{2}}][S_{1i}(\chi_0-\tilde{\chi}_0)+S_{0i}(\chi_{-1}-\chi_1)-S_{-1i}(\chi_0+\tilde{\chi}_0)+s_{0i}(\chi_{-1}+\chi_1)], \tag{6.2}$$

$$\sigma_{pz}\Psi_i=a_i[S_{1i}\chi_1+S_{0i}\tilde{\chi}_0-S_{-1i}\chi_{-1}+s_{0i}\chi_0], \tag{6.3}$$

so that in the primed coordinate system the components of the proton polarization vector[18] defined by

$$\mathbf{P}'=\sum_i\Psi_i{}^*\boldsymbol{\sigma}_p\Psi_i/\sum_i\Psi_i{}^*\Psi_i, \tag{7}$$

are given by

$$P_x'\bar{\sigma}=(2)^{\frac{1}{2}}\sum_i|a_i|^2\,\mathrm{Re}\{(S_{1i}{}^*+S_{-1i}{}^*)S_{0i}-(S_{1i}{}^*-S_{-1i}{}^*)s_{0i}\}, \tag{7.1}$$

$$P_y'\bar{\sigma}=(2)^{\frac{1}{2}}\sum_i|a_i|^2\,\mathrm{Im}\{(S_{1i}{}^*-S_{-1i}{}^*)S_{0i}-(S_{1i}{}^*+S_{-1i}{}^*)s_{0i}\}, \tag{7.2}$$

$$P_z'\bar{\sigma}=\sum_i|a_i|^2\{|S_{1i}|^2-|S_{-1i}|^2+2\,\mathrm{Re}(S_{0i}{}^*s_{0i})\}. \tag{7.3}$$

The differential cross section may be expressed as

$$\sigma(\theta,\varphi)=\mathcal{C}(k)\bar{\sigma}(\theta,\varphi),$$
$$\bar{\sigma}(\theta,\varphi)=\sum_i|a_i|^2[\sum_m|S_{mi}|^2+|s_{0i}|^2]. \tag{8}$$

with the conventions

$$\sum_i|a_i|^2=1,\quad \mathcal{C}(k)=2\omega e^2/\hbar cv, \tag{8.1}$$

which corresponds to the probability of the initial state being unity. For unpolarized deuterons

$$|a_1|^2=|a_2|^2=|a_3|^2=\tfrac{1}{3}. \tag{8.2}$$

The relation of the energy-dependent factor in front of the summation sign in (8) to $B(k)$ used by Austern[2] may be seen by noting that

$$B(k)=\frac{N^2}{4\gamma^2}\left(\frac{2\omega e^2}{3\hbar cv}\right), \tag{8.3}$$

where the quantity in parentheses is the factor in (8) resulting on insertion of (8.1) and (8.2); the factor N^2/γ^2 arises on account of the convention of multiplying radial integrals by γ/N in Austern's notation and the factor $\frac{1}{4}$ has its origin in the convention introduced by Rarita and Schwinger[2] and adopted by Austern[2] of employing in the radial integrals those arising from $2\mathfrak{A}$ rather than $\mathfrak{A}$.

From Eqs. (7.1)–(7.3) it is seen that the results may be considerably simplified by defining the elements of another amplitude matrix $S_{mi}{}^\xi$ by

$$S_{0i}{}^\xi=s_{0i}, \tag{9.1}$$

$$S_{1i}{}^\xi=(2)^{-\frac{1}{2}}(S_{-1i}-S_{1i}), \tag{9.2}$$

$$S_{2i}{}^\xi=-i(2)^{-\frac{1}{2}}(S_{-1i}+S_{1i}), \tag{9.3}$$

$$S_{3i}{}^\xi=S_{0i}. \tag{9.4}$$

[18] For discussion of the completeness of the description of the polarization state of a particle of spin $\frac{1}{2}$ by means of P', see G. Breit and J. S. McIntosh, in *Encyclopedia of Physics* (Springer-Verlag, Berlin, 1959), Vol. 41, Part I, p. 466.

In this case one gets for the polarizations

$$P_x'\bar{\sigma}=2\sum_i|a_i|^2[\mathrm{Re}\{(S_{0i}^{\xi})^*S_{1i}^{\xi}\}+\mathrm{Im}\{(S_{2i}^{\xi})^*S_{3i}^{\xi}\}], \quad (9.5)$$

$$P_y'\bar{\sigma}=2\sum_i|a_i|^2[\mathrm{Re}\{(S_{0i}^{\xi})^*S_{2i}^{\xi}\}+\mathrm{Im}\{(S_{3i}^{\xi})^*S_{1i}^{\xi}\}], \quad (9.6)$$

$$P_z'\bar{\sigma}=2\sum_i|a_i|^2[\mathrm{Re}\{(S_{0i}^{\xi})^*S_{3i}^{\xi}\}+\mathrm{Im}\{(S_{1i}^{\xi})^*S_{2i}^{\xi}\}]. \quad (9.7)$$

For the differential cross section one obtains

$$\sigma(\theta,\varphi)=\mathfrak{C}(k)\sum_{i,j}|a_j|^2|S_{ji}^{\xi}|^2. \quad (9.8)$$

The S^{ξ}-matrix form of the equations could have been obtained directly by describing the final state in terms of combinations of spin functions that transform like the components of a vector. This may be done by introducing spin functions ξ_j $(j=0,1,2,3)$ by means of

$$\chi_\mu=\sum_j\xi_j\mathfrak{M}_{j\mu}, \quad (10.1)$$

where

$$\mathfrak{M}=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -(2)^{-\frac{1}{2}} & 0 & (2)^{-\frac{1}{2}}\\ 0 & -i(2)^{-\frac{1}{2}} & 0 & -i(2)^{-\frac{1}{2}}\\ 0 & 0 & 1 & 0\end{pmatrix}. \quad (10.2)$$

Here the columns correspond to $\tilde{\chi}_0$, χ_1, χ_0, χ_{-1} starting on the left and the rows to $j=0$, 1, 2, 3 reading down. The expression for the final state resulting from substate $\mathfrak{g}_i$ is according to Eq. (5.4),

$$\sum_m\chi_m S_{mi}+\tilde{\chi}_0 s_{0i}=\sum_j\xi_j S_{ji}^{\xi}, \quad (10.3)$$

where the S^{ξ} matrix is defined by

$$S^{\xi}=\begin{pmatrix}S_{0i}^{\xi}\\ S_{1i}^{\xi}\\ S_{2i}^{\xi}\\ S_{3i}^{\xi}\end{pmatrix}=\mathfrak{M}S=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -(2)^{-\frac{1}{2}} & 0 & (2)^{-\frac{1}{2}}\\ 0 & -i(2)^{-\frac{1}{2}} & 0 & -i(2)^{-\frac{1}{2}}\\ 0 & 0 & 1 & 0\end{pmatrix}\begin{pmatrix}s_{0i}\\ S_{1i}\\ S_{0i}\\ S_{-1i}\end{pmatrix}=\begin{pmatrix}s_{0i}\\ (2)^{-\frac{1}{2}} & (S_{-1i}-S_{1i})\\ -i(2)^{-\frac{1}{2}} & (S_{-1i}+S_{1i})\\ & S_{0i}\end{pmatrix}, \quad (10.4)$$

which is just the definition used in Eqs. (9.1)–(9.4).

III. INTRODUCTION OF THE TENSOR COUPLING

As is well known, the inclusion of a tensor term S_{12} in the potential has the consequence that states of the same total angular momentum and parity but different orbital angular momentum become coupled to each other, so that the expression for S_{mi} given in Eq. (3.2) has to be modified. Furthermore, independently of the form of the interaction energy or the existence of a static interaction potential, the real eigenphase shift description is[19] the most general one in the nucleon-nucleon problem below the meson production threshold.

The modification to Eq. (3.2) required by coupling of states with different L can be worked out employing the expression for the wave function in the final state taking the coupling into account. In the notation of Breit, Ehrman, and Hull[16] the outgoing wave modification of the triplet wave arising from the plane wave $e^{ikz'}\chi_m$ outside the nuclear interaction region is

$$(e^{ikz'}\chi_m)\rightarrow\frac{(4\pi)^{\frac{1}{2}}}{2i\rho}\sum_{LJ}(2L+1)^{\frac{1}{2}}i^L\begin{pmatrix}L & J\\ 0 & m\end{pmatrix}\times[-H_L^*\mathcal{Y}_m^{LJ}+\sum_{L'}U_{LL'}^{J}\mathcal{Y}_m^{L'J}H_{L'}], \quad (11)$$

where

$$H_L=G_L+iF_L;\quad \rho=kr. \quad (11.1)$$

For uncoupled states the matrix U^J consists of a single element $\exp(2i\delta_J)$ while for coupled states it is a two by two unitary symmetric matrix. The first and second terms in square brackets of (11) represent, respectively, converging and diverging waves. The first term in the brackets is the same as in the absence of interaction. The matrix U in that case is a unit matrix and the second term is then $H_L\mathcal{Y}_m^{LJ}$. For the ingoing wave modification the $H_L\mathcal{Y}_m^{LJ}$ in the brackets must be kept unchanged and the p-n interaction affects the first term. A consideration of the complex conjugate of the solution of the wave equation determines the form of the combination in the square bracket and hence the ingoing wave modification of χ_m multiplied by the plane wave $\exp(ikz')$

$$\psi_f^{(-)}=\frac{(4\pi)^{\frac{1}{2}}}{2i\rho}\sum_{LJ}(2L+1)^{\frac{1}{2}}i^L\begin{pmatrix}L & J\\ 0 & m\end{pmatrix}\times[H_L\mathcal{Y}_m^{LJ}-\sum_{L'}U_{LL'}^{J}\mathcal{Y}_m^{L'J}H^*_{L'}]. \quad (11.2)$$

This wave function together with its continuation into the region of nuclear interaction forms the $\psi_f^{(-)}$ in the required generalization of Eq. (3). For a diagonal U^J, (11.2) is equivalent to the employment of (1.1) in (1). The modification caused by the off-diagonal elements of U^J is conveniently expressed in terms of the Wigner[20] parametrization of U in the form of Blatt and Biedenharn.[20] This gives

$$U=\begin{pmatrix}c_\epsilon^2\exp(2i\delta_\alpha)+s_\epsilon^2\exp(2i\delta_\beta), & c_\epsilon s_\epsilon[\exp(2i\delta_\alpha)-\exp(2i\delta_\beta)]\\ c_\epsilon s_\epsilon[\exp(2i\delta_\alpha)-\exp(2i\delta_\beta)], & s_\epsilon^2\exp(2i\delta_\alpha)+c_\epsilon^2\exp(2i\delta_\beta)\end{pmatrix}, \quad (11.3)$$

$$c_\epsilon=\cos\epsilon,\quad s_\epsilon=\sin\epsilon, \quad (11.4)$$

with δ_α, δ_β having the significance of eigenphase shifts. The convention used in (11.3) is that the first row and column refer to the smaller of the two coupled L. The

[19] G. Breit, *University of Pennsylvania Bicentennial Conference* (University of Pennsylvania Press, Philadelphia, Pennsylvania, 1941).

[20] E. P. Wigner, Proc. Natl. Acad. Sci. U. S. **32**, 302 (1946); J. M. Blatt and L. C. Biedenharn, Phys. Rev. **86**, 399 (1952).

fact that the eigenvalues of U are $\exp(2i\delta_\alpha)$, $\exp(2i\delta_\beta)$ shows after a short calculation that the linear combinations

$$\Psi_\lambda=[-\exp(-i\varphi_\lambda)\mathcal{Y}_m^{\lambda,J}+\sum_{\lambda'}U_{\lambda,\lambda'}\times\exp(i\varphi_{\lambda'})\mathcal{Y}_m^{\lambda',J}]/r \quad (11.5)$$

$$\begin{pmatrix}\lambda=J-1,\ J+1\\ \lambda'=J-1,\ J+1\end{pmatrix}$$

$$\varphi_\lambda=kr-\lambda\pi/2 \quad (11.5')$$

may be expressed as

$$\begin{pmatrix}\Psi_L\\ \Psi_{L+2}\end{pmatrix}\sim 2i\begin{pmatrix}c_\epsilon\exp(i\delta_\alpha), & -s_\epsilon\exp(i\delta_\beta)\\ s_\epsilon\exp(i\delta_\alpha), & c_\epsilon\exp(i\delta_\beta)\end{pmatrix}\begin{pmatrix}\psi^\alpha\\ \psi^\beta\end{pmatrix}, \quad (11.6)$$

where

$$\begin{aligned}\psi^\alpha&=[c_\epsilon\sin(\varphi_{J-1}+\delta_\alpha)\mathcal{Y}_m^{J-1,J}\\ &\qquad+s_\epsilon\sin(\varphi_{J+1}+\delta_\alpha)\mathcal{Y}_m^{J+1,J}]/r,\\ \psi^\beta&=[-s_\epsilon\sin(\varphi_{J-1}+\delta_\beta)\mathcal{Y}_m^{J-1,J}\\ &\qquad+c_\epsilon\sin(\varphi_{J+1}+\delta_\beta)\mathcal{Y}_m^{J+1,J}]/r.\end{aligned} \quad (11.7)$$

The coefficients of the spin-angular functions in the last equation are real and hence the continuation of ψ^α, ψ^β by means of the wave equation to smaller distances gives eigensolutions of the form

$$\begin{aligned}\psi^\alpha&=(u_J{}^\alpha\mathcal{Y}_m^{J-1,J}+v_J{}^\alpha\mathcal{Y}_m^{J+1,J})/r,\\ \psi^\beta&=(u_J{}^\beta\mathcal{Y}_m^{J-1,J}+v_J{}^\beta\mathcal{Y}_m^{J+1,J})/r,\end{aligned} \quad (12)$$

in which $u_J{}^\alpha$, $\cdots$, $v_J{}^\beta$ are real. Comparison of (12) with (11.7) shows that outside the interaction region

$$\begin{aligned}u_J{}^\alpha&=c_\epsilon[F_{J-1}\cos\delta_\alpha+G_{J-1}]\sin\delta_\alpha],\\ v_J{}^\alpha&=s_\epsilon[F_{J+1}\cos\delta_\alpha+G_{J+1}\sin\delta_\alpha],\\ u_J{}^\beta&=-s_\epsilon[F_{J-1}\cos\delta_\beta+G_{J-1}\sin\delta_\beta],\\ v_J{}^\beta&=c_\epsilon[F_{J+1}\cos\delta_\beta+G_{J+1}\sin\delta_\beta],\end{aligned} \quad (12.1)$$

which together with (12) identifies the functions when the radial equation for the $u_J{}^\alpha$, $\cdots$ $v_J{}^\beta$ following from (12) are solved. Since the square bracket in (11) is asymptotically $r\Psi_\lambda$ one has available through the asymptotic forms of the Ψ_λ in (11.6) expressions in terms of the eigensolutions ψ^α, ψ^β for the outgoing wave modification of a plane wave. Similarly introducing

$$\Psi_\lambda{}^*=[-\exp(i\varphi_\lambda)\mathcal{Y}_m^{\lambda J}+\sum U^J{}_{\lambda,\lambda'}{}^*\times\exp(-i\varphi_\lambda)\mathcal{Y}_m^{\lambda' J}]/r, \quad (11.5')$$

one has available the negative of the square bracket expression divided by r in the expansion for $\psi_f{}^{(-)}$ in (11.2) On the other hand, from (11.6) and (12) one has

$$\begin{pmatrix}\Psi_L{}^*\\ \Psi_{L+2}{}^*\end{pmatrix}\sim -2i\begin{pmatrix}c_\epsilon\exp(-i\delta_\alpha), & -s_\epsilon\exp(-i\delta_\beta)\\ s_\epsilon\exp(-i\delta_\alpha), & c_\epsilon\exp(-i\delta_\beta)\end{pmatrix}\times\begin{pmatrix}\psi^\alpha\\ \psi^\beta\end{pmatrix}, \quad (12.2)$$

and consequently the part of $\psi_f{}^{(-)}$ containing waves with angular momentum J is

$$\begin{aligned}(\psi_f{}^{(-)})_J=\frac{(4\pi)^{\frac{1}{2}}}{kr}&\left[(2L+1)^{\frac{1}{2}}\begin{pmatrix}L & J\\ 0 & m\end{pmatrix}i^L\right]_{L=J-1}\\ &\times(c_\epsilon e^{-i\delta_\alpha}\psi^\alpha-s_\epsilon e^{-i\delta_\beta}\psi^\beta]\\ &+\left[(2L+1)^{\frac{1}{2}}\begin{pmatrix}L & J\\ 0 & m\end{pmatrix}i^L\right]_{L=J+1}\\ &\times[s_\epsilon e^{-i\delta_\alpha}\psi^\alpha+c_\epsilon e^{-i\delta_\beta}\psi^\beta].\end{aligned} \quad (12.3)$$

This quantity has to be used in the calculation of S_{mi} by substitution of $\mathfrak{A}$ in place of H' on the left side of (3) and the replacement of

$$(4\pi)^{\frac{1}{2}}i^L(2L+1)^{\frac{1}{2}}\begin{pmatrix}L & J\\ 0 & m\end{pmatrix}\times\exp(-i\delta^{L_J})\mathcal{Y}_m^{LJ}(')\frac{\mathfrak{F}^{L_J}(kr)}{kr}, \quad (12.4)$$

the complex conjugate of which occurs in Eq. (3), by the right-hand side of (12.3). According to Eq. (12) the ψ^α and ψ^β each contain two terms with two different angular-spin functions but both of them belong to the same total angular momentum J and the same magnetic quantum number m representing the projection of J. According to the Wigner-Eckart theorem the ratio of contributions of these terms to the matrix element S_{mi} is therefore the same for different m and i.

It will be noted that according to (11.7) the functions ψ^α and ψ^β are normalized to unit radial density at large r and may, therefore, be taken to be the eigenfunctions corresponding to the eigenphases δ_α, δ_β. The contribution to S_{mi} from a coupled state consists accordingly of four parts corresponding, respectively, to $u_J{}^\alpha$, $v_J{}^\alpha$, $u_J{}^\beta$, $v_J{}^\beta$ in (12). Each part contains contributions from the ${}^3S_1$ and ${}^3D_1$ parts of the ground state. Since these two parts have the same J the ratio of their matrix elements is independent of m and the contribution to S_{mi} due to a pair of coupled states takes the form

$$\begin{aligned}[S_{mi}]_J=&C_{mi}{}^J(J-1)(c_\epsilon e^{i\delta_\alpha})[L_{u\alpha}{}^J+\Re_J L_{v\alpha}{}^J]\\ &+C_{mi}{}^J(J+1)(s_\epsilon e^{i\delta_\alpha})[L_{u\alpha}{}^J/\Re_J+L_{v\alpha}{}^J]\\ &-C_{mi}{}^J(J-1)(s_\epsilon e^{i\delta_\beta})[L_{u\beta}{}^J+\Re_J L_{v\beta}{}^J]\\ &+C_{mi}{}^J(J+1)(c_\epsilon e^{i\delta_\beta})[L_{u\beta}{}^J/\Re_J+L_{v\beta}{}^J].\end{aligned} \quad (13)$$

The successive terms arise here from the four parts of (12.3). The radial integrals are the L and the designations $u\alpha$, $\cdots$, $v\beta$ indicate the origin in the functions $u_J{}^\alpha$, $\cdots$ of (12). The coefficients $C_{mi}{}^J(L)$ have arisen from corresponding coefficients in (12.2) combined with those arising from angular integrations and sums over spin coordinates.

Expressing quantities in the ξ representation as in Eqs. (9.1) to (9.4) or in Eq. (10.4) one has for $E1$ transitions to $({}^3P_2+{}^3F_2)$ states taking $i=1$ by analogy

with Eq. (9.2)

$$[C_{11}^2(1)]^\xi = -i\cos\theta\sin\theta\cos^2\varphi,$$
$$[C_{11}^2(3)]^\xi = i[3(2)^{\frac{1}{2}}/5]\cos\theta\sin\theta\cos^2\varphi. \quad (13.1)$$

For $M1$ transitions to $({}^3S_1{}^*+{}^3D_1{}^*)$ states, taking $i=1$ and in analogy with Eq. (9.4)

$$[C_{31}^1(1)]^\xi = -i\cos\theta, \quad [C_{31}^1(2)]^\xi = i(2)^{-\frac{1}{2}}\cos\theta. \quad (13.2)$$

Similarly for $E2$ transitions to $({}^3D_3+{}^3G_3)$ states, taking $i=2$ and using Eq. (9.3)

$$[C_{22}^3(2)]^\xi = -\tfrac{1}{6}i\sin 2\theta\cos 2\varphi,$$
$$[C_{22}^3(4)]^\xi = -i[3(2)^{-\frac{1}{2}}/14]\sin 2\theta\cos 2\varphi. \quad (13.3)$$

The radial integrals $L_{u\alpha}{}^J, \cdots, L_{v\beta}{}^J$ as well as integrals I_0, I_1 for ${}^3P_0$, ${}^3P_1$; M_S, M_D for $M1$ leading to ${}^1S_0$, ${}^1D_2$; M'_D for $M1$ to ${}^3D_2$ as well as Q_{D2} for $E2$ to ${}^3D_2$ are used in the same general convention as in the paper of Austern[2] for coupled states the normalization of the radial functions was such that

$$\langle(u_J{}^\alpha)^2+(v_J{}^\alpha)^2\rangle\sim\tfrac{1}{2}, \quad \langle(u_J{}^\beta)^2+(v_J{}^\beta)^2\rangle\sim\tfrac{1}{2}, \quad (13.4)$$

as implied by (12.1) and for the ground state

$$\int_0^\infty (U^2+W^2)dr = 1, \quad (13.5)$$

the wave function for the latter being

$$(\psi_d)_m = (4\pi)^{-\frac{1}{2}}[U+8^{-\frac{1}{2}}S_{12}W]\chi_m/r, \quad (13.6)$$

and S_{12} the usual tensor force operator. For $L=J-1$ and $E1$ transitions the convention is to have the coefficient of the product in the integrand containing U, u, and a power of r equal to γ where γ is defined through the asymptotic form of U at large r, viz $U\sim\text{const}e^{-\gamma r}$; for $E2$ the corresponding coefficient is $\gamma\omega/(4c)$, for $M1$ the coefficient is $(\gamma\hbar/Mc)(\mu_p+\mu_n-\frac{1}{2})$. For coupled case integrals with $L=J+1$ the same coefficients are used for the product of W, v, and a power of r as just mentioned for $L=J-1$ for the product of U, u, and a power of r. In those cases for which the final state reached by the U and W parts of the ground state turn out to be the same, the radial integrals are combined. For example, for $E1$ transition to $({}^3P_2+{}^3F_2)$ states

$$I_{u\alpha}{}^2 = \int_0^\infty (\gamma r)[U-W/5\sqrt{2}]u_2{}^\alpha dr, \quad (13.7)$$

$$I_{u\beta}{}^2 = \int_0^\infty (\gamma r)[U-W/5\sqrt{2}]u_2{}^\beta dr, \quad (13.8)$$

$$I_{v\alpha}{}^2 = \int_0^\infty (\gamma r)Wv_2{}^\alpha dr, \quad (13.9)$$

$$I_{v\beta}{}^2 = \int_0^\infty (\gamma r)Wv_2{}^\beta dr. \quad (13.10)$$

The factor $[-(2^{-\frac{1}{2}}/5]$ is the ratio of the matrix elements ${}^3D_1\rightarrow{}^3P_2/{}^3S_1\rightarrow{}^3P_2$. The other radial integrals in Tables III and IV are similarly defined. The coefficients multiplying the integrands in the uncoupled cases arising through $E1$, $M1$ and $E2$ transitions are the same as in their respective coupled cases. The coefficient $\gamma(\hbar/Mc)(\mu_p-\mu_n)$ is used to multiply the integrand for singlet transitions. The additional radial integrals which occur in the text are defined as follows:

$$I_0 = \int_0^\infty (\gamma r)[U-\sqrt{2}W]\mathfrak{F}^1{}_0(kr)dr,$$

$$I_1 = \int_0^\infty (\gamma r)[U+W/\sqrt{2}]\mathfrak{F}^1{}_1(kr)dr,$$

$$M'_D = \gamma\zeta\int_0^\infty W\mathfrak{F}^2{}_2(kr)dr,$$

$$M_{ur}{}^1 = \gamma\zeta\int_0^\infty Uu_1{}^r dr,$$

$$M_{vr}{}^1 = \gamma\zeta\int_0^\infty Wv_1{}^r dr,$$

$$Q_{vr}{}^1 = \gamma q\int_0^\infty (U-W/\sqrt{2})v_1{}^r r^2 dr,$$

$$Q_{ur}{}^1 = \gamma q\int_0^\infty Wu_1{}^r r^2 dr,$$

$$Q_{ur}{}^3 = \gamma q\int_0^\infty (U-\sqrt{2}W/7)u_3{}^r r^2 dr,$$

$$Q_{vr}{}^3 = \gamma q\int_0^\infty Wv_3{}^r r^2 dr,$$

$$Q_{D_2} = \gamma q\int_0^\infty (U+W/\sqrt{2})\mathfrak{F}^2{}_2(kr)r^2 dr,$$

$$M_S = \gamma(\hbar/Mc)(\mu_p-\mu_n)\int_0^\infty U\mathfrak{F}^1{}_0(kr)dr,$$

$$M_D = \gamma(\hbar/Mc)(\mu_p-\mu_n)\int_0^\infty W\mathfrak{F}^1{}_2(kr)dr,$$

where $\zeta=(\hbar/Mc)(\mu_p+\mu_n-\frac{1}{2})$; $q=(\omega/4c)$, and $\tau=(\alpha,\beta)$ in all the equations defining the radial matrix elements. In Eq. (13) there occurs the ratio

$$\mathcal{R}_J = (\mathcal{Y}_m{}^{J+1,J}|H'|\mathcal{Y}_i{}^{l1})/(\mathcal{Y}_m{}^{J-1,J}|H'|\mathcal{Y}_i{}^{l'1}), \quad (14)$$

where $l=2$ and $l'=0$ except for the $E2$ transition $({}^3S_1+{}^3D_1)\rightarrow({}^3D_1+{}^3S_1)$ for which $l=0$ and $l'=2$. $\mathcal{R}_J$ is independent of m and i according to the Wigner-Eckart theorem. It has been found convenient to

TABLE I. Values of $\mathfrak{R}_J$ for different transitions.

Multipole	Final state	J	$\mathfrak{R}_J$
$E1$	$({}^3P_2+{}^3F_2)$	2	$3^{\frac{1}{2}}/5$
$E2$	$({}^3D_3+{}^3G_3)$	3	$3(6)^{\frac{1}{2}}/7$
	$({}^3S_1+{}^3D_1)$	1	1
$M1$	$({}^3S_1+{}^3D_1)$	1	$-\frac{1}{2}$

introduce this ratio so as to have one rather than two sets of coefficients $C_{mi}{}^J$. In (13) two terms contain $\mathfrak{R}_J$ and two others contain $\mathfrak{R}_J{}^{-1}$. The reason for this is that the $C_{mi}{}^J(J-1)$ are standardized conveniently in terms of the ${}^3S_1$ part of the ground state while the $C_{mi}{}^J(J+1)$ are more readily expressed in terms of the ${}^3D_1$ part. Thus for $E1$ transitions the ${}^3P_2+{}^3F_2$ coupled state has the $L=J+1$ part of the continuum state, i.e., the ${}^3F_2$ component, combining only with ${}^3D_1$. The $\mathfrak{R}_J$ have values as in Table I.

IV. ELLIPTICALLY POLARIZED GAMMA RAYS

The S matrix of Eq. (3.2) was worked out for a state of polarization

$$(\mathcal{E}_x,\mathfrak{IC}_y)=(\mathcal{E},\mathcal{E})e^{i\kappa z}. \tag{15.1}$$

In the present discussion this matrix will be called $S^{(1)}$. By a similar procedure the elements of a matrix $S^{(2)}$ corresponding to the perpendicular polarization

$$(\mathcal{E}_y,\mathfrak{IC}_x)=(\mathcal{E},-\mathcal{E})e^{i\kappa z}, \tag{15.2}$$

may be worked out and the relation of $S^{(1)}$ to $S^{(2)}$ will be discussed presently. Since the only restriction on the direction of the x axis necessary so far was that of being perpendicular to $\boldsymbol{\kappa}$, there is no loss of generality in arranging for it to be along one of the principal axes of the polarization ellipse. Superposing (15.1) and (15.2) with a 90° phase difference, an elliptically polarized wave

$$\begin{aligned}(\mathcal{E}_x,\mathcal{E}_y,\mathcal{E}_z)&=(\cos\chi,\, i\sin\chi,\, 0)\mathcal{E}e^{i\kappa z},\\ (\mathfrak{IC}_x,\mathfrak{IC}_y,\mathfrak{IC}_z)&=(-i\sin\chi,\,\cos\chi,\,0)\mathcal{E}e^{i\kappa z},\end{aligned} \tag{15.3}$$

results. Here χ is a real angle which is 0 or π for linear polarization along x, $\pi/2$, or $3\pi/2$ for linear polarization along y, $\pi/4$, and $-\pi/4$ for opposite directions of circular polarization, etc.

In Eqs. (15.1), (15.2), (15.3) the usual convention is followed of writing on the right-hand side only the part of the field strength which matters for absorption, the complex conjugate of the part written being omitted. The time dependence, not explicitly indicated, is $e^{-i\omega t}$ for the term written therefore. This time dependence shows that for $\chi=\pi/4$ the electric vector rotation is right handed around the positive z axis and that the angular momentum projection available in the radiation field is positive. Reference to Eq. (137) and Fig. 2 of an earlier paper[21] shows conditions for elliptic polarizations somewhat more generally. The relation of wave intensity to $|\mathcal{E}|$ is independent of χ in this notation. Taking $\mathcal{E}$ to be the same in (15.1), (15.2), and (15.3) and denoting by $S^{(\chi)}$ the S matrix for the state of polarization described by (15.3) one has

$$S^{(\chi)}=S^{(1)}\cos\chi+iS^{(2)}\sin\chi. \tag{15.4}$$

An obvious orthogonal transformation can be carried out so as to have the principal axes at an arbitrary angle with respect to the x axis.

The matrix $S^{(2)}$ can be obtained from $S^{(1)}$ by the following consideration. The quantity $\mathfrak{A}$ needed for (15.2) will be called $\mathfrak{A}^{(2)}$ while the $\mathfrak{A}$ corresponding to (15.1) which was used mostly so far will be called $\mathfrak{A}^{(1)}$. The $(\mathfrak{E},\mathfrak{IC})$ vectors for case (2) are obtained from case (1) by a rotation through 90° in the direction from x to y. The quantity $\mathfrak{A}^{(2)}$ considered as a function of the azimuthal angle φ is therefore obtainable as

$$\mathfrak{A}^{(2)}(\varphi)=\mathfrak{A}^{(1)}(\varphi-\tfrac{1}{2}\pi), \tag{15.5}$$

as is clear geometrically and readily verifiable by noting that $\mathfrak{A}^{(1)}$, $\mathfrak{A}^{(2)}$ have the forms $\mathfrak{A}^{(1)}=A_x+B_y=A\cos\varphi+B\sin\varphi$, $\mathfrak{A}^{(2)}=A_y-B_x=A\sin\varphi-B\cos\varphi$. In these relations φ is the azimuthal angle of a point in coordinate space, while the vectors $\mathbf{A}$, $\mathbf{B}$ are coordinate space vectors coupled to $\mathfrak{E}$ and $\mathfrak{IC}$ in the Hamiltonian. Therefore, if in the evaluation of (3) one arranges for the points entering the integrals on the right side to have values such that

$$\varphi^{(1)}=\varphi^{(2)}-\tfrac{1}{2}\pi, \tag{15.6}$$

then the integrals will be the same except for such changes as occur in the $\mathfrak{g}_i$ on account of (15.6). Here $\varphi^{(1)}$, $\varphi^{(2)}$ are azimuthal angles of two *different* vectors referred to the x axis of the original coordinate system. In order to secure (15.6) it suffices to have the azimuthal angles of the proton emission line in the comparison of $S^{(1)}$ with $S^{(2)}$ correlated by a relation identical in form with (15.6). This amounts in fact to rotating all vectors with $(\mathfrak{E},\mathfrak{IC})$ so that the integrals would remain unchanged if it were not for the changes in the $\mathfrak{g}_i$ which by definition are referred to the original coordinate system.

Two points $\mathbf{r}^{(1)}$, $\mathbf{r}^{(2)}$ with azimuthal angles related as in (15.6) have coordinates related by

$$x^{(1)}=y^{(2)},\quad y^{(1)}=-x^{(2)},\quad z^{(1)}=z^{(2)}. \tag{16}$$

By means of these equations points $\mathbf{r}^{(1)}$ used in the evaluation of $S^{(1)}$ are mapped into a duplicate of the original space as points $\mathbf{r}^{(2)}$ in the evaluation of $S^{(2)}$. For the latter evaluation there enter in Eq. (3) as a result of the mapping $\mathfrak{g}_i(x^{(2)},y^{(2)},z^{(2)})$. On account of the vector property of the functional form of the $\mathfrak{g}_i$ it follows from (16) that

$$\begin{aligned}&\mathfrak{g}_i(\mathbf{r}^{(1)})=\mathfrak{g}_2(\mathbf{r}^{(2)}),\quad \mathfrak{g}_2(\mathbf{r}^{(1)})=-\mathfrak{g}_1(\mathbf{r}^{(2)}),\\ &\qquad\qquad\qquad\qquad \mathfrak{g}_3(\mathbf{r}^{(1)})=\mathfrak{g}_3(\mathbf{r}^{(2)}).\end{aligned} \tag{16.1}$$

Quantities occurring in the right-hand sides of these equations occur directly in (3.2) in the evaluation of

[21] G. Breit, Revs. Modern Phys. **5**, 91 (1933).

TABLE II. Values of coefficients T_1, T_2, T_3, T_4 in Eq. (17) and of symbols that will be used in place of $X_J{}^c$ as listed in the last column.

Multipole	Final state	c	T_1 or T_3	T_2 or T_4	$X_J{}^c$
$E1$	$({}^3P_2+{}^3F_2)$	a	1	$-3(2)^{\frac{1}{2}}/5$	$I_2{}^a$
		b	1	$9(2)^{-\frac{1}{2}}/5$	$I_2{}^a$
$E2$	$({}^3S_1+{}^3D_1)$	a	-1	$2^{-\frac{1}{2}}$	$Q_1{}^a$
		b	1	$2^{\frac{1}{2}}$	$Q_1{}^b$
$E2$	$({}^3D_3+{}^3G_3)$	a	-1	$9(2)^{-\frac{1}{2}}/7$	$Q_3{}^a$
		b	1	$12(2)^{-\frac{1}{2}}/7$	$Q_3{}^b$
$M1$	$({}^3S_1+{}^3D_1)$	a	-1	$-2^{-\frac{1}{2}}/2$	$M_1{}^a$
		b	1	$-2^{-\frac{1}{2}}$	$M_1{}^b$

$S_{mi}{}^{(2)}$ while those on the left give on account of (15.6) components of $S_{mi}{}^{(1)}$. It follows that

$$S_{m1}{}^{(1)}(\varphi^{(1)})=S_{m2}{}^{(2)}(\varphi^{(2)}),\quad S_{m2}{}^{(1)}(\varphi^{(1)})=-S_{m1}{}^{(2)}(\varphi^{(2)}),\quad S_{m3}{}^{(1)}(\varphi^{(1)})=S_{m3}{}^{(2)}(\varphi^{(2)}). \quad (16.2)$$

The simplicity of the answer is seen to be due to the vector nature of the $\mathfrak{g}_i$. The same result is obtained by expressing S_{mi} for both cases in terms of components of $\mathcal{O}$ as in (5.2′) and the analogous case of $M1$ transitions. A straightforward calculation gives the same result as in (16.2). Since $S^{(1)}$ is what has previously been called S both $S^{(1)}$ and $S^{(2)}$ are available for (15.4) and all quantities are available for an arbitrary state of polarization of incident gamma rays.

V. FORMULAS FOR THE AMPLITUDES IN THE CARTESIAN VECTOR TOTAL SPIN REPRESENTATION

In this section, the explicit expressions for the S-matrix elements for outgoing protons are given. For transitions to coupled states it will be convenient to use the following abbreviations:

$$X_J{}^c=-i[(T_1(X_{u\alpha}+\mathcal{R}_JX_{v\alpha})c_\epsilon e^{i\delta_\alpha}+T_2(X_{u\alpha}/\mathcal{R}_J+X_{v\alpha})s_\epsilon e^{i\delta_\alpha}-T_3(X_{u\beta}+\mathcal{R}_JX_{v\beta})s_\epsilon e^{i\delta_\beta}+T_4(X_{u\beta}/\mathcal{R}_J+X_{v\beta})c_\epsilon e^{i\delta_\beta}], \quad (17)$$

where the coefficients T_1, T_2, T_3, T_4 have values as in Table II. In using (17) and Table II the values of $\mathcal{R}_J$ are understood to be inserted by means of Table I. The designations a, b refer to the origin of the quantity either in $m=\pm1$ in the case of a or $m=0$ in the case of b. The elements of the S matrix can now be expressed as

$$2s_{01}{}^\xi=[3(2)^{-\frac{1}{2}}/2]M_De(iK_2)\sin^2\theta\sin2\varphi, \quad (18.1)$$

$$2s_{02}{}^\xi=M_Se(iK_0)+3(2)^{-\frac{1}{2}}M_De(iK_2)\times(\sin^2\theta\sin^2\varphi-\tfrac{1}{3}), \quad (18.2)$$

$$2s_{03}{}^\xi=[3(2)^{-\frac{1}{2}}/2]M_De(iK_2)\sin2\theta\sin\varphi, \quad (18.3)$$

$$2S_{11}{}^\xi=\sin\theta\{I_2{}^a\cos\theta\cos^2\varphi+(2^{-\frac{1}{2}}/5)Q_1{}^a-(i/6)Q_{D2}\times e(i\delta^2{}_2)\cos2\varphi+Q_3{}^a[(1/3)\cos^2\varphi-\cos^2\theta\cos^2\varphi-(1/15)]+M_1{}^a+[3i(2)^{-\frac{1}{2}}/2]M_D{}'e(i\delta_2{}^2)\cos2\varphi\}, \quad (18.4)$$

$$2S_{12}{}^\xi=(1/2)\sin\theta\sin2\varphi\{I_2{}^a\cos\theta-(i/3)Q_{D2}e(i\delta^2{}_2)+Q_3{}^a[(1/3)-\cos^2\theta]+3i(2)^{-\frac{1}{2}}M_D{}'e(i\delta^2{}_2)\} \quad (18.5)$$

$$2S_{13}{}^\xi=\cos\varphi\{(i/2)I_1e(i\delta^1{}_1)+(1/2)I_2{}^a\cos2\theta-[(2)^{-\frac{1}{2}}/5]Q_1{}^a\cos\theta+(i/6)Q_{D2}e(i\delta^2{}_2)\cos\theta-Q_3{}^a\cos\theta[(4/15)-\sin^2\theta]+M_1{}^a\cos\theta+[3i(2)^{-\frac{1}{2}}/2]M_D{}'e(i\delta^2{}_2)\cos\theta\}, \quad (18.6)$$

$$2S_{21}{}^\xi=(1/2)\sin\theta\sin2\varphi\{-I_2{}^a+(i/3)Q_{D2}e(i\delta^2{}_2)\cos\theta+(2/3)Q_3{}^a\cos\theta-3i(2)^{-\frac{1}{2}}M_D{}'e(i\delta^2{}_2)\cos\theta\}, \quad (18.7)$$

$$2S_{22}{}^\xi=\sin\theta\{-(i/2)I_1e(i\delta^1{}_1)+(1/2)I_2{}^a\cos2\varphi-(i/3)Q_{D2}e(i\delta^2{}_2)\cos\theta(1+\cos^2\varphi)+Q_3{}^a[(2/3)\sin^2\varphi-(1/3)]\cos\theta-3i(2)^{-\frac{1}{2}}\times M_D{}'e(i\delta^2{}_2)\cos\theta\sin^2\varphi\}, \quad (18.8)$$

$$2S_{23}{}^\xi=\sin\varphi\{[-(i/2)I_1e(i\delta^1{}_1)-(1/2)I_2{}^a]\cos\theta+[(2)^{-\frac{1}{2}}/5]Q_1{}^a-(i/6)Q_{D2}e(i\delta^2{}_2)\cos2\theta+Q_3{}^a[(1/3)\cos^2\theta-(1/15)]-M_1{}^a-[3i(2)^{-\frac{1}{2}}/2]M'{}_De(i\delta^2{}_2)\cos2\theta\}, \quad (18.9)$$

$$2S_{31}{}^\xi=-(i/3)I_0e(i\delta^1{}_0)+I_2{}^b(\sin^1\theta\cos^2\varphi-\tfrac{1}{3})+[(2)^{-\frac{1}{2}}/5]Q_1{}^b\cos\theta+Q_3{}^b\cos\theta\times(\sin^2\theta\cos^2\varphi-1/5)+M_1{}^b\cos\theta, \quad (18.10)$$

$$2S_{32}{}^\xi=(1/2)\sin^2\theta\sin2\varphi\{I_2{}^b+Q_3{}^b\cos\theta\}, \quad (18.11)$$

$$2S_{33}{}^\xi=\sin\theta\cos\varphi\{I_2{}^b\cos\theta+[(2)^{-\frac{1}{2}}/5]Q_1{}^b+Q_3{}^b(\cos^2\theta-1/5)-M_1{}^b\}, \quad (18.12)$$

where

$$e(i\delta^L{}_J)\equiv\exp(i\delta^L{}_J). \quad (18.13)$$

Equations (18.1)–(18.12) have been checked independently by using Racah coefficients. In order to obtain the amplitudes and polarization for the outgoing neutron, it is merely necessary to change the sign of the $E1$ terms in Eqs. (18.4) to (18.12) and of the first terms on the right side of Eqs. (9.5) to (9.7). The change in sign in Eqs. (9.5) to (9.7) arises because the singlet spin function is antisymmetric between neutron and proton. The same results for angular distribution and polarization of the outgoing neutron can, however, be obtained simply by changing the sign of the $E1$ and $M1$ spin-flip terms in Eqs. (18.1)–(18.12) and employing the Eqs. (9.5) to (9.7) as they are.

VI. THE SEMIPHENOMENOLOGICAL TWO-NUCLEON POTENTIAL

The basis of the two-nucleon potentials used in the present work is the static potential derived by Gartenhaus[22] on the basis of meson theory in the form proposed by Chew.[23] This potential gives a good fit to the low-energy data. However, it was soon found by Gammel and Thaler[24] that the fit at higher energies is very poor. Signell and Marshak[7] showed that an apparent improvement could be obtained by adding to the Gartenhaus potential a short-range isotopic-spin-independent intrinsically attractive spin-orbit potential of a form

[22] S. Garthenhaus, Phys. Rev. **100**, 900 (1955).
[23] G. Chew, Phys. Rev. **95**, 285 (1954); **95**, 1669 (1954).
[24] J. Gammel and R. Thaler, Phys. Rev. **103**, 1874 (1956).

TABLE III. Radial matrix elements in units of 10^{-13} cm and phase shifts in degrees from Potential I.

E_γ (Mev)[a]	22.2	32.2	62.2	102.2	152.2	177.2
I_0/N	0.4119	0.2214	0.06376	0.01900	0.008030	0.007800
I_1/N	0.9168	0.6842	0.3894	0.2377	0.1501	0.1229
$I_{u\alpha}{}^2/N$	0.6957	0.4765	0.1784	0.06934	0.02298	0.01271
$I_{u\beta}{}^2/N$	0.1468	0.1350	0.09468	0.06429	0.04680	0.04040
$I_{v\alpha}{}^2/N$	−0.04666	−0.06042	−0.06825	−0.06402	−0.06627	−0.06865
$I_{v\beta}{}^2/N$	0.1778	0.1765	0.1503	0.1172	0.08606	0.07980
Q_{D2}/N	0.1263	0.1090	0.05861	0.02349	0.003131	−0.002791
$Q_{u\alpha}{}^2/N$	−0.009515	−0.01008	−0.009650	−0.009809	−0.01148	−0.01159
$Q_{u\beta}{}^1/N$	−0.001016	−0.001636	−0.003518	−0.004953	−0.005652	−0.005781
$Q_{v\alpha}{}^1/N$	0.005424	0.007031	0.01039	0.01078	0.01008	0.00976
$Q_{v\beta}{}^1/N$	0.0971	0.0842	0.06643	0.05520	0.04414	0.03962
$Q_{u\alpha}{}^3/N$	0.08865	0.07131	0.05013	0.04095	0.03564	0.03412
$Q_{u\beta}{}^3/N$	0.06334	0.06158	0.04568	0.02656	0.01041	0.00532
$Q_{v\alpha}{}^3/N$	−0.01582	−0.02299	−0.03613	−0.04333	−0.04623	−0.04769
$Q_{v\beta}{}^3/N$	0.02344	0.02825	0.03459	0.03536	0.03360	0.03301
M'_D/N	0.01876	0.02170	0.02252	0.01865	0.01386	0.01194
$M_{u\alpha}{}^1/N$	−0.007066	−0.007005	−0.006470	−0.005742	−0.004845	−0.004693
$M_{u\beta}{}^1/N$	−0.01180	−0.01318	−0.01460	−0.01436	−0.01337	−0.01288
$M_{v\alpha}{}^1/N$	0.007214	0.007033	0.006467	0.005728	0.005020	0.004706
$M_{v\beta}{}^1/N$	0.01269	0.01379	0.01453	0.01417	0.01340	0.01296
M_s/N	0.09462	0.06569	0.03330	0.01927	0.01236	0.01054
M_D/N	0.05682	0.06420	0.06899	0.06420	0.05439	0.04924
$\delta^1{}_0$	9.246	11.37	10.57	5.633	−1.614	−5.239
$\delta^1{}_1$	−8.012	−11.32	−17.96	−23.77	−29.39	−31.87
$\delta^\alpha{}_2$	4.806	6.738	9.031	7.602	3.790	1.793
$\delta^\beta{}_2$	−0.3603	−0.8021	−2.111	−3.571	−5.916	−7.342
ϵ_2	−11.49	−15.06	−20.31	−24.56	−33.48	−39.78
$\delta^2{}_2$	5.968	11.61	26.45	38.52	45.28	46.84
$\delta^\alpha{}_1$	69.16	58.18	38.05	21.85	8.096	2.670
$\delta^\beta{}_1$	−5.373	−9.059	−17.71	−25.69	−32.84	−35.66
ϵ_1	3.294	2.911	2.044	1.328	0.8368	0.6629
$\delta^\alpha{}_3$	−0.9110	−1.977	−5.307	−8.239	−10.86	−11.89
$\delta^\beta{}_3$	0.4105	1.297	4.749	8.010	10.69	11.46
ϵ_3	−35.74	−41.91	−50.75	−56.96	−61.45	−63.22
K_0	40.99	33.64	18.20	4.385	−7.901	−12.81
K_2	1.578	2.952	6.873	11.42	15.42	16.72

[a] The number N is the normalization constant defined as in the paper by Austern. It is such that in the present notation $\lim(U/N)=e^{-\gamma r}$ for $r\to\infty$. The value of N^2 needed for the calculation of cross section and polarization for Potential I is 7.491×10^{12} cm^{-1}. The numbers tabulated refer to the quantities L_0, L_1, $\cdots$ by Austern except for the additional ones listed here.

first used by Case and Pais.[25] The potential thus obtained may be written in the form

$$V=AV_C(x)+BV_T(x)S_{12}+CV_{\mathrm{L.S}}(x)\mathbf{L}\cdot\mathbf{S}, \quad (19)$$

where

$$V_{LS}=\frac{d}{(r/r_0)d(r/r_0)}\left[\frac{\exp(-r/r_0)}{r/r_0}\right],$$

$$r_0=1.07\times10^{-13}\text{ cm}, \quad (19.1)$$

and $V_C(x)$, $V_T(x)$ are dimensionless functions of $x=\mu r$, $\mu=\mu_0=m_\pi c/\hbar$, $r=|\mathbf{r}_p-\mathbf{r}_n|$. The original Signell-Marshak values correspond essentially to

$$A/Mc^2=1.065835,\quad B/Mc^2=1.065835,\quad \mu=\mu_0,$$
$$C=30\text{ Mev}. \quad (19.2)$$

The work referred to previously[8] has shown that improvements in the fit to experimental data may be obtained by adjusting the depth parameters A, B, and C and the range $1/\mu$ of the Signell-Marshak potential and including infinite repulsive cores for certain states without changing the shape of the central, tensor, and spin-orbit potentials.[26] Two such improved potentials were used in the present work; they will be referred to as Potentials I and II.

The following parameters were used for both potentials.

For triplet-odd states the Signell-Marshak values as in (19.2) were used with

$$\text{Core radius } x_C=0.408 \quad \text{(triplet-odd)}. \quad (20.1)$$

The repulsive core is necessary in these states as otherwise the $^3P_2$ state would be bound. For the singlet even states the values

$$A/(Mc^2)=0.78875,\quad \mu=0.84\mu_0,$$
$$\text{no core (singlet-even)} \quad (20.2)$$

were employed. The *distinction* between Potentials I and II entered in the treatment of *triplet-even* states. Since there is no direct evidence for the existence of

[25] K. M. Case and A. Pais, Phys. Rev. **80**, 203 (1950).

[26] Actually there was a slight change in the shape of the spin-orbit potential since a rounded cutoff was used for $r<r_c$, $r_c=\hbar/Mc=0.21\times10^{-13}$ cm, instead of the straight cutoff used by Signell and Marshak. However, the effect of this change is negligible since the spin-orbit force is important only in the triplet odd states and in these states an infinite repulsive core was used out to $x=0.408$.

TABLE IV. Radial matrix elements in units of 10^{-13} cm and additional phase shifts in degrees from Potential II.

E_γ (Mev)[a]	22.2	32.2	62.2	102.2	152.2	177.2
I_0/N	0.4163	0.2174	0.06210	0.02606	0.01395	0.01138
I_1/N	0.9076	0.6785	0.3852	0.2303	0.1422	0.1159
$I_{u\alpha}{}^2/N$	0.6924	0.4719	0.1745	0.06755	0.02156	0.00931
$I_{u\beta}{}^2/N$	0.1462	0.1339	0.09312	0.06321	0.04557	0.03895
$I_{v\alpha}{}^2/N$	−0.04244	−0.05595	−0.06641	−0.06358	−0.06396	−0.06504
$I_{v\beta}{}^2/N$	0.1593	0.1613	0.1458	0.1175	0.08400	0.07658
Q_{D2}/N	0.1276	0.1079	0.05935	0.02344	0.003673	−0.001460
$Q_{u\alpha}{}^1/N$	−0.008687	−0.009948	−0.01126	−0.01048	−0.009270	−0.008877
$Q_{u\beta}{}^1/N$	−0.001076	−0.001694	−0.003601	−0.005181	−0.006095	−0.006302
$Q_{v\alpha}{}^1/N$	0.007283	0.006859	0.01016	0.01054	0.009526	0.009067
$Q_{v\beta}{}^1/N$	0.09887	0.08601	0.06414	0.05808	0.04895	0.04762
$Q_{u\alpha}{}^3/N$	0.06230	0.05615	0.03902	0.03259	0.03017	0.02932
$Q_{u\beta}{}^3/N$	0.08938	0.07457	0.05000	0.02674	0.008868	0.003142
$Q_{v\alpha}{}^3/N$	−0.01985	−0.02504	−0.03869	−0.04928	−0.05403	−0.05483
$Q_{v\beta}{}^3/N$	0.01471	0.02040	0.02545	0.02548	0.02256	0.02074
M'_D/N	0.01750	0.02056	0.02148	0.01733	0.01198	0.01087
$M_{u\alpha}{}^1/N$	−0.006292	−0.006077	−0.005335	−0.004482	−0.003687	−0.003329
$M_{u\beta}{}^1/N$	−0.01135	−0.01280	−0.01410	−0.01383	−0.01291	−0.01240
$M_{v\alpha}{}^1/N$	0.008512	0.006001	0.005278	0.004485	0.003673	0.003327
$M_{v\beta}{}^1/N$	0.01155	0.01283	0.01376	0.01383	0.01304	0.01240
M_s/N	0.09113	0.06247	0.03056	0.01681	0.01006	0.008313
M_D/N	0.05313	0.06120	0.06722	0.06133	0.05014	0.04490
$\delta^2{}_2$	5.964	11.52	25.45	36.03	41.43	42.51
$\delta^\alpha{}_1$	67.52	56.22	35.46	18.75	4.560	−1.032
$\delta^\beta{}_1$	−6.263	−10.54	−20.70	−30.25	−39.03	−42.54
ϵ_1	3.938	2.284	0.6551	−0.9242	−2.426	−3.067
$\delta^\alpha{}_3$	−0.7140	−1.504	−4.876	−8.072	−11.21	−12.59
$\delta^\beta{}_3$	0.9041	2.137	7.745	13.74	19.19	21.10
ϵ_3	−55.38	−54.22	−61.85	−68.83	−74.23	−76.41

[a] The value of N^2 needed for the calculation of cross section and polarization for Potential II is 7.681×10^{12} cm^{-1}.

the spin-orbit potential for these states the spin-orbit potential depth parameter was given the value $C=0$ for Potential I and the value 30 Mev as in (19.2) for Potential II. The central and tensor depth parameters were adjusted to give correct binding energy and scattering length in both cases. This resulted[8] in

$$A/Mc^2=B/Mc^2=1.08017,\quad C=0,\quad \mu=\mu_0,\quad \text{no core (Potential I)} \tag{20.3}$$

$$A/Mc^2=B/Mc^2=1.17723,\quad C=30\text{ Mev},\quad \mu=\mu_0,\quad \text{no core (Potential II)}. \tag{20.4}$$

The amount of D wave in the ground-state wave function predicted by these two potentials is 6.8% for Potential I and 6.1% for Potential II. Numerical values of the radial integrals are given in Tables III and IV. The radial matrix elements are defined essentially in accordance with the convention used by Austern,[2] i.e., as in Eqs. (13.7) to (13.10) of the text above. The values entered in the tables are not directly the matrix elements of Eqs. (13.7) to (13.10) but these quantities divided by a number N defined in the footnote to Tables III and IV. In the zero-range approximation $N^2=2\gamma$.

VII. NUMERICAL CALCULATIONS

As mentioned in the Introduction, five approximations as listed in Table V were used in order to investigate the relative importance of the various transitions. The calculations were carried out using the amplitudes given in Sec. V. Two procedures were followed. In the first, numerical values of the matrix elements and phase-shifts were substituted into the amplitudes and formulas for $\sigma(\theta,\varphi)$ and $\sigma P'(\theta,\varphi)$ obtained in terms of θ and φ. In the second procedure, the amplitudes were reduced completely to complex numbers by substituting values for θ and φ and $\sigma(\theta,\varphi)$ and $\sigma P'(\theta,\varphi)$ obtained numerically. The values of θ and φ used in the second procedure were always chosen so as to provide a complete numerical check on the formulas.

The components of polarization were computed along the directions of the axes of the primed coordinate system shown in Fig. 1.

In approximation A and B,

$$\sigma=a_E+b_E\sin^2\theta(1+\cos2\varphi), \tag{21.1}$$

$$\sigma P_x'=B_E\sin\theta\sin2\varphi, \tag{21.2}$$

$$\sigma P_y'=B_E\sin\theta\cos\theta(1+\cos2\varphi), \tag{21.3}$$

TABLE V. Transitions added in successive approximations.

Transitions	Approximations
$E1$: $(^3S_1+^3D_1)\rightarrow{}^3P_0$, $^3P_1$, $(^3P_2+^3F_2)$	A[a] and B
$M1$: $(^3S_1+^3D_1)\rightarrow{}^1S_0$, $^1D_2$	C
$M1$: $(^3S_1+^3D_1)\rightarrow(^3S_1+^3D_1)$, $^3D_2$	D
$E2$: $(^3S_1+^3D_1)\rightarrow(^3D_1+^3S_1)$, $^3D_2$, $(^3D_3+^3G_3)$	E

[a] For A the $^3P_2-{}^3F_2$ coupling is neglected. This approximation uses radial functions calculated with coupling and substitutes the P and F parts of eigenstates originating adiabatically from pure P and F functions for these functions.

TABLE VI. Angular distribution and polarization parameters in microbarns/steradian for protons in approximations A and B. Suffixes 1 and 2 correspond to Potentials I and II.

	Approximation A						Approximation B					
E_γ (Mev)	a_{1E}	b_{1E}	B_{1E}	a_{2E}	b_{2E}	B_{2E}	a_{1E}	b_{1E}	B_{1E}	a_{2E}	b_{2E}	B_{2E}
22.2	4.68	50.7	5.55	4.30	51.6	5.46	4.92	51.0	5.25	4.52	51.9	5.18
32.2	5.06	28.1	5.28	4.81	28.4	5.19	5.42	28.3	4.87	5.15	28.6	4.80
62.2	4.46	6.40	3.00	4.42	6.34	2.96	5.31	6.74	2.87	5.28	6.67	2.84
102.2	3.09	1.40	1.50	3.01	1.42	1.48	3.95	1.79	1.60	3.86	1.85	1.58
152.2	1.90	0.170	0.715	1.77	0.190	0.675	2.69	0.639	0.942	2.51	0.696	0.887
177.2	1.58	0.000	0.546	1.47	0.000	0.489	2.27	0.672	0.811	2.13	0.665	0.748

The z' component of **P** vanishes in these approximations. The quantities a_E, b_E, and B_E are tabulated for protons in Table VI for both the potentials. The subscripts on the coefficients indicate their origin in electric and magnetic transitions. In some of the latter improved approximations these designations are omitted in view of the complexity of the origin of the coefficients.

In approximation C,

$$\sigma=a_E+b_E\sin^2\theta(1+\cos2\varphi)+a_M+b_M\sin^2\theta\times(1-\cos2\varphi), \quad (22.1)$$

$$\sigma P_x'=B_E\sin\theta\sin^2\varphi+E_{EM}\sin\theta\cos\theta\sin2\varphi, \quad (22.2)$$

$$\sigma P_y'=B_E\sin\theta\cos\theta(1+\cos2\varphi)+A_{EM}\sin\theta+E_{EM}\sin\theta\cos2\varphi, \quad (22.3)$$

$$\sigma P_z'=G_{EM}\sin^2\theta\sin2\varphi, \quad (22.4)$$

with additional parameters for protons as in Table VII.

In approximation D

$$\sigma=a+b\sin^2\theta+c\cos\theta+f\sin^2\theta\cos^2\varphi. \quad (23.1)$$

These parameters are given for protons in Table VIII. The three components of polarization in approximation D can be expressed as

$$\sigma P_x'=(F\sin\theta+E\sin\theta\cos\theta)\sin2\varphi, \quad (23.2)$$

$$\sigma P_y'=A\sin\theta+B\sin\theta\cos\theta+(E\sin\theta+F\sin\theta\cos\theta)\times\cos2\varphi, \quad (23.3)$$

$$\sigma P_z'=G\sin^2\theta\sin2\varphi, \quad (23.4)$$

with the parameters for protons given in Table IX.

In approximation E,

$$\sigma=a+b\sin^2\theta\pm c\cos\theta\pm d\cos\theta\sin^2\theta+e\sin^2\theta\cos^2\theta+\cos2\varphi(f\sin^2\theta+d\cos\theta\sin^2\theta+e\sin^2\theta\cos^2\theta), \quad (24.1)$$

which gives for the total cross section

$$\sigma_T=4\pi a+\frac{8\pi}{3}b+\frac{8\pi e}{15}. \quad (24.2)$$

The + sign (− sign) refers to protons (neutrons).

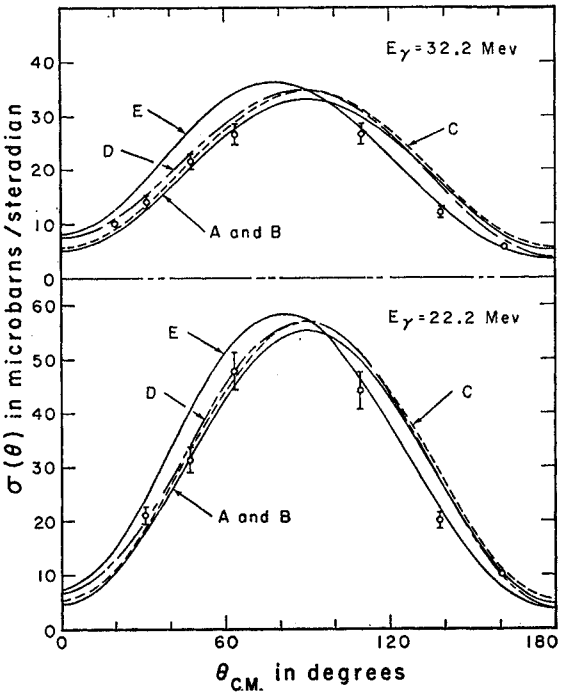

FIG. 2. Differential cross section for the D$(\gamma,n)p$ reaction with unpolarized gamma rays of energy 22.2 and 32.2 Mev in the laboratory system for Potential I. The experimental points are due to L. Allen, Jr.[3]

TABLE VII. Additional angular distribution and polarization parameters in microbarns/steradian for proton in approximation C. Suffixes 1 and 2 correspond to Potentials I and II.

E_γ Mev	a_{1M}	b_{1M}	A_{1EM}	E_{1EM}	G_{1EM}	a_{2M}	b_{2M}	A_{2EM}	E_{2EM}	G_{2EM}
22.2	0.308	0.737	−4.69	−1.22	−3.48	0.297	0.676	−4.57	−1.29	−3.32
32.2	0.0969	0.838	−2.99	0.330	−1.87	0.0896	0.779	−2.89	0.290	−1.75
62.2	0.0210	0.886	−1.40	1.38	−0.762	0.0375	0.834	−1.33	1.36	−0.715
102.2	0.0939	0.769	−0.853	1.33	−0.565	0.0990	0.690	−0.791	1.26	−0.547
152.2	0.124	0.579	−0.618	1.03	−0.486	0.118	0.483	−0.547	0.929	−0.460
177.2	0.121	0.492	−0.533	0.874	−0.473	0.112	0.399	−0.469	0.776	−0.438

TABLE VIII. Angular distribution parameters in microbarns/steradian for protons in approximation D. Suffixes 1 and 2 correspond to Potentials I and II.

E_γ Mev	a_1	b_1	c_1	f_1	a_2	b_2	c_2	f_2
22.2	5.35	51.7	1.49	50.2	4.93	52.5	1.36	51.2
32.2	5.68	29.1	1.85	27.6	5.40	29.3	1.72	27.9
62.2	5.58	7.56	2.04	5.89	5.54	7.44	1.94	5.87
102.2	4.29	2.51	1.63	1.06	4.18	2.50	1.54	1.20
152.2	3.04	1.16	1.21	0.105	2.84	1.13	1.08	0.262
177.2	2.62	1.11	1.09	0.231	2.44	1.01	0.960	0.315

These parameters and the total cross section are shown in Table X.

The three components of the polarization in approximation E can be expressed as

$$\sigma P_x' = (L\sin\theta + M\sin\theta\cos\theta + N\sin\theta\cos^2\theta)\sin 2\varphi, \quad (24.3)$$

$$\sigma P_y' = A\sin\theta + B\sin\theta\cos\theta + C\sin\theta\cos^2\theta + D\sin\theta\cos^3\theta + \cos 2\varphi(E\sin\theta + F\sin\theta\cos\theta + C\sin\theta\cos^2\theta + D\sin\theta\cos^3\theta), \quad (24.4)$$

$$\sigma P_z' = (G\sin^2\theta + H\cos\theta\sin^2\theta)\sin 2\varphi. \quad (24.5)$$

These coefficients are shown in Table XI. Since there is a difference between the proton and neutron coefficients for polarization, they have been distinguished by the designations (p) or (n). These parameters are not all independent since $E+C=M$. The results for unpolarized γ rays are obtained by integrating over φ. In this case P_x' and P_z' vanish as is required by parity conservation.

The different approximations for $\sigma(\theta)$ for the case of unpolarized γ rays using Potential I are shown for $E_\gamma = 22.2$ and 32.2 Mev in Fig. 2, for 62.2 and 102.2 in Fig. 3, and for 152.2 and 177.2 Mev in Fig. 4 together with some available experimental values.[27] The corresponding results for $P(\theta)$ are shown in Figs. 5, 6, and 7. No experimental values are available in this case.

[27] The published values in Allen's paper are raised by 9% in accordance with advice from Professor A. O. Hanson of the University of Illinois, based on his correspondence with Dr. L. Allen and J. E. Leiss and employment of a new National Bureau of Standards photon flux calibration. Thanks are due to the three gentlemen just mentioned for looking up old records and ascertaining the best absolute intensity calibration. The possibility of such an error in the calibration has already been noted in Allen's paper (reference 3).

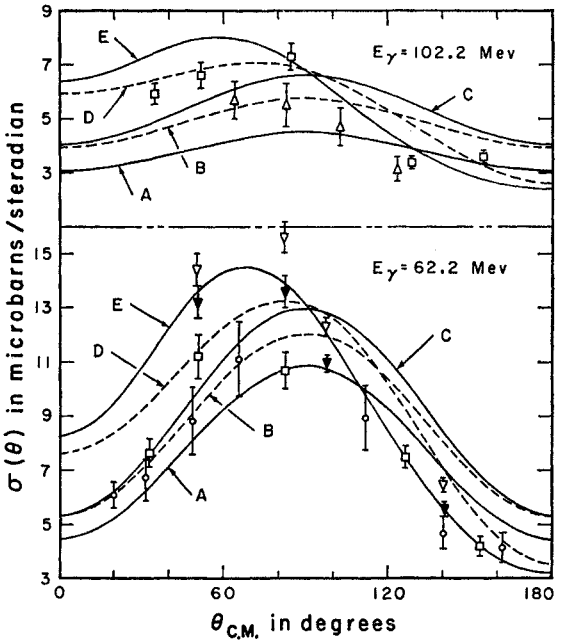

FIG. 3. Differential cross section for the $D(\gamma,n)p$ reaction with unpolarized gamma rays of energy 62.2 and 102.2 Mev in the laboratory system for Potential I. The experimental points of various investigators are represented as follows[3]: Circles for those of L. Allen, Jr., at 66 Mev; squares for those of E. A. Whalin, B. D. Schriever, and A. O. Hanson at 65 and 105 Mev; Triangles for those of J. C. Keck and A. V. Tollestrup at 105 Mev; open and solid inverted triangles for those of J. A. Galey at 60 and 65 Mev, respectively.

Although graphs of the results for the two intermediate energies, 62.2 and 102.2 Mev, have been published previously,[4] the inadvertent omission of $E2(^3D_1 \rightarrow {}^3D_J)$, $J=1$, 2, 3 transitions together with an incorrect sign of $M1(^3D_1 \rightarrow {}^3D_2)$ matrix element combined with a 9% increase[27] in Allen's cross sections produces a nonnegligible difference in the graphs which are being, therefore, published in revised form.

The calculations using Potential II, as is evident from the tables, give essentially the same results as those for Potential I. It is found that most of the difference enters in approximation E and that this difference increases with increasing energy. In Figs. 8 and 9 the results for angular distribution and polarization of the two potentials at 152 and 177 Mev γ-ray energies in approximation E are compared.

TABLE IX. Polarization parameters in microbarns/steradian for protons in approximation D. Suffixes 1 and 2 correspond to Potentials I and II.

E_γ Mev	A_1	B_1	E_1	F_1	G_1	A_2	B_2	E_2	F_2	G_2
22.2	−3.85	5.01	−0.076	5.59	−2.13	−3.73	4.95	−0.157	5.41	−1.98
32.2	−2.56	4.64	1.19	5.10	−0.446	−2.55	4.59	1.01	5.01	−0.393
62.2	−1.55	2.66	1.80	3.07	0.493	−1.54	2.65	1.68	3.03	0.474
102.2	−1.11	1.39	1.62	1.80	0.222	−1.05	1.38	1.50	1.78	0.155
152.2	−0.778	0.738	1.31	1.15	−0.130	−0.670	0.693	1.18	1.08	−0.179
177.2	−0.658	0.615	1.15	1.01	−0.243	−0.571	0.565	1.02	0.930	−0.252

TABLE X. Angular distribution parameters and total cross section for protons and neutrons in approximation E. The parameters are in microbarns/steradian and σ_T is in microbarns. Suffixes 1 and 2 correspond to Potentials I and II.

E_γ (Mev)	a_1	b_1	c_1	d_1	e_1	f_1	$\sigma_T(1)$	a_2	b_2	c_2	d_2	e_2	f_1	$\sigma_T(2)$
22.2	5.44	51.5	1.93	15.8	1.22	50.2	502	5.01	52.4	1.75	16.2	1.27	51.2	504
32.2	5.81	29.0	2.38	11.1	1.08	27.5	318	5.51	29.2	2.19	11.2	1.10	27.8	316
62.2	5.73	7.37	2.52	4.45	0.680	5.84	135	5.66	7.28	2.42	4.23	0.630	5.83	133
102.2	4.40	2.37	1.98	2.00	0.403	1.02	75.9	4.27	2.39	1.93	1.76	0.340	1.18	74.3
152.2	3.13	1.06	1.49	1.02	0.219	0.0823	48.6	2.89	1.07	1.41	0.821	0.173	0.258	4.56
177.2	2.70	1.01	1.37	0.880	0.174	0.210	42.7	2.48	0.974	1.28	0.706	0.140	0.317	39.6

VIII. DISCUSSION OF THE RESULTS

The results for σ in approximation A are reasonably consistent with those of de Swart and Marshak for a slightly different potential.[5] The values of **P** with unpolarized 62-Mev gamma rays are qualitatively similar to those of Czyż and Sawicki from a less accurate calculation.[10] The results reported on here compare very well with those of de Swart, Czyż, and Sawicki[12] if the $M1$ to triplet transitions are not included and certain other interference terms also not considered by them are removed from results reported on here.

Comparison of approximations C and D for σ shows appreciable effects of the inclusion of $M1$ transitions to triplet states above 60 Mev and for **P** these effects are seen to be major. Similarly the effect of including $E2$ is appreciable for σ even at 32 Mev and is non-negligible for **P** at 62 Mev.

With regard to the comparison of σ with experimental values, the calculated results at the lower energies appear to be somewhat too large. It may be relevant to note that for some of the experimental data the results of the various investigators do not always agree within the limits of the claimed experimental error.

At 152 Mev the results agree rather well with experiment while at 177 Mev the calculated values are too small. However, it is felt that comparison with experiment at these high energies is not very significant since the semiphenomenological potentials give very poor values for scattering at these energies and the form of the electromagnetic interaction used is probably not

TABLE XI. Polarization parameters in microbarns/steradian for protons and neutrons in approximation E. Suffixes 1 and 2 correspond to Potentials I and II.

E_γ (Mev)	22.2	32.2	62.2	102.2	152.2	177.2
$A_1(p)$	−3.87	−2.55	−1.58	−1.21	−0.915	−0.809
$A_1(n)$	−5.50	−3.44	−1.22	−0.492	−0.321	−0.257
$B_1(p)$	4.66	4.48	2.60	1.24	0.560	0.441
$B_1(n)$	6.03	5.35	2.92	1.54	0.912	0.802
$C_1(p)$	0.738	0.819	0.845	0.753	0.590	0.553
$C_1(n)$	−0.738	−0.819	−0.845	−0.753	−0.590	−0.553
$D_1(p)$	−0.0103	−0.0241	−0.0027	0.055	0.085	0.0897
$D_1(n)$	−0.0103	−0.0241	−0.0027	0.055	0.085	0.0897
$E_1(p)$	−0.170	1.07	1.60	1.41	1.13	0.984
$E_1(n)$	−2.28	−0.411	1.16	1.24	0.934	0.763
$F_1(p)$	5.22	4.95	2.96	1.72	1.14	1.03
$F_1(n)$	5.57	5.02	2.85	1.51	0.823	0.699
$G_1(p)$	−2.21	−0.561	0.409	0.220	−0.0821	−0.186
$G_1(n)$	−4.74	−3.19	−1.93	−1.35	−0.889	−0.760
$H_1(p)$	−0.477	−0.295	−0.106	−0.119	−0.157	−0.162
$H_1(n)$	0.789	0.636	0.434	0.349	0.274	0.233
$L_1(p)$	5.46	5.10	3.07	1.75	1.05	0.897
$L_1(n)$	5.07	4.71	2.83	1.65	1.01	0.874
$M_1(p)$	0.567	1.89	2.45	2.17	1.72	1.54
$M_1(n)$	−3.02	−1.23	0.311	0.490	0.343	0.211
$N_1(p)$	−0.252	−0.170	−0.111	0.027	0.174	0.222
$N_1(n)$	0.490	0.288	0.0186	−0.092	−0.101	−0.086
$A_2(p)$	−3.82	−2.62	−1.65	−1.22	−0.834	−0.730
$A_2(n)$	−5.33	−3.17	−1.03	−0.365	−0.260	−0.209
$B_2(p)$	4.59	4.40	2.52	1.16	0.484	0.370
$B_2(n)$	5.95	5.23	2.88	1.56	0.947	0.825
$C_2(p)$	0.874	0.966	0.938	0.789	0.535	0.443
$C_2(n)$	−0.874	−0.966	−0.938	−0.789	−0.535	−0.4434
$D_2(p)$	0.0123	0.0076	0.0409	0.0967	0.0930	0.0837
$D_2(n)$	0.0123	0.0076	0.0409	0.0967	0.0930	0.0837
$E_2(p)$	−3.14	0.816	1.42	1.24	0.973	0.839
$E_2(n)$	−2.26	−0.236	1.30	1.28	0.885	0.712
$F_2(p)$	5.12	4.83	2.90	1.69	1.08	0.955
$F_2(n)$	5.50	4.91	2.76	1.41	0.723	0.601
$G_2(p)$	−2.05	−0.492	0.414	0.181	−0.115	−0.185
$G_2(n)$	−4.60	−3.01	−1.85	−1.28	−0.805	−0.693
$H_2(p)$	−0.428	−2.64	−0.112	−0.143	−0.230	−0.262
$H_2(n)$	0.751	0.597	0.395	0.341	0.292	0.281
$L_2(p)$	5.34	4.97	3.02	1.70	0.970	0.812
$L_2(n)$	5.02	4.66	2.83	1.66	0.971	0.835
$M_2(p)$	0.560	1.77	2.36	2.03	1.51	1.28
$M_2(n)$	−3.14	−1.20	0.359	0.495	0.350	0.269
$N_2(p)$	−0.232	−0.164	−0.0790	0.0871	0.207	0.227
$N_2(n)$	0.493	0.261	−0.0227	−0.155	−0.155	−0.151

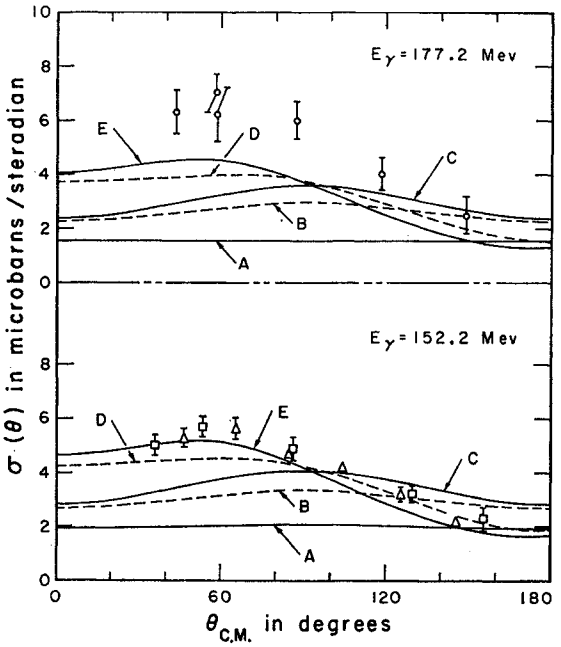

FIG. 4. Differential cross section for the $D(\gamma,n)p$ reaction with unpolarized gamma rays of energy 152.2 and 177.2 Mev in the laboratory system for Potential I. The experimental points of the various investigators are represented as follows[3]: squares for those of E. A. Whalin, B. D. Schriever and A. O. Hanson at 149 Mev; triangles for those of J. C. Keck and A. V. Tollestrup at 155 Mev; circles for those of D. R. Dixon and K. C. Bandtel at 182 Mev.

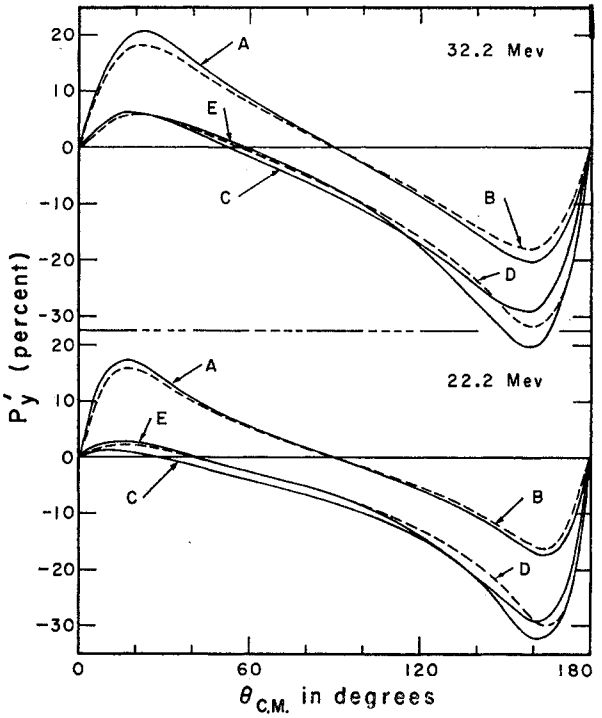

FIG. 5. Percentage polarization of protons from the D$(\gamma,n)p$ reaction with unpolarized gamma rays of energy 22.2 and 32.2 Mev in the laboratory system for Potential I.

valid so close to or above the meson threshold.[28] Furthermore, the effect of including retardation terms may produce appreciable effects at the higher energies.[29] Agreement with experiment appears to be better than could be expected considering uncertainties in the theory.

The relatively small difference between values calculated by means of Potentials I and II appears noteworthy in corroborating the view that the spin-orbit potential in triplet-even states may be absent. These findings are in agreement with the fact that the two potentials agree about equally well with scattering data.[8] It may be noted that at 22 and 32 Mev the results for σ depend almost entirely on $E1$ transitions, so that experiments on the polarization at these energies would be most useful in order to investigate the form of the $M1$ interaction.

ACKNOWLEDGMENTS

The authors desire to acknowledge the help of Dr. M. H. Hull, Jr., Dr. K. D. Pyatt, Jr., and Dr. C. R.

[28] Some of the more recent discussions of this problem may be found in papers by R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951); Phys. Rev. **81**, 710 (1951); J. G. Brennan and R. G. Sachs, Phys. Rev. **88**, 824 (1952); and L. Foldy, Phys. Rev. **92**, 178 (1953).

[29] S. H. Hsieh, Progr. Theoret. Phys. (Kyoto) **21**, 185, 211 (1959); M. Matsumoto, Progr. Theoret. Phys. (Kyoto) **23**, 597 (1960).

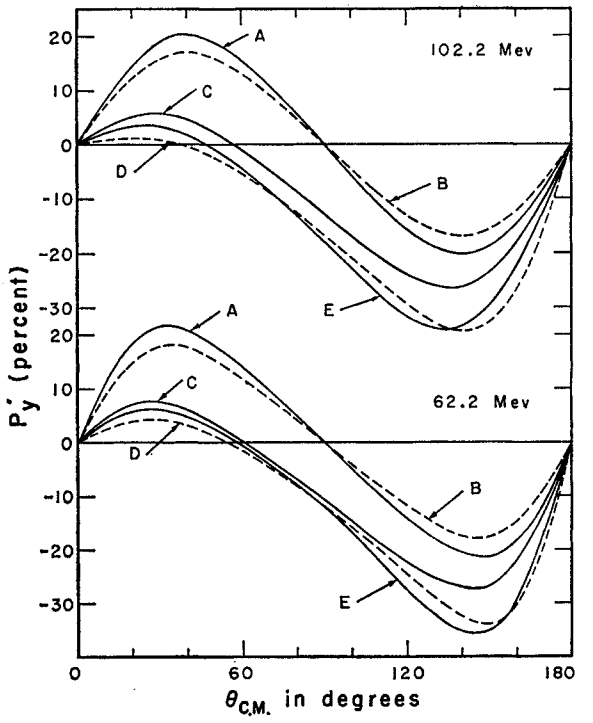

FIG. 6. Percentage polarization of protons from the D$(\gamma,n)p$ reaction with unpolarized gamma rays of energy 62.2 and 102.2 Mev in the laboratory system for Potential I.

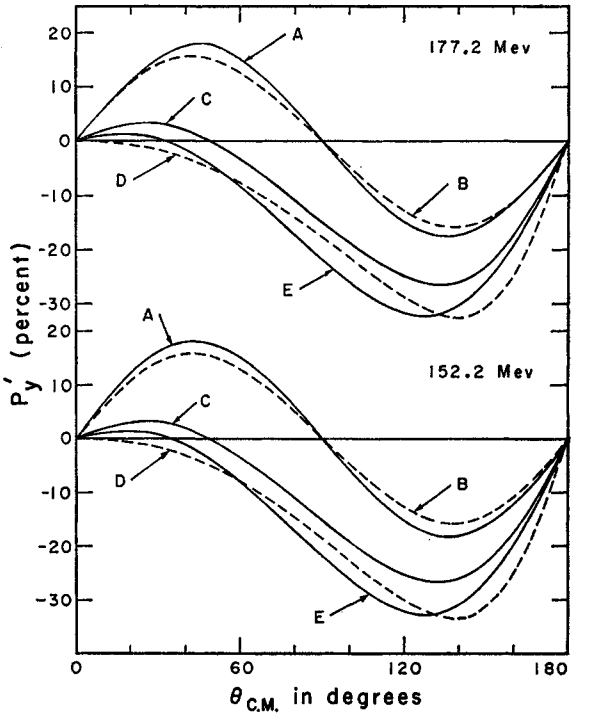

FIG. 7. Percentage polarization of protons from the D $(\gamma,n)p$ reaction with unpolarized gamma rays of energy 152.2 and 177.2 Mev in the laboratory system for Potential I.

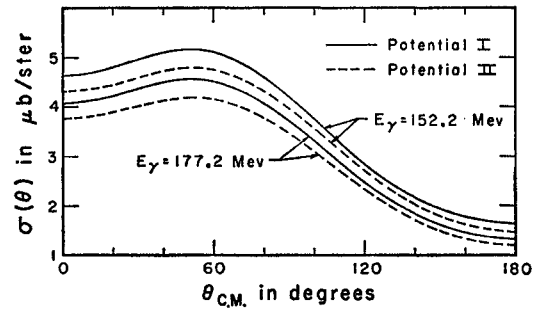

FIG. 8. Comparison of differential cross sections in approximation E for Potentials I and II. The energies of the unpolarized gamma rays are 152.2 and 177.2 Mev in the laboratory system.

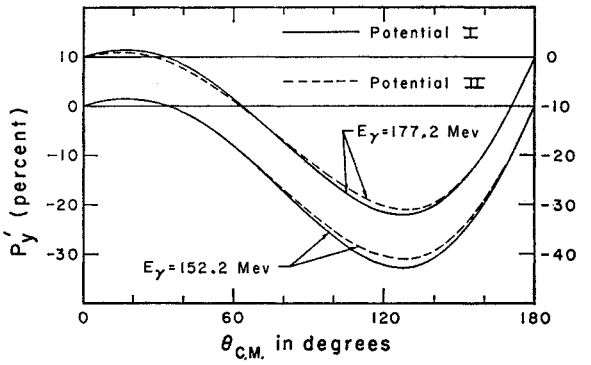

FIG. 9. Comparison of proton polarization in approximation E for Potentials I and II. The ordinates for the two unpolarized gamma-ray energies 152.2 and 177.2 Mev have been displaced to prevent overlapping of the graphs. The scale on the left- and right-hand sides belong to 155.2- and 177.2-Mev gamma-ray energies, respectively.

Fischer for providing the program used in the computation of the radial wave functions by means of the IBM 704 machine and to Dr. C. R. Fischer and Dr. K. D. Pyatt, Jr., for making the machine runs, to Dr. Loyal Durand for his participation in the early phases of this problem, to Miss J. Gibson for help in part of the numerical work, and to Mr. T. C. Degges for occasional computational assistance. A grant of IBM-704 machine time by the International Business Machine Corporation is gratefully acknowledged.

APPENDIX. RELATION BETWEEN n-p SCATTERING ENERGY AND EQUIVALENT GAMMA-RAY ENERGY

The n-p radial functions used in this work were computed for specified neutron kinetic energies in the laboratory scattering system. In this appendix a brief outline is given of the transformation to equivalent γ-ray energies in the laboratory photodisintegration system.

The photodisintegration will first be considered. Primed symbols refer to the center-of-mass (zero total momentum) system (K'), unprimed symbols to the laboratory system. The relative velocity of K and K' is $c\beta$, the system K' moving in the direction of the incident γ ray with respect to K.

For the γ-ray frequency one finds

$$\nu'=\nu(1-\beta)^{\frac{1}{2}}(1+\beta)^{-\frac{1}{2}}, \tag{A1}$$

where

$$\beta=\alpha/(1+\alpha); \quad \alpha=h\nu/M_Dc^2, \tag{A2}$$

and M_D is the deuteron mass

$$M_D=2M-(\epsilon/c^2). \tag{A3}$$

If one keeps only the first order term in β in Eq. (A1) one obtains the expression for the classical Doppler shift. However, β is of the order v^2/c^2 as seen from (A2) so that a consistent approximation is not obtained unless the particles are treated relativistically. A straightforward calculation shows that the energies of the nucleons in the center-of-mass system are given by

$$E_p'=E_n'=(1/2)M_Dc^2(1+2\alpha)^{\frac{1}{2}}. \tag{A4}$$

Considering the scattering, the relation between the neutron kinetic energy in the laboratory system of the scattering experiment and its total energy in the center-of-mass system turns out to be

$$T_{\text{lab}}{}^n/Mc^2=2[(E_n'/Mc^2)^2-1]. \tag{A5}$$

By kinetic energy is here meant the difference between the total and rest mass energies. Using Eq. (A4) one finds

$$h\nu=\epsilon(1-\epsilon/4Mc^2)/(1-\epsilon/2Mc^2) +(1/2)T_{\text{lab}}{}^n/(1-\epsilon/2Mc^2). \tag{A6}$$

Since $\epsilon/2Mc^2\cong 1/850$, the effects of this parameter are negligible and for practical purposes one may write

$$h\nu=\epsilon+T_{\text{lab}}{}^n/2. \tag{A7}$$