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# Inertial Moment of Large Many-Fermion Systems with Repulsive Interactions*

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The interaction effects on the moment of inertia of a large many-body fermion system moving under periodic boundary conditions have been explicitly derived in the second order of particle-particle coupling for the case of nonsingular repulsive interparticle forces. The derivation is facilitated by the use of a new Hamiltonian constructed by canonical transformation. This transformation is determined by requiring that it remove the compensating effects of pair excitation and that the new interaction operators of first and second order in the cranking field be also of first order in particle-particle coupling. A graphical analysis is presented in which those classes of diagrams which yield corrections vanishing relative to the rigid moment in the limit $L\rightarrow\infty$ are distinguished. Some remarks on the related problem of diamagnetism are made. The possible application of the method of canonical transformation to related many-body problems is indicated.

## INTRODUCTION

THE question of whether particle-particle interactions for which perturbation theory converges might alter the value of the noninteracting moment of inertia of a large Fermi gas was recently investigated by Amado and Brueckner.[1] It was found[1] that interaction effects cancelled in the lowest order of perturbation theory independent of potential form (tacitly assuming a nonsingular potential) in the case of periodic boundary conditions. In subsequent discussions of the cranking moment of such a system,[2] the formal machinery necessary for a systematic perturbation-theoretic analysis of interaction effects was simplified extensively and it was shown that simple effective mass corrections were just compensated by particle-hole correlation in every order,[2] a result which considerably extended the previous first-order calculation. However, since the random-phase diagrams[3] associated with the usual pair approximation[2,4] to collective excitation form only a subset of all graphs which might be expected to arise in a systematic treatment of particle-particle interaction, it becomes of interest to investigate the contribution to the moment of the residuum of diagrams which lie outside this approximation. For example, cancellation of these graphs (or, properly, their contribution to the inertial moment) is essential in extending the result of reference 1 to the next order in particle-particle coupling. Of course, such an extension would lend weight to the contention[5] that cancellation of interaction effects persists to all finite orders of perturbation theory.

The demonstration below that cancellation of interaction effects does *not* occur in the second order in particle-particle coupling forms the substance of this paper. Our result is independent of the details of the potential provided it be nonsingular and repulsive. A secondary result of some formal interest is the presentation of a method of canonical transformation[6] leading to a weak-field Hamiltonian from which compensating pair diagrams have been eliminated. This Hamiltonian yields all the fourth-order corrections (second order in particle-particle coupling and second order in the weak external field) in the second order of perturbation theory.

After some formal preliminaries (Sec. II), we proceed to a derivation of the transformed Hamiltonian (Sec. III) and the second-order interaction corrections to the inertial moment (Sec. IV). A graphical analysis of vanishing and nonvanishing classes of higher order corrections is presented in Sec. V, and some useful comments on the relationship of this work to the problem of diamagnetism are made in Sec. VI.

## FORMAL PRELIMINARIES

The Hamiltonian describing an interacting many-fermion system coupled to a weak external field may customarily be written in the form[7] (taking $\hbar=1$),

$$H=H_t-\lambda H_i, \tag{2.1}$$

$$H_t=H_0+\xi H_v, \tag{2.2}$$

$$H_0=\sum_{\mathbf{k}\sigma}\epsilon_{\mathbf{k}}c_{\mathbf{k}\sigma}{}^{\dagger}c_{\mathbf{k}\sigma}, \tag{2.3}$$

$$H_v=\frac{1}{2\Omega}\sum_{\mathbf{p}'\mathbf{q}'\mathbf{p}\mathbf{q}}\sum_{\sigma\sigma'}v_{\mathbf{p}'\mathbf{q}';\mathbf{q}\mathbf{p}}c_{\mathbf{p}'\sigma}{}^{\dagger}c_{\mathbf{q}'\sigma'}{}^{\dagger}c_{\mathbf{p}\sigma'}c_{\mathbf{q}\sigma}, \tag{2.4}$$

$$H_i=\sum_{\mathbf{k}\neq\mathbf{k}',\sigma}a_{\mathbf{k}'\mathbf{k}}c_{\mathbf{k}'\sigma}{}^{\dagger}c_{\mathbf{k}\sigma}, \tag{2.5}$$

where $\Omega$ is the quantization volume and $\epsilon_{\mathbf{k}}$, the kinetic

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[1] R. D. Amado and K. A. Brueckner, Phys. Rev. **115**, 778 (1959).

[2] R. M. Rockmore, Phys. Rev. **116**, 469 (1959), and Phys. Rev. **118**, 1645 (1960). We shall refer to these papers as I and II, respectively.

[3] K. Sawada, K. Brueckner, N. Fukuda, and R. Brout, Phys. Rev. **108**, 507 (1957).

[4] R. M. Rockmore, Phys. Rev. **114**, 941 (1959).

[5] A. Bohr and A. B. Migdal (private communication to R. Amado and K. Brueckner).

[6] Our canonical transformation is closely related to an analogous transformation in coordinate space introduced by Gross in his treatment of the collective rotations of nuclei [E. P. Gross, Nuclear Phys. **14**, 389 (1959)].

[7] Our notation follows that of I, II, and K. Sawada, Phys. Rev. **119**, 2090 (1960).

energy, is given by $\epsilon_k = k^2/2M$; the parameters $\lambda$, $\xi$ are characteristic of particle-field and particle-particle coupling, respectively. Our notation for the matrix element $v_{\mathbf{p}'\mathbf{q}';\mathbf{q}\mathbf{p}}$ is understood to imply $\mathbf{p}'+\mathbf{q}'=\mathbf{p}+\mathbf{q}$. In the case of the cranking interaction, the Fourier transform of the external field, $a_{\mathbf{k}'\mathbf{k}}$, is replaced with that of the $z$ component of the angular momentum, $(L_z)_{\mathbf{k}'\mathbf{k}} = (L_z)_{\mathbf{k}+\mathbf{p},\mathbf{k}}(\delta_{\mathbf{pr}}-\delta_{\mathbf{ps}})$ in the weak-field interaction, $H_i$. The two types of terms $(L_z)_{\mathbf{k}+\mathbf{r},\mathbf{k}}$ and $(L_z)_{\mathbf{k}+\mathbf{s},\mathbf{k}}$ contribute incoherently to the inertial moment,[8] so that it is sufficient to consider only

$$(H_i)_{xy} = \sum_{r\neq 0}\sum_{\mathbf{k}\sigma} (L_z)_{\mathbf{k}+\mathbf{r},\mathbf{k}} c_{\mathbf{k}+\mathbf{r},\sigma}{}^{\dagger} c_{\mathbf{k}\sigma}$$

$$= \sum_{r\neq 0}\sum_{\mathbf{k}\sigma} (L_z)_{\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{k}-\frac{1}{2}\mathbf{r}} c_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger} c_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma}; \quad (2.6)$$

consequently, our discussion will apply to $\mathcal{I}_{xy}$. Whenever convenient we also make the usual separation of $c_{\mathbf{p}\sigma}{}^{\dagger}(c_{\mathbf{p}\sigma})$ into particle and hole operators according to the definition

$$c_{\mathbf{p}\sigma}{}^{\dagger} = a_{\mathbf{p}\sigma}{}^{\dagger}, \quad p > P_F \qquad (2.7a)$$

$$= b_{\mathbf{p}\sigma}, \quad p < P_F. \qquad (2.7b)$$

One may divide $H_v$ into two parts, $H_v^{(1)}$ and $H_v^{(2)}$, with

$$H_v^{(1)} = \frac{1}{2\Omega}\sum_{\mathbf{p}'\mathbf{q}'\mathbf{p}\mathbf{q}\sigma} \mathcal{V}_{\mathbf{p}'\mathbf{q}';\mathbf{q}\mathbf{p}} c_{\mathbf{p}'\sigma}{}^{\dagger} c_{\mathbf{q}'\sigma}{}^{\dagger} c_{\mathbf{p}\sigma} c_{\mathbf{q}\sigma}, \qquad (2.8)$$

$$\mathcal{V}_{\mathbf{ij};\mathbf{kl}} = \tfrac{1}{4}(v_{\mathbf{ij};\mathbf{kl}} - v_{\mathbf{ij};\mathbf{lk}} - v_{\mathbf{ji};\mathbf{kl}} + v_{\mathbf{ji};\mathbf{lk}}), \qquad (2.9)$$

$$H_v^{(2)} = \frac{1}{2\Omega}\sum_{\mathbf{p}'\mathbf{q}'\mathbf{p}\mathbf{q}(\sigma\neq\sigma')} \mathcal{V}_{\mathbf{p}'\mathbf{q}';\mathbf{q}\mathbf{p}}{}' c_{\mathbf{p}'\sigma}{}^{\dagger} c_{\mathbf{q}'\sigma'}{}^{\dagger} c_{\mathbf{p}\sigma'} c_{\mathbf{q}\sigma}, \qquad (2.10)$$

$$\mathcal{V}_{\mathbf{ij};\mathbf{kl}}{}' = \tfrac{1}{4}(v_{\mathbf{ij};\mathbf{kl}} + v_{\mathbf{ji};\mathbf{lk}} + v_{\mathbf{kl};\mathbf{ij}} + v_{\mathbf{lk};\mathbf{ji}}). \qquad (2.11)$$

Note that in the zero-range approximation, $\mathcal{V}(\mathbf{r},\mathbf{r}') = -(4\pi|a|/M)\delta(\mathbf{r}-\mathbf{r}')$, only the latter part, $H_v^{(2)}$, survives.

In summary, one has to deal with the Hamiltonian

$$H = H_0 + \xi H_v^{(1)} + \xi H_v^{(2)} - \lambda (H_i)_{xy}, \qquad (2.12)$$

where

$$H_0 = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} a_{\mathbf{k}\sigma}{}^{\dagger} a_{\mathbf{k}\sigma} - \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} b_{\mathbf{k}\sigma}{}^{\dagger} b_{\mathbf{k}\sigma}, \qquad (2.13)$$

$$H_v^{(1)} = \frac{1}{2\Omega}\sum_{\mathbf{p}'\mathbf{q}'\mathbf{p}\mathbf{q}\sigma} \mathcal{V}_{\mathbf{p}'\mathbf{q}';\mathbf{q}\mathbf{p}} (a_{\mathbf{p}'\sigma}{}^{\dagger} + b_{\mathbf{p}'\sigma})(a_{\mathbf{q}'\sigma}{}^{\dagger} + b_{\mathbf{q}'\sigma})(a_{\mathbf{p}\sigma} + b_{\mathbf{p}\sigma}{}^{\dagger})(a_{\mathbf{q}\sigma} + b_{\mathbf{q}\sigma}{}^{\dagger}), \qquad (2.14)$$

$$H_v^{(2)} = \frac{1}{2\Omega}\sum_{\mathbf{p}'\mathbf{q}'\mathbf{p}\mathbf{q}}\sum_{\sigma\neq\sigma'} \mathcal{V}_{\mathbf{p}'\mathbf{q}';\mathbf{q}\mathbf{p}}{}' (a_{\mathbf{p}'\sigma}{}^{\dagger} + b_{\mathbf{p}'\sigma})(a_{\mathbf{q}\sigma} + b_{\mathbf{q}\sigma}{}^{\dagger})(a_{\mathbf{q}'\sigma'}{}^{\dagger} + b_{\mathbf{q}'\sigma'})(a_{\mathbf{p}\sigma'} + b_{\mathbf{p}\sigma'}{}^{\dagger}), \qquad (2.15)$$

$$(H_i)_{xy} = \sum_{r\neq 0}\sum_{\mathbf{k}\sigma} (L_z)_{\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{k}-\frac{1}{2}\mathbf{r}} (a_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger} + b_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma})(a_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma} + b_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger}), \qquad (2.16)$$

with

$$(L_z)_{\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{k}-\frac{1}{2}\mathbf{r}} = \frac{(-1)^{rL/2\pi}}{i}\frac{k_y}{(k_x+\frac{1}{2}r)-(k_x-\frac{1}{2}r)} = \frac{(-1)^{\Delta l}}{i}\frac{k_y}{r}. \qquad (2.17)$$

The "principal part"[2] of $(H_i)_{xy}$ is defined as

$$(H_i)_{xy}|_{\text{(principal part)}} = \sum_{r\neq 0}\sum_{\mathbf{k}\sigma} (L_z)_{\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{k}-\frac{1}{2}\mathbf{r}} (a_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger} b_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger} - b_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma} a_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma}) \equiv (H_i{}^P)_{xy}. \qquad (2.16a)$$

We also set down the equations of motion,[9]

$$[a_{\mathbf{p}\sigma}{}^{\dagger} b_{\mathbf{q}\sigma}{}^{\dagger}, H_0 + \xi H_v^{(1)}] = -(\tilde{\epsilon}_{\mathbf{p}} - \tilde{\epsilon}_{\mathbf{q}}) a_{\mathbf{p}\sigma}{}^{\dagger} b_{\mathbf{q}\sigma}{}^{\dagger} - A_{\mathbf{q},\mathbf{p}}{}^{\sigma}$$
$$+ \sum_{\mathbf{q}'} (a_{\mathbf{p}\sigma}{}^{\dagger} a_{\mathbf{q}'\sigma}{}^{\dagger} F_{\mathbf{q},\mathbf{q}'}{}^{\sigma\dagger} + a_{\mathbf{p}\sigma}{}^{\dagger} F_{\mathbf{q},\mathbf{q}'}{}^{\sigma\dagger} b_{\mathbf{q}'\sigma} + b_{\mathbf{q}\sigma}{}^{\dagger} b_{\mathbf{q}'\sigma} F_{\mathbf{p},\mathbf{q}'}{}^{\sigma} + b_{\mathbf{q}\sigma}{}^{\dagger} F_{\mathbf{p},\mathbf{q}'}{}^{\sigma} a_{\mathbf{q}'\sigma}), \qquad (2.18)$$

$$[a_{\mathbf{p}\sigma}{}^{\dagger} a_{\mathbf{q}\sigma}, H_0 + \xi H_v^{(1)}] = -(\tilde{\epsilon}_{\mathbf{p}} - \tilde{\epsilon}_{\mathbf{q}}) a_{\mathbf{p}\sigma}{}^{\dagger} a_{\mathbf{q}\sigma}$$
$$+ \sum_{\mathbf{q}'} (a_{\mathbf{p}\sigma}{}^{\dagger} a_{\mathbf{q}'\sigma}{}^{\dagger} F_{\mathbf{q},\mathbf{q}'}{}^{\sigma\dagger} + a_{\mathbf{p}\sigma}{}^{\dagger} F_{\mathbf{q},\mathbf{q}'}{}^{\sigma\dagger} b_{\mathbf{q}'\sigma} - F_{\mathbf{p},\mathbf{q}'}{}^{\sigma} a_{\mathbf{q}'\sigma} a_{\mathbf{q}\sigma} - b_{\mathbf{q}'\sigma}{}^{\dagger} F_{\mathbf{p},\mathbf{q}'}{}^{\sigma} a_{\mathbf{q}\sigma}), \qquad (2.19)$$

$$[b_{\mathbf{q}\sigma}{}^{\dagger} b_{\mathbf{p}\sigma}, H_0 + \xi H_v^{(1)}] = -(\epsilon_{\mathbf{p}} - \epsilon_{\mathbf{q}}) b_{\mathbf{q}\sigma}{}^{\dagger} b_{\mathbf{p}\sigma}$$
$$+ \sum_{\mathbf{q}'} (F_{\mathbf{q},\mathbf{q}'}{}^{\sigma\dagger} b_{\mathbf{q}'\sigma} b_{\mathbf{p}\sigma} + a_{\mathbf{q}'\sigma}{}^{\dagger} F_{\mathbf{q},\mathbf{q}'}{}^{\sigma\dagger} b_{\mathbf{p}\sigma} - b_{\mathbf{q}\sigma}{}^{\dagger} b_{\mathbf{q}'\sigma}{}^{\dagger} F_{\mathbf{p},\mathbf{q}'}{}^{\sigma} - b_{\mathbf{q}\sigma}{}^{\dagger} F_{\mathbf{p},\mathbf{q}'}{}^{\sigma} a_{\mathbf{q}'\sigma}), \qquad (2.20)$$

[8] Since the momentum transfers $\mathbf{r}$ and $\mathbf{s}$ are orthogonal.

[9] See K. Sawada, reference 7.

where

$$\tilde{\epsilon}_{\mathbf{p}}=\epsilon_{\mathbf{p}}+\frac{2\xi}{\Omega}\sum_{q<P_F}\mathcal{V}_{\mathbf{pq};\mathbf{pq}}, \tag{2.21}$$

$$A_{\mathbf{q},\mathbf{p}}{}^{\sigma}=\frac{2\xi}{\Omega}\sum_{\mathbf{p}'\mathbf{q}'}\mathcal{V}_{\mathbf{p}'\mathbf{q};\mathbf{q}'\mathbf{p}}c_{\mathbf{p}'\sigma}{}^{\dagger}c_{\mathbf{q}'\sigma}, \tag{2.22}$$

$$F_{\mathbf{r},\mathbf{s}}{}^{\sigma}=\frac{\xi}{\Omega}\sum_{\mathbf{pq}}c_{\mathbf{p}\sigma}{}^{\dagger}c_{\mathbf{q}\sigma}{}^{\dagger}\mathcal{V}_{\mathbf{pq};\mathbf{rs}}, \tag{2.23}$$

which we will find useful in our calculation of non-random-phase corrections to $(\mathcal{I}_{xy})_{\text{rigid}}$.

## WEAK-FIELD TRANSFORMATION; TRANSFORMED HAMILTONIAN

Consider the unitary weak-field transformation,

$$U(\lambda)=\exp(i\lambda\mathcal{S}), \tag{3.1}$$

where the Hermitian operator $\mathcal{S}$ is assumed independent of $\lambda$. Applied to the Hamiltonian, $H=H_T-\lambda(H_i)_{xy}$, it yields the new Hamiltonian,

$$H'=U(\lambda)^{\dagger}HU(\lambda) \tag{3.2}$$

$$=H_T-\lambda H_1-\lambda^2H_2, \tag{3.3}$$

where we neglect terms $O(\lambda^3)$. On expanding the right-hand side of (3.2) in multiple commutators, one finds

$$H_1=(H_i)_{xy}-i[\mathcal{S},H_T], \tag{3.4}$$

$$H_2=i[\mathcal{S},(H_i)_{xy}]+\tfrac{1}{2}[\mathcal{S},[\mathcal{S},H_T]] \tag{3.5}$$

$$=\tfrac{1}{2}i[\mathcal{S},\,(H_i)_{xy}+H_1]. \tag{3.6}$$

Again neglecting terms $O(\lambda^3)$, one may write the ground-state energy of $H$ in the form,

$$E_0{}^{\xi}(\lambda)=E_0{}^{\xi}(0)-\tfrac{1}{2}\lambda^2\mathcal{I}_{xy}, \tag{3.7}$$

where

$$\mathcal{I}_{xy}=2\langle\mathbf{0}|H_1[H_T(\xi)-E_0{}^{\xi}(0)]^{-1}H_1|\mathbf{0}\rangle+2\langle\mathbf{0}|H_2|\mathbf{0}\rangle; \tag{3.8}$$

$|\mathbf{0}\rangle$ denotes the eigenvector of $H_T(\xi)$ of lowest energy. So far no detailed assumptions regarding $\mathcal{S}$ have been made.

We now assume for $\mathcal{S}$ the form,[10]

$$\mathcal{S}=\sum_{\mathbf{k}r\sigma(r\neq0)}f(\mathbf{k}+\tfrac{1}{2}\mathbf{r},\,\mathbf{k}-\tfrac{1}{2}\mathbf{r})c_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger}c_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma}. \tag{3.9}$$

One finds

$$iH_1=iH_1'+iH_1''=\{i(H_i)_{xy}+[\mathcal{S},\,H_0+\xi H_v{}^{(1)}]\}+[\mathcal{S},H_v{}^{(2)}], \tag{3.10}$$

with[11]

$$\begin{aligned}
iH_1'=&\sum_{r\neq0}\sum_{\mathbf{k}\sigma}(a_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger}b_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger}-\text{H.c.})\Big\{(-1)^{\Delta l}\frac{k_y}{r}-(\tilde{\epsilon}_{\mathbf{k}+\frac{1}{2}\mathbf{r}}-\tilde{\epsilon}_{\mathbf{k}-\frac{1}{2}\mathbf{r}})f(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_>,\,\mathbf{k}-\tfrac{1}{2}\mathbf{r}_<)\\
&-\frac{2\xi}{\Omega}\sum_{\mathbf{k}'}[\mathcal{V}_{\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{k}'-\frac{1}{2}\mathbf{r};\mathbf{k}-\frac{1}{2}\mathbf{r},\mathbf{k}'+\frac{1}{2}\mathbf{r}}-\mathcal{V}_{\mathbf{k}-\frac{1}{2}\mathbf{r},-\mathbf{k}'+\frac{1}{2}\mathbf{r};\mathbf{k}+\frac{1}{2}\mathbf{r},-\mathbf{k}'-\frac{1}{2}\mathbf{r}}]f(\mathbf{k}'+\tfrac{1}{2}\mathbf{r}_>,\,\mathbf{k}'-\tfrac{1}{2}\mathbf{r}_<)\Big\}\\
&+\sum_{r\neq0}\sum_{\mathbf{k}\sigma}a_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger}a_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma}\Big\{(-1)^{\Delta l}\frac{k_y}{r}-(\tilde{\epsilon}_{\mathbf{k}+\frac{1}{2}\mathbf{r}}-\tilde{\epsilon}_{\mathbf{k}-\frac{1}{2}\mathbf{r}})f(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_>,\,\mathbf{k}-\tfrac{1}{2}\mathbf{r}_>)\\
&-\frac{2\xi}{\Omega}\sum_{\mathbf{k}'}[\mathcal{V}_{\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{k}'-\frac{1}{2}\mathbf{r};\mathbf{k}-\frac{1}{2}\mathbf{r},\mathbf{k}'+\frac{1}{2}\mathbf{r}}-\mathcal{V}_{\mathbf{k}-\frac{1}{2}\mathbf{r},-\mathbf{k}'+\frac{1}{2}\mathbf{r};\mathbf{k}+\frac{1}{2}\mathbf{r},-\mathbf{k}'-\frac{1}{2}\mathbf{r}}]f(\mathbf{k}'+\tfrac{1}{2}\mathbf{r}_>,\,\mathbf{k}'-\tfrac{1}{2}\mathbf{r}_<)\Big\}\\
&-\sum_{r\neq0}\sum_{\mathbf{k}\sigma}b_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger}b_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma}\Big\{(-1)^{\Delta l}\frac{k_y}{r}-(\tilde{\epsilon}_{\mathbf{k}+\frac{1}{2}\mathbf{r}}-\tilde{\epsilon}_{\mathbf{k}-\frac{1}{2}\mathbf{r}})f(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_<,\,\mathbf{k}-\tfrac{1}{2}\mathbf{r}_<)\\
&-\frac{2\xi}{\Omega}\sum_{\mathbf{k}'}[\mathcal{V}_{\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{k}'-\frac{1}{2}\mathbf{r};\mathbf{k}-\frac{1}{2}\mathbf{r},\mathbf{k}'+\frac{1}{2}\mathbf{r}}-\mathcal{V}_{\mathbf{k}-\frac{1}{2}\mathbf{r},-\mathbf{k}'+\frac{1}{2}\mathbf{r};\mathbf{k}-\frac{1}{2}\mathbf{r},-\mathbf{k}'-\frac{1}{2}\mathbf{r}}]f(\mathbf{k}'+\tfrac{1}{2}\mathbf{r}_>,\,\mathbf{k}'-\tfrac{1}{2}\mathbf{r}_<)\Big\}\\
&+\sum_{r\neq0}\sum_{\mathbf{k}\sigma}f(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_>,\,\mathbf{k}-\tfrac{1}{2}\mathbf{r})\sum_{\mathbf{q}'}[(a_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger}a_{\mathbf{q}'\sigma}{}^{\dagger}F_{\mathbf{k}-\frac{1}{2}\mathbf{r},\mathbf{q}'}{}^{\sigma\dagger}+a_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger}F_{\mathbf{k}-\frac{1}{2}\mathbf{r},\mathbf{q}'}{}^{\sigma\dagger}b_{\mathbf{q}'\sigma})-\text{H.c.}]\\
&+\sum_{r\neq0}\sum_{\mathbf{k}\sigma}f(\mathbf{k}+\tfrac{1}{2}\mathbf{r},\,\mathbf{k}-\tfrac{1}{2}\mathbf{r}_<)\sum_{\mathbf{q}'}[(b_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger}b_{\mathbf{q}'\sigma}{}^{\dagger}F_{\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{q}'}{}^{\sigma}+b_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger}F_{\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{q}'}{}^{\sigma}a_{\mathbf{q}'\sigma})-\text{H.c.}],
\end{aligned} \tag{3.11}$$

[10] In the analogous transformation in I the pair approximation to $\mathcal{S}$ was used.

[11] The notation $\mathbf{a}_>(\mathbf{a}_<)$ implies that the vector $\mathbf{a}$ satisfies the spherical inequality $|\mathbf{a}|>P_F(|\mathbf{a}|<P_F)$.

and

$$iH_1''=\sum_{r\neq 0}\sum_{\mathbf{k}\sigma\sigma'(\sigma\neq\sigma')}\{[f(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_>,\mathbf{k}-\tfrac{1}{2}\mathbf{r})\sum_{\mathbf{q}'}\bar{F}_{\mathbf{k}-\frac{1}{2}\mathbf{r},\mathbf{q}'}{}^{\sigma}a_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma'}{}^{\dagger}c_{\mathbf{q}'\sigma'} - f(\mathbf{k}+\tfrac{1}{2}\mathbf{r},\mathbf{k}-\tfrac{1}{2}\mathbf{r}_<)\sum_{\mathbf{q}'}\bar{F}_{\mathbf{q}',\mathbf{k}+\frac{1}{2}\mathbf{r}}{}^{\sigma}c_{\mathbf{q}'\sigma'}{}^{\dagger}b_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma'}{}^{\dagger}]-\text{H.c.}\},\quad(3.12)$$

where

$$\bar{F}_{\mathbf{p}'\mathbf{q}}{}^{\sigma}=\frac{\xi}{\Omega}\sum_{\mathbf{p}\mathbf{q}'}\mathcal{V}_{\mathbf{p}'\mathbf{q}';\mathbf{q}\mathbf{p}}{}'c_{\mathbf{q}'\sigma}{}^{\dagger}c_{\mathbf{p}\sigma}.\quad(3.13)$$

$f(\mathbf{k}+\frac{1}{2}\mathbf{r}_>,\mathbf{k}-\frac{1}{2}\mathbf{r}_<)$ is determined by setting[12]

$$(-1)^{\Delta l}\frac{k_y}{r}-(\tilde{\epsilon}_{\mathbf{k}+\frac{1}{2}\mathbf{r}}-\tilde{\epsilon}_{\mathbf{k}-\frac{1}{2}\mathbf{r}})f(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_>,\mathbf{k}-\tfrac{1}{2}\mathbf{r}_<) -\frac{2\xi}{\Omega}\sum_{\mathbf{k}'}[\mathcal{V}_{\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{k}'-\frac{1}{2}\mathbf{r};\mathbf{k}-\frac{1}{2}\mathbf{r},\mathbf{k}'+\frac{1}{2}\mathbf{r}}-\mathcal{V}_{\mathbf{k}-\frac{1}{2}\mathbf{r},-\mathbf{k}'+\frac{1}{2}\mathbf{r};\mathbf{k}+\frac{1}{2}\mathbf{r},-\mathbf{k}'-\frac{1}{2}\mathbf{r}}]f(\mathbf{k}'+\tfrac{1}{2}\mathbf{r}_>,\mathbf{k}'-\tfrac{1}{2}\mathbf{r}_<)=0.\quad(3.14)$$

We shall also take[12]

$$\begin{aligned}(-1)^{\Delta l}\frac{k_y}{r}-(\epsilon_{\mathbf{k}+\frac{1}{2}\mathbf{r}}-\epsilon_{\mathbf{k}-\frac{1}{2}\mathbf{r}})f(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_>,\mathbf{k}-\tfrac{1}{2}\mathbf{r}_>)&=0,\\(-1)^{\Delta l}\frac{k_y}{r}-(\epsilon_{\mathbf{k}+\frac{1}{2}\mathbf{r}}-\epsilon_{\mathbf{k}-\frac{1}{2}\mathbf{r}})f(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_<,\mathbf{k}-\tfrac{1}{2}\mathbf{r}_<)&=0.\end{aligned}\quad(3.15)$$

Note that *the integral equation satisfied by $f(\mathbf{k}+\frac{1}{2}\mathbf{r}_>,\mathbf{k}-\frac{1}{2}\mathbf{r}_<)$ is precisely that obtained previously in the "pair" approximation; it expresses the condition that the compensating pair graphs be transformed away.* The unitary transformation $U(\lambda)$ merely displaces the weak-field Hamiltonian $H$ to the "center of oscillation" of its phonon-like pairs; the resulting Hamiltonian $H'$, in contrast to previous work,[2] still provides an exact dynamical description of the system. The solution to Eqs. (3.14–15) may be written down at once,

$$f(\mathbf{k}+\tfrac{1}{2}\mathbf{r},\mathbf{k}-\tfrac{1}{2}\mathbf{r})=(-1)^{\Delta l}\frac{k_y}{r}\frac{1}{(\epsilon_{\mathbf{k}+\frac{1}{2}\mathbf{r}}-\epsilon_{\mathbf{k}-\frac{1}{2}\mathbf{r}})}.\quad(3.16)$$

Note that as a result of the choice for $f(\mathbf{k}+\frac{1}{2}\mathbf{r}_>,\mathbf{k}-\frac{1}{2}\mathbf{r}_<)$ given above,[13] in calculating with $H_1$ in second order (in $\xi$), one need keep only its effective parts,

$$(iH_1')_{\text{effective}}=\frac{\xi}{\Omega}\sum_{r\neq0}\sum_{\mathbf{k}\sigma}\{f(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_>,\mathbf{k}-\tfrac{1}{2}\mathbf{r})\sum_{\mathbf{p}\mathbf{q}\mathbf{q}'}\mathcal{V}_{\mathbf{p}\mathbf{q};\mathbf{k}-\frac{1}{2}\mathbf{r},\mathbf{q}'}(a_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger}a_{\mathbf{q}'\sigma}{}^{\dagger}b_{\mathbf{q}\sigma}{}^{\dagger}b_{\mathbf{p}\sigma}{}^{\dagger}-\text{H.c.}) +f(\mathbf{k}+\tfrac{1}{2}\mathbf{r},\mathbf{k}-\tfrac{1}{2}\mathbf{r}_<)\sum_{\mathbf{p}\mathbf{q}\mathbf{q}'}\mathcal{V}_{\mathbf{p}\mathbf{q};\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{q}'}(a_{\mathbf{p}\sigma}{}^{\dagger}a_{\mathbf{q}\sigma}{}^{\dagger}b_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger}b_{\mathbf{q}'\sigma}{}^{\dagger}-\text{H.c.})\},\quad(3.17)$$

and

$$(iH_1'')_{\text{effective}}=\frac{\xi}{\Omega}\sum_{r\neq0}\sum_{\mathbf{k}\sigma\sigma'(\sigma\neq\sigma')}\{f(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_>,\mathbf{k}-\tfrac{1}{2}\mathbf{r})\sum_{\mathbf{p}\mathbf{q}\mathbf{q}'}\mathcal{V}_{\mathbf{p}\mathbf{q};\mathbf{k}-\frac{1}{2}\mathbf{r},\mathbf{q}'}{}'(a_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma'}{}^{\dagger}b_{\mathbf{p}\sigma'}{}^{\dagger}a_{\mathbf{q}'\sigma}{}^{\dagger}b_{\mathbf{q}\sigma}{}^{\dagger}-\text{H.c.}) -\sum_{r\neq0}\sum_{\mathbf{k}\sigma\sigma'(\sigma\neq\sigma')}f(\mathbf{k}+\tfrac{1}{2}\mathbf{r},\mathbf{k}-\tfrac{1}{2}\mathbf{r}_<)\sum_{\mathbf{p}\mathbf{q}\mathbf{q}'}\mathcal{V}_{\mathbf{p}\mathbf{q};\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{q}'}{}'(a_{\mathbf{p}\sigma'}{}^{\dagger}b_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma'}{}^{\dagger}a_{\mathbf{q}\sigma}{}^{\dagger}b_{\mathbf{q}'\sigma}{}^{\dagger}-\text{H.c.})\}.\quad(3.18)$$

[12] These conditions are dictated largely by perturbation-theoretic considerations; there remains the possibility of framing them by variational means or of requiring that the functions $f(\mathbf{k}+\mathbf{r},\mathbf{k})$ satisfy different "equations of motion." Thus one might demand that $f(\mathbf{k}+\mathbf{r}_>,\mathbf{k}_<)$ satisfy

$$(E_{k+r}-E_k)f(\mathbf{k}+\mathbf{r}_>,\mathbf{k}_<)+2\sum_{\mathbf{k}'}\frac{\xi}{\Omega}[\mathcal{K}_{\mathbf{k}+\mathbf{r},\mathbf{k}';\mathbf{k},\mathbf{k}'+\mathbf{r}}-\mathcal{K}_{\mathbf{k},-\mathbf{k}';\mathbf{k}+\mathbf{r},-\mathbf{k}'-\mathbf{r}}]f(\mathbf{k}'+\mathbf{r}_>,\mathbf{k}_<')=(-1)^{\Delta l}\frac{k_y}{r},$$

where $E_{\mathbf{k}}$ and $(\mathcal{K})$ denote the single-particle energy and $K$ matrix, respectively. This last choice would seem to be appropriate to the case of singular particle-particle interaction.

[13] The similarity to the transformation of Gross (reference 6) becomes apparent on writing $f(\mathbf{k}_>,\mathbf{k}_<')=\langle n|L_z|0\rangle(E_n-E_0)$, where $|n\rangle=a_{\mathbf{k}}{}^{\dagger}b_{\mathbf{k}'}{}^{\dagger}|0\rangle$.

We consider now the terms in $H_2$ which are $O(\xi^0)$,

$$
\begin{aligned}
H_2|_{(\xi=0)} &= \tfrac{1}{2}i[\mathcal{S},(H_i)_{xy}] \\
&= \tfrac{1}{2}i \sum_{\mathbf{k}\mathbf{k}'\sigma} [f(\mathbf{k}+\tfrac{1}{2}\mathbf{r},\, \mathbf{k}-\tfrac{1}{2}\mathbf{r})(L_z)_{\mathbf{k}-\frac{1}{2}\mathbf{r},\mathbf{k}'} - (L_z)_{\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{k}-\frac{1}{2}\mathbf{r}} f(\mathbf{k}-\tfrac{1}{2}\mathbf{r},\, \mathbf{k}')] c_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma}{}^{\dagger} c_{\mathbf{k}'\sigma}.
\end{aligned}
\tag{3.19}
$$

Its nondiagonal terms ($\mathbf{k}'\neq\mathbf{k}+\frac{1}{2}\mathbf{r}$) make no contribution to the ground state energy.[14] If we $\mathcal{S}$ order the remaining diagonal operators ($\mathbf{k}'=\mathbf{k}+\frac{1}{2}\mathbf{r}$), we obtain

$$
H_2|_{(\xi=0)} \longrightarrow \tfrac{1}{2}(\mathcal{J}_{xy})_{\text{rigid}} + (\text{diagonal terms correcting } H_0). \tag{3.20}
$$

The diagonal operators appearing in (3.20) may in turn be dropped, since they give rise to corrections to the energy $O(\lambda^4)$. We are thus led to

$$
H_2 \longrightarrow \tfrac{1}{2}(\mathcal{J}_{xy})_{\text{rigid}} + \tfrac{1}{2}i[\mathcal{S},H_1]. \tag{3.21}
$$

## SECOND-ORDER CORRECTIONS TO THE INERTIAL MOMENT

The second-order corrections to the inertial moment are given by the expression,

$$
\begin{aligned}
\Delta\mathcal{J}_{xy} &\equiv \mathcal{J}_{xy} - (\mathcal{J}_{xy})_{\text{rigid}} \\
&= 2\langle 0|H_1'H_0^{-1}H_1'|0\rangle + 2\langle 0|H_1''H_0^{-1}H_1''|0\rangle - 2i\langle 0|\xi H_v{}^{(1)}H_0^{-1}[\mathcal{S},H_1']|0\rangle - 2i\langle 0|\xi H_v{}^{(2)}H_0^{-1}[\mathcal{S},H_1'']|0\rangle,
\end{aligned}
\tag{4.1}
$$

a result easily derivable from (3.8) and (3.21). $|0\rangle$ denotes the unperturbed vacuum state vector. A straightforward, albeit lengthy, calculation yields

$$
\Delta\mathcal{J}_{xy} = \sum_{j=1}^{4} \Delta\mathcal{J}_{xy}{}^{(j)}, \tag{4.2}
$$

where

$$
\begin{aligned}
\Delta\mathcal{J}_{xy}{}^{(1)} = \frac{8\xi^2}{\Omega^2} \sum_{\substack{r\mathbf{klst} \\ (l>k_F;\, s,\, t<k_F)}} \Bigg\{ & f^2(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_>,\, \mathbf{k}-\tfrac{1}{2}\mathbf{r}_<) \frac{(\mathcal{V}_{\mathrm{st};\,\mathrm{k}-\frac{1}{2}\mathrm{r},\,\mathrm{l}}{}^2 - \mathcal{V}_{\mathrm{st};\,\mathrm{k}+\frac{1}{2}\mathrm{r},\,\mathrm{l}}{}^2)}{\epsilon_{\mathrm{k}+\frac{1}{2}\mathrm{r}} + \epsilon_{\mathrm{l}} - \epsilon_{\mathrm{s}} - \epsilon_{\mathrm{t}}} + \mathcal{V}_{\mathrm{st};\,\mathrm{k}+\frac{1}{2}\mathrm{r},\,\mathrm{l}-\frac{1}{2}\mathrm{r}} \mathcal{V}_{\mathrm{st};\,\mathrm{k}-\frac{1}{2}\mathrm{r},\,\mathrm{l}+\frac{1}{2}\mathrm{r}} f(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_>,\, \mathbf{k}-\tfrac{1}{2}\mathbf{r}) \\
& \times \left[ \frac{f(\mathbf{l}+\frac{1}{2}\mathbf{r}_>,\, \mathbf{l}-\frac{1}{2}\mathbf{r})}{\epsilon_{\mathrm{k}+\frac{1}{2}\mathrm{r}} + \epsilon_{\mathrm{l}+\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{s}} - \epsilon_{\mathrm{t}}} - \frac{f(\mathbf{l}-\frac{1}{2}\mathbf{r}_>,\, \mathbf{l}+\frac{1}{2}\mathbf{r})}{\epsilon_{\mathrm{k}+\frac{1}{2}\mathrm{r}} + \epsilon_{\mathrm{l}-\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{s}} - \epsilon_{\mathrm{t}}} \right] - \mathcal{V}_{\mathrm{s}-\frac{1}{2}\mathrm{r},\,\mathrm{t};\,\mathrm{k}-\frac{1}{2}\mathrm{r},\,\mathrm{l}} \mathcal{V}_{\mathrm{s}+\frac{1}{2}\mathrm{r},\,\mathrm{t};\,\mathrm{k}+\frac{1}{2}\mathrm{r},\,\mathrm{l}} f(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_>,\, \mathbf{k}-\tfrac{1}{2}\mathbf{r}) \\
& \times \left[ \frac{f(\mathbf{s}+\frac{1}{2}\mathbf{r},\, \mathbf{s}-\frac{1}{2}\mathbf{r}_<)}{\epsilon_{\mathrm{k}+\frac{1}{2}\mathrm{r}} + \epsilon_{\mathrm{l}} - \epsilon_{\mathrm{s}-\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{t}}} - \frac{f(\mathbf{s}-\frac{1}{2}\mathbf{r},\, \mathbf{s}+\frac{1}{2}\mathbf{r}_<)}{\epsilon_{\mathrm{k}+\frac{1}{2}\mathrm{r}} + \epsilon_{\mathrm{l}} - \epsilon_{\mathrm{s}+\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{t}}} \right] - \mathcal{V}_{\mathrm{s},\,\mathrm{t}-\frac{1}{2}\mathrm{r};\,\mathrm{k}-\frac{1}{2}\mathrm{r},\,\mathrm{l}} \mathcal{V}_{\mathrm{s},\,\mathrm{t}+\frac{1}{2}\mathrm{r};\,\mathrm{k}+\frac{1}{2}\mathrm{r},\,\mathrm{l}} f(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_>,\, \mathbf{k}-\tfrac{1}{2}\mathbf{r}_<) \\
& \times \left[ \frac{\mathbf{f}(\mathbf{t}+\frac{1}{2}\mathbf{r}_>,\, \mathbf{t}-\frac{1}{2}\mathbf{r})}{\epsilon_{\mathrm{k}+\frac{1}{2}\mathrm{r}} + \epsilon_{\mathrm{l}} - \epsilon_{\mathrm{s}} - \epsilon_{\mathrm{t}-\frac{1}{2}\mathrm{r}}} - \frac{f(\mathbf{t}-\frac{1}{2}\mathbf{r}_>,\, \mathbf{t}+\frac{1}{2}\mathbf{r})}{\epsilon_{\mathrm{k}+\frac{1}{2}\mathrm{r}} + \epsilon_{\mathrm{l}} - \epsilon_{\mathrm{s}} - \epsilon_{\mathrm{t}+\frac{1}{2}\mathrm{r}}} \right] + \frac{\mathcal{V}_{\mathrm{st};\,\mathrm{k}-\frac{1}{2}\mathrm{r},\,\mathrm{l}}{}^2 f^2(\mathbf{k}+\frac{1}{2}\mathbf{r}_>,\, \mathbf{k}-\frac{1}{2}\mathbf{r}_>)(\epsilon_{\mathrm{k}-\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{k}+\frac{1}{2}\mathrm{r}})}{(\epsilon_{\mathrm{k}+\frac{1}{2}\mathrm{r}} + \epsilon_{\mathrm{l}} - \epsilon_{\mathrm{s}} - \epsilon_{\mathrm{t}})(\epsilon_{\mathrm{k}-\frac{1}{2}\mathrm{r}} + \epsilon_{\mathrm{l}} - \epsilon_{\mathrm{s}} - \epsilon_{\mathrm{t}})} \Bigg\},
\end{aligned}
\tag{4.3}
$$

$$
\begin{aligned}
\Delta\mathcal{J}_{xy}{}^{(2)} = \frac{8\xi^2}{\Omega^2} \sum_{\substack{r\mathbf{klst} \\ (s,\, t>k_F;\, l<k_F)}} \Bigg\{ & f^2(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_>,\, \mathbf{k}-\tfrac{1}{2}\mathbf{r}_<) \frac{(\mathcal{V}_{\mathrm{st};\,\mathrm{k}+\frac{1}{2}\mathrm{r},\,\mathrm{l}}{}^2 - \mathcal{V}_{\mathrm{st};\,\mathrm{k}-\frac{1}{2}\mathrm{r},\,\mathrm{l}}{}^2)}{\epsilon_{\mathrm{s}} + \epsilon_{\mathrm{t}} - \epsilon_{\mathrm{k}-\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{l}}} + \mathcal{V}_{\mathrm{st};\,\mathrm{k}+\frac{1}{2}\mathrm{r},\,\mathrm{l}-\frac{1}{2}\mathrm{r}} \mathcal{V}_{\mathrm{st};\,\mathrm{k}-\frac{1}{2}\mathrm{r},\,\mathrm{l}+\frac{1}{2}\mathrm{r}} f(\mathbf{k}+\tfrac{1}{2}\mathbf{r},\, \mathbf{k}-\tfrac{1}{2}\mathbf{r}_<) \\
& \times \left[ \frac{f(\mathbf{l}+\frac{1}{2}\mathbf{r},\, \mathbf{l}-\frac{1}{2}\mathbf{r}_<)}{\epsilon_s + \epsilon_t - \epsilon_{\mathrm{k}-\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{l}-\frac{1}{2}\mathrm{r}}} - \frac{f(\mathbf{l}-\frac{1}{2}\mathbf{r},\, \mathbf{l}+\frac{1}{2}\mathbf{r}_<)}{\epsilon_s + \epsilon_t - \epsilon_{\mathrm{k}-\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{l}+\frac{1}{2}\mathrm{r}}} \right] - \mathcal{V}_{\mathrm{s}+\frac{1}{2}\mathrm{r},\,\mathrm{t};\,\mathrm{k}+\frac{1}{2}\mathrm{r},\,\mathrm{l}} \mathcal{V}_{\mathrm{s}-\frac{1}{2}\mathrm{r},\,\mathrm{t};\,\mathrm{k}-\frac{1}{2}\mathrm{r},\,\mathrm{l}} f(\mathbf{k}+\tfrac{1}{2}\mathbf{r},\, \mathbf{k}-\tfrac{1}{2}\mathbf{r}_<) \\
& \times \left[ \frac{f(\mathbf{s}+\frac{1}{2}\mathbf{r}_>,\, \mathbf{s}-\frac{1}{2}\mathbf{r})}{\epsilon_{\mathrm{s}+\frac{1}{2}\mathrm{r}} + \epsilon_{\mathrm{t}} - \epsilon_{\mathrm{k}-\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{l}}} - \frac{f(\mathbf{s}-\frac{1}{2}\mathbf{r}_>,\, \mathbf{s}+\frac{1}{2}\mathbf{r})}{\epsilon_{\mathrm{s}-\frac{1}{2}\mathrm{r}} + \epsilon_{\mathrm{t}} - \epsilon_{\mathrm{k}-\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{l}}} \right] - \mathcal{V}_{,\,\mathrm{s}\,\mathrm{t}+\frac{1}{2}\mathrm{r};\,\mathrm{k}+\frac{1}{2}\mathrm{r},\,\mathrm{l}} \mathcal{V}_{\mathrm{s},\,\mathrm{t}-\frac{1}{2}\mathrm{r};\,\mathrm{k}-\frac{1}{2}\mathrm{r},\,\mathrm{l}} f(\mathbf{k}+\tfrac{1}{2}\mathbf{r},\, \mathbf{k}-\tfrac{1}{2}\mathbf{r}_<) \\
& \times \left[ \frac{f(\mathbf{t}+\frac{1}{2}\mathbf{r}_>,\, \mathbf{t}-\frac{1}{2}\mathbf{r})}{\epsilon_{\mathrm{s}} + \epsilon_{\mathrm{t}+\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{k}-\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{l}}} - \frac{f(\mathbf{t}-\frac{1}{2}\mathbf{r}_>,\, \mathbf{t}+\frac{1}{2}\mathbf{r})}{\epsilon_{\mathrm{s}} + \epsilon_{\mathrm{t}-\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{k}-\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{l}}} \right] + \frac{\mathcal{V}_{\mathrm{st};\,\mathrm{k}+\frac{1}{2}\mathrm{r},\,\mathrm{l}}{}^2 f^2(\mathbf{k}+\frac{1}{2}\mathbf{r}_<,\, \mathbf{k}-\frac{1}{2}\mathbf{r}_<)(\epsilon_{\mathrm{k}+\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{k}-\frac{1}{2}\mathrm{r}})}{(\epsilon_{\mathrm{s}} + \epsilon_{\mathrm{t}} - \epsilon_{\mathrm{k}-\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{l}})(\epsilon_{\mathrm{s}} + \epsilon_{\mathrm{t}} - \epsilon_{\mathrm{k}+\frac{1}{2}\mathrm{r}} - \epsilon_{\mathrm{l}})} \Bigg\},
\end{aligned}
\tag{4.4}
$$

$$
\Delta\mathcal{J}_{xy}{}^{(3)} = \tfrac{1}{2}\Delta\mathcal{J}_{xy}{}^{(1)}(\mathcal{V}\rightarrow\mathcal{V}'), \tag{4.5}
$$

$$
\Delta\mathcal{J}_{xy}{}^{(4)} = \tfrac{1}{2}\Delta\mathcal{J}_{xy}{}^{(2)}(\mathcal{V}\rightarrow\mathcal{V}'). \tag{4.6}
$$

[14] This is a consequence of the rule that the momentum transfer taken up from the external cranking field at one vertex must be given up at the other; otherwise stated, ground state corrections must involve zero net momentum transfer (by conservation of momentum).

We propose to show that two terms in each of the $\Delta \mathcal{I}_{xy}^{(j)}$ have the same $L$ dependence as the inertial moment so that $\lim_{L\to\infty}(\Delta\mathcal{I}_{xy}^{(j)}/(\mathcal{I}_{xy})_{\rm rigid})\neq 0$ ($j=1,\cdots 4$). Since the major contribution to each term comes from small $r$, it is appropriate to discuss the dependence on $r$ of their respective summands as $r\to 0$. Accordingly we make a Taylor expansion of the nonsingular factors in each summand about $r=0$. This expansion need not be carried very far, since those terms behaving like $r^n$, $n\geq 0$, are vanishingly small compared to the rigid moment in the limit of large volume. [Note that we have $r\propto 1/L$ and that corrections with the same $L$ dependence as the rigid moment must be $O(1/r^2)$.] Finally, only the even powers of $r$ in this expansion need be considered since the sums $\sum_{r\neq 0} r^{2m+1}$ ($m$ integral) vanish by consideration of symmetry. We also require the relation[15]

$$\lim_{r\to 0}\eta(|\mathbf{k}\pm\tfrac{1}{2}\mathbf{r}|-k_F)\eta(k_F-|\mathbf{k}\mp\tfrac{1}{2}\mathbf{r}|)\to\pm\mathbf{n}\cdot\mathbf{r}\eta(\pm\mathbf{n}\cdot\mathbf{r})\delta(k-k_F), \tag{4.7}$$

where $\mathbf{n}=\mathbf{k}/k$. In our coordinate system[2] we have $\mathbf{n}\cdot\mathbf{r}=n_x r$. It follows immediately from (4.7) that

$$(-1)^{\Delta l}f(\mathbf{k}\pm\tfrac{1}{2}\mathbf{r}_>,\ \mathbf{k}\mp\tfrac{1}{2}\mathbf{r}_<)\cong\frac{Mk_y}{r}\delta(k-k_F)\eta(\mathbf{n}\cdot\mathbf{r}). \tag{4.8}$$

It is easy to show, by symmetrizing with respect to $r$ and using momentum conservation, that the sum of all terms in (4.3–6) for which both arguments of *each* $f$ lie on the same side of the Fermi surface vanish. (We will prove this explicitly to all orders in the following section.) In addition, the pairs of cross terms proportional to $f(\mathbf{a}+\tfrac{1}{2}\mathbf{r}_>,\ \mathbf{a}-\tfrac{1}{2}\mathbf{r}_<)f(\mathbf{b}\pm\tfrac{1}{2}\mathbf{r}_>,\ \mathbf{b}\mp\tfrac{1}{2}\mathbf{r}_<)$, where $\mathbf{a}\neq\mathbf{b}$, individually are $O(1/r^2)$ by virtue of (4.8), their respective differences are $O(r^0)$, and consequently are negligible compared to the rigid moment in the limit of large $L$. The remaining cross terms may be reduced by means of (4.8) together with momentum conservation. Thus, one has

$$\begin{aligned}\sum_{j=1}^{4}\Delta\mathcal{I}_{xy}^{(j)}\big|_{(L\to\infty)}={}&-\frac{M^2k_F^2}{\pi^2\Omega}\sum_{r\neq 0}\frac{1}{r^2}\langle n_y^2\rangle_{\rm av}\frac{d}{dk}\sum_{\substack{\mathrm{lst}\\(l>k_F;\,s,\,t<k_F)}}\left[\frac{(2\mathcal{V}_{\mathrm{st;kl}}^2+\mathcal{V}_{\mathrm{st;kl}}'^2)}{\epsilon_{\mathrm{k}}+\epsilon_{\mathrm{l}}-\epsilon_{\mathrm{s}}-\epsilon_{\mathrm{t}}}\right]_{(k=k_F)}\\&+\frac{M^2k_F^2}{\pi^2\Omega}\sum_{r\neq 0}\frac{1}{r^2}\langle n_y^2\rangle_{\rm av}\frac{d}{dk}\sum_{\substack{\mathrm{lst}\\(l<k_F;\,s,\,t,\,>k_F)}}\left[\frac{(2\mathcal{V}_{\mathrm{st;kl}}^2+\mathcal{V}_{\mathrm{st;kl}}'^2)}{\epsilon_{\mathrm{s}}+\epsilon_{\mathrm{t}}-\epsilon_{\mathrm{k}}-\epsilon_{\mathrm{l}}}\right]_{(k=k_F)}\\&+\frac{Mk_F^3}{\pi^2\Omega}\sum_{r\neq 0}\frac{1}{r^2}\langle n_y^2\rangle_{\rm av}\sum_{\substack{\mathrm{lst}\\(l>k_F;\,s,\,t,\,<k_F)}}\left[\frac{(2\mathcal{V}_{\mathrm{st;kl}}^2+\mathcal{V}_{\mathrm{st;kl}}'^2)}{(\epsilon_{\mathrm{k}}+\epsilon_{\mathrm{l}}-\epsilon_{\mathrm{s}}-\epsilon_{\mathrm{t}})^2}\right]\\&+\frac{Mk_F^3}{\pi^2\Omega}\sum_{r\neq 0}\frac{1}{r^2}\langle n_y^2\rangle_{\rm av}\sum_{\substack{\mathrm{lst}\\(l<k_F;\,s,\,t>k_F)}}\left[\frac{(2\mathcal{V}_{\mathrm{st;kl}}^2+\mathcal{V}_{\mathrm{st;kl}}'^2)}{(\epsilon_{\mathrm{s}}+\epsilon_{\mathrm{t}}-\epsilon_{\mathrm{k}}-\epsilon_{\mathrm{l}})^2}\right]_{(k=k_F)}.\end{aligned} \tag{4.9}$$

These terms may be written somewhat more simply as

$$\lim_{L\to\infty}\frac{\Delta\mathcal{I}_{xy}}{(\mathcal{I}_{xy})_{\rm rigid}}=\frac{M}{k_F}\left(\frac{dV_p^{(2)}}{dk_F}-\frac{dV_h^{(2)}}{dk_F}\right)+(W_p^{(2)}(k_F)+W_h^{(2)}(k_F)), \tag{4.10}$$

where

$$V_p^{(2)}(k)=\sum_{\substack{\mathrm{lst}\\(k,\,l>k_F;\,s,\,t<k_F)}}\frac{(2v_{\mathrm{st;kl}}^2+v_{\mathrm{st;kl}}v_{\mathrm{st;lk}})}{\epsilon_{\mathrm{s}}+\epsilon_{\mathrm{t}}-\epsilon_{\mathrm{k}}-\epsilon_{\mathrm{l}}}, \tag{4.11}$$

$$V_h^{(2)}(k)=\sum_{\substack{\mathrm{lst}\\(k,\,l<k_F;\,s,\,t>k_F)}}\frac{(2v_{\mathrm{st;kl}}^2+v_{\mathrm{st;kl}}v_{\mathrm{st;lk}})}{\epsilon_{\mathrm{k}}+\epsilon_{\mathrm{l}}-\epsilon_{\mathrm{s}}-\epsilon_{\mathrm{t}}}, \tag{4.12}$$

$$W_p^{(2)}(k)=\sum_{\substack{\mathrm{lst}\\(k,\,l>k_F;\,s,\,t<k_F)}}\frac{(2v_{\mathrm{st;kl}}^2+v_{\mathrm{st;kl}}v_{\mathrm{st;lk}})}{(\epsilon_{\mathrm{k}}+\epsilon_{\mathrm{l}}-\epsilon_{\mathrm{s}}-\epsilon_{\mathrm{t}})^2}, \tag{4.13}$$

$$W_h^{(2)}(k)=\sum_{\substack{\mathrm{lst}\\(k,\,l<k_F;\,s,\,t>k_F)}}\frac{(2v_{\mathrm{st;kl}}^2+v_{\mathrm{st;kl}}v_{\mathrm{st;lk}})}{(\epsilon_{\mathrm{s}}+\epsilon_{\mathrm{t}}-\epsilon_{\mathrm{k}}-\epsilon_{\mathrm{l}})^2}; \tag{4.14}$$

In the neglect of exchange terms, $V_p^{(2)}(k)$ is twice the particle "rearrangement" energy[16] and $V_h^{(2)}(k)$ twice a $K$-matrix term in the hole energy[16]; the terms proportional to $(dV_p^{(2)}/dk_F-dV_h^{(2)}/dk_F)$ thus represent effective mass corrections. The terms $(W_p^{(2)}+W_h^{(2)})$ arise from propagator changes.

[15] See, for example, appendix A of D. F. DuBois, Ann. Phys. **7**, 174 (1959).
[16] K. A. Brueckner and D. T. Goldman, Phys. Rev. **117**, 207 (1960).

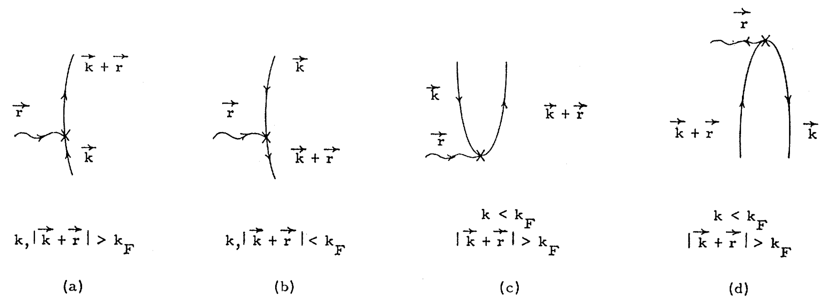


FIG. 1. Graphs for (a) particle deflection and (b) hole deflection in the external (cranking) field ($s$-vertex insertion). Graphs for (c) pair creation and (d) pair annihilation in the external (cranking) field ($p$-vertex insertion).

## REMARKS ON THE PERSISTENCE OF INTERACTION CORRECTIONS IN HIGHER ORDERS

In this section we shall, by a detailed graphical analysis (in the Schrödinger representation) indicate those classes of graphs $O(1/r^2)$[17] which vanish in all orders[18] as well as those which contribute to a nonvanishing interaction correction to the rigid moment in all orders. We consider first the former class. There are three sets of these which vanish by virtue of momentum conservation: (a) those in which

$$(H_i(r)-H_i^p(r))_{xy} = \frac{(-1)^{\Delta l}}{ir}\sum_{\mathbf{k}\sigma} k_y(a_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma}^\dagger a_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma} - b_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma}^\dagger b_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma}) \tag{5.1}$$

acts twice, (b) a set in which $(H_i(r)-H_i^p(r))_{xy}$ acts only once, and (c) a set in which $(H_i^p)_{xy}$ acts twice.

We note, first of all, that $(H_i(r)-H_i^p(r))_{xy}$ describes processes in which particles are scattered into particles, holes into holes, by the external cranking field, i.e., the interaction (5.1) does not alter the nature of the internal line[19] in which it is inserted. A graphic representation of such an insertion is given in Figs. 1(a), (b). We shall term the scattering vertices produced by (5.1) $s$ vertices. [Similarly, we shall term the vertices which arise from the interaction $H_i^p$, $p$ vertices. These vertices are the characteristic ones of pair creation and annihilation in an external field; they are graphically represented in Figs. 1(c), (d).]

The following prescription[20] will serve to generate all members of class (a). First, draw all distinct ground-state graphs. In any one of these, insert two $s$ vertices in all possible distinct ways. Thus, for example, in second order, such double insertion [in Fig. 2(a)] will yield twelve distinct graphs [of which Fig. 2(b) is one]. Each cranking vertex will give a factor $1/r$; the limit $r\rightarrow 0$ may be taken at once in the remaining factors in the matrix element of each of the twelve diagrams.[21] Further, each cranking vertex is proportional to the $y$ component of the momentum of the internal line in which it is inserted times the particle number of the line ($+1$ for particles, $-1$ for holes). Then, holding the first insertion [from (5.1)] fixed, one finds that the sum of all graphs obtained by making the second insertion on a cut which splits the graph into earlier and later pieces must, since as many particle lines as hole lines are so cut, vanish by momentum conservation. The sum of insertions on a cut which intersects the two particle and two hole lines whose momentum sum vanishes will be termed a null set of graphs. Thus the graphs of class (a) are obtained by making all distinct $s$-vertex insertions on all distinct pairs of cuts in all ground-state graphs. These graphs, appropriately grouped, then sum to sets of null graphs and vanish. Figures 2(b)–(e) form such a null set. These statements are also equivalent to the replacement, for a given value of $r$, of (5.1) by

$$(H_i(r)-H_i^p(r))_{xy}' = \frac{(-1)^{\Delta l}}{ir}\sum_{\mathbf{k}\sigma} k_y(a_{\mathbf{k}\sigma}^\dagger a_{\mathbf{k}\sigma} - b_{\mathbf{k}\sigma}^\dagger b_{\mathbf{k}\sigma}), \tag{5.2}$$

and the subsequent incorporation of this operator into the kinetic energy, so that one has

$$H_0\rightarrow H_0' = \sum_{\mathbf{k}\sigma(k>k_F)}\left[\epsilon_{\mathbf{k}} - \frac{\lambda(-1)^{\Delta l}}{ir}(\mathbf{k})_y\right]a_{\mathbf{k}\sigma}^\dagger a_{\mathbf{k}\sigma} - \sum_{\mathbf{k}\sigma(k<k_F)}\left[\epsilon_{\mathbf{k}} - \frac{\lambda(-1)^{\Delta l}}{ir}(\mathbf{k})_y\right]b_{\mathbf{k}\sigma}^\dagger b_{\mathbf{k}\sigma}. \tag{5.3}$$

Diagrams of class (a) are then given by

$$\text{Contribution}_{(a)} = \tfrac{1}{2}\lambda^2\frac{\partial^2}{\partial\lambda^2}E_0^{\xi}(\lambda)\bigg|_{(\lambda=0)}; \tag{5.4}$$

their sum vanishes since $[H_T, (H_i(r)-H_i^p(r))_{xy}']=0$.

In class (b) a $p$-vertex or pair-chain insertion is made

[17] These contributions are of the order of the cranking moment (see reference 2).

[18] I.e., to within terms which vanish relative to the rigid moment in the limit $L\rightarrow\infty$.

[19] By an "internal line," we shall mean a particle (hole) line joining two particle-particle vertices, each of which involves a nonzero momentum transfer. (Thus a given internal line may bear any number of particle-particle vertices each making zero momentum transfer to the given internal line.)

[20] The following discussion is easily generalized to apply to the case of any number of zero-momentum insertions in a given internal line.

[21] The addition of a set of four of these graphs whose sum is zero by conservation of momentum (see discussion below) will enable us to group the resulting *sixteen* graphs into sets of four, each set of which then vanishes by virtue of momentum conservation.

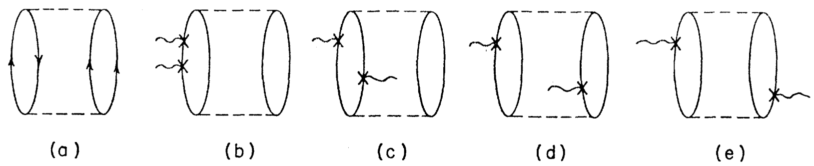


FIG. 2. (a) Ground-state energy graph in second order. (b)–(e) A null set of graphs in class (a) in second order.

in the one and an $s$ insertion is made in the other of two *distinct* cuts in a ground-state graph. Again these graphs vanish by momentum conservation for the sum of $s$ insertions for a given such $p$-vertex or pair-chain insertion. It is necessary that the $p$ vertex lie "outside" the remainder of the diagram in order that the diagram be $O(1/r^2)$. [This follows from (4.7).] A complicated example of class (b) is shown in Fig. 3.

Class (c) is that class of corrections obtained by inserting $p$ vertices in any two different internal lines in an arbitrary ground-state energy graph. So that these graphs may have the same $L$ dependence as the rigid moment, the respective $p$ vertices must lie above or below the uppermost or lowermost particle-particle interaction vertex.[22] [When both $p$ vertices lie above the rest of the diagram, the sum of the two diagrams, each with weight $\frac{1}{2}$, obtained by exchanging the relative positions of the $p$ vertices, decouples the denominators of each pair. (Addition of the time reversed diagram, with both $p$ vertices below the rest of the diagram, restores the weight to 1.)] Then it is easy to see that for a particular insertion of a $p$ vertex in an arbitrary line, the other $p$ vertex may be inserted in any *other* line in two ways, with the vertex either above or below the rest of the diagram. These two conjugate insertions are identical in the limit as $r \to 0$ but differ in sign, since in the former case the vertex inserted corresponds to, say, a pair annihilation in the cranking field, while in the latter case it corresponds to pair creation by the cranking field. Hence, their contribution is at most $O(r^0)$ and vanishes relative to the rigid moment in the limit $L \to \infty$. We summarize this: In any arbitrary graph, if an internal line has been distorted by the insertion of *only one* pair-creation (-annihilation) cranking vertex, then this graph is cancelled[18] by the conjugate graph obtained by inserting (in the same internal line) instead, a pair-annihilation (-creation) vertex. Two conjugate graphs in second order are shown in Fig. 4.

We now discuss the three classes of graphs which contribute to a nonvanishing interaction correction. The first set [set ($\alpha$)] consists in all those graphs obtained by first making one $s$ insertion in all possible ways along a cut which splits an arbitrary ground-state graph into earlier and later pieces, and then making a $p$ insertion in all possible ways in all the remaining uninserted internal lines intersected by the cut. The $p$ vertex must lie "outside" the remainder of the graph. These graphs do not sum to null sets; they may be grouped in triads with sums proportional to $(-k_y^2)$ where $\mathbf{k}$ is the momentum of the internal line bearing the $p$-vertex insertion. An example of such a triad is shown in Fig. 5.

Class ($\beta$) consists in those graphs obtained by the insertion of two $p$ vertices in the *same* internal line in any ground-state energy graph. There are two distinct graphs for each doubly inserted internal line corresponding to crossed and uncrossed $p$ vertices (see Fig. 6). Since there is only *one* condition (4.7) on the double insertion, these graphs are $O(1/r^3)$ when both cranking vertices lie outside (one above and one below) the particle-particle interaction vertices, and $O(1/r^2)$ when only one $p$ vertex lies outside the rest of the diagram. The sum of the latter graphs is such as to decouple the energy denominator associated with the pair from the rest of the diagram. The crossed and uncrossed graphs are also conjugate in that they differ in sign. This difference in sign may be understood in the following way. Consider the respective closed loops in two conjugate graphs determined by the double insertion. Note that the portion of the loop which is an undeformed internal line has the same sense in both graphs. In the case of the uncrossed graph, if that internal line is a particle (hole) line then the rest of the graph may be viewed as a nonlocal mass correction in the particle (hole) line[16] of the pair loop which yields the rigid moment; in the case of the crossed graph, if that internal line is a particle (hole) line, then the rest of the graph may be viewed as a nonlocal mass correction in the hole (particle) line[16] of the pair loop which yields the rigid moment. Since self-mass insertions in

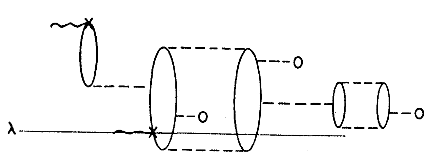


FIG. 3. A higher order graph of class (b). The set of graphs obtained by making $s$-vertex insertions on the cut λ is a null set.

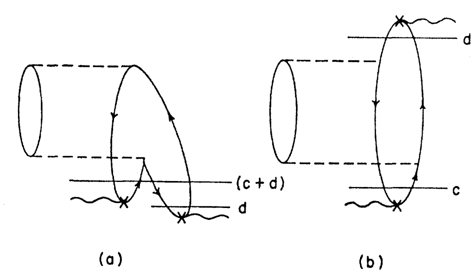


FIG. 4. (a) and (b) are conjugate graphs of class (c) in second order.

[22] By (4.7) a $p$ vertex is $O(r^0)$. A $p$ vertex which lies "outside" the remainder of a graph contributes a denominator $O(r)$.

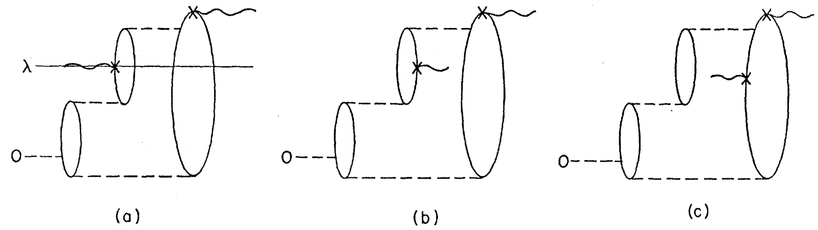


FIG. 5. Graphs (a), (b), and (c) form a triad of class ($\alpha$).

particle and hole lines differ in sign, it follows that diagrams conjugate in the above sense also differ in sign. The sum of these conjugate graphs is a correction of the order of the cranking moment and in the second order in particle-particle coupling is given by (4.9). In Fig. 6 the closed loop in each instance has been defined by choosing the internal line to be a particle line.

The third set of graphs ($\gamma$) which are likely to contribute to the persistence of interaction effects in higher orders are obtained by modifying the interaction between pair loops of small net momentum $r$ in typical pair-chain diagrams.[2] In order that these diagrams be $O(1/r^2)$, it is essential to restrict the nonlocality of the modified interactions. In a "chain" of $N$ linked pairs, there must be $N$ denominators each $O(r)$ so as to yield a term $\propto L^2$ in the limit $L\rightarrow\infty$. An example of such a modified pair "chain" is shown in Fig. 7.

Finally, it is discernible that the last two classes of diagrams discussed above which yield persistent interaction effects on the rigid moment in higher orders are simply the nonlocal modifications of those diagrams which have been previously found to compensate.[2]

## CONCLUDING REMARKS

We have explicitly derived an interaction correction to the inertial moment of a large many-body fermion system moving under periodic boundary conditions in the second order of particle-particle interaction. It follows necessarily from this result that the conjectured[5] cancellation of interaction effects in perturbation theory independent of potential form does not take place. (This result patently disagrees with a recent prediction of Wentzel[23] based on some earlier considerations of Blatt, Butler, and Schafroth[24] on the statistical mechanics of rotating buckets.) We think it may prove instructive to relate our work on the cranking moment of a system with periodic boundary conditions to earlier work of Wentzel[25] on the diamagnetism of a dense electron gas. In the latter case, the choice,

$$A_y=\lambda\cos rx,\quad A_x=A_z=0,\quad (r/k_F\ll 1) \tag{6.1}$$

for the vector potential of the external magnetic field, led to the interaction Hamiltonian

$$H^d=H_1{}^d+H_2{}^d, \tag{6.2}$$

with

$$H_1{}^d=-\frac{e\lambda}{2Mc}\sum_{\mathbf{k}\sigma}k_y(c_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma}{}^\dagger c_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma}+\text{H.c.}), \tag{6.3}$$

and[26]

$$H_2{}^d=\frac{e^2\lambda^2}{4Mc^2}N. \tag{6.4}$$

Neglecting $H_2{}^d$ momentarily, one may compare $H_1{}^d$ with the corresponding terms in the cranking interaction,[27]

$$H^c\equiv-\lambda(H_i(r))_{xy}$$
$$=-\frac{\lambda}{ir}\sum_{\mathbf{k}\sigma}k_y(c_{\mathbf{k}+\frac{1}{2}\mathbf{r},\sigma}{}^\dagger c_{\mathbf{k}-\frac{1}{2}\mathbf{r},\sigma}-\text{H.c.}). \tag{6.5}$$

Let us call the contributions to the ground-state energy $O(\lambda^2)$ resulting from a perturbation-theoretic treatment of $H_1{}^d$ and $H^c$, $E_1{}^d(\lambda)$ and $E^c(\lambda)$, respectively. Then we

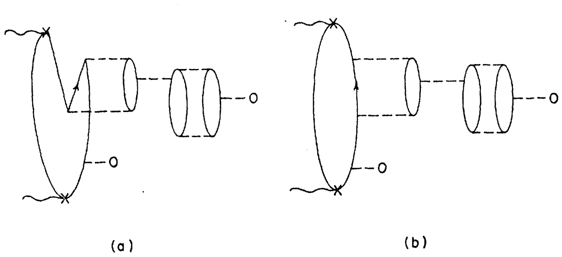


FIG. 6. An example of conjugate graphs in class ($\beta$).

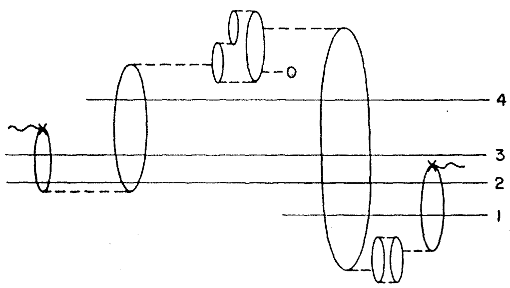


FIG. 7. An example of a pair-chain $O(1/r^2)$ with two modified interactions.

[23] G. Wentzel, Phys. Rev. Letters **7**, 349 (1960).

[24] J. M. Blatt, S. T. Butler, and M. R. Schafroth, Phys. Rev. **100**, 481 (1955). Their "theorem" does not appear to have a rigorous basis; further, it is not clear whether it is indeed applicable at $T=0$. (The remarks of Wentzel[23] imply that it is.)

[25] G. Wentzel, Phys. Rev. **108**, 1593 (1957).

[26] Only the diagonal part of $H_2{}^d$ matters.

[27] The difference between $H_1{}^d$ and $H^c$ with regard to the phase of $\lambda$ is irrelevant since the general choice, $2A_y=\lambda e^{irx}+\lambda^* e^{-irx}$, leads to $E(\lambda)\propto|\lambda|^2$.

have

$$[E_1{}^d(\lambda)/r^2E^c(\lambda)]=e^2/4M^2c^2. \tag{6.6}$$

The "rigid" diamagnetic energy, $-H_2{}^d$, then corresponds to the rigid inertial moment. [In the absence of particle-particle interaction, one finds[28]

$$\lim_{(r/k_F)\to 0} E_1{}^d(\lambda)+e^2\lambda^2N/4Mc^2=0, \tag{6.7}$$

so that the leading term in an expansion of $E_1{}^d(\lambda)+H_2{}^d$ in powers of $(r/k_F)$ gives the Landau susceptibility.[25]] In the case of Coulombic interaction, a Landau susceptibility[29] then implies a rigid inertial moment and conversely. Note that our investigation has been restricted here to the case of *nonsingular* particle-particle interactions; we regard our results as appropriate to this situation *only*. In the case of singular particle-particle interactions, e.g., Coulomb interaction, it will be necessary to include the damping[30] of the potential matrix elements in order to obtain convergent results.[29] One might carry this through properly by making the usual pair approximation[2,4] in the *transformed* Hamiltonian. It is readily seen that such a procedure extends the random-phase approximation well beyond its previous range[2,25] of utility in a wide range of problems involving interaction with weak external fields, e.g., the scattering of photons by an electron gas,[31] the polarization of a dense electron gas by substitutional impurities.[32] However, further consideration of these problems together with the modifications they entail in our method[33] is left to subsequent communications.


## ACKNOWLEDGMENTS

It is a pleasure to thank Dr. R. Amado, Dr. N. Hugenholtz, Dr. K. Sawada, and Dr. J. Weneser for helpful discussions.


[28] See the discussion in Sec. 4 of reference 25.

[29] We comprehend the possibility of correction to the Landau susceptibility arising from additional terms in the energy $O(r^2/k_F{}^2)$. [See H. Kanazawa and N. Matsudaira, Progr. Theoret. Phys. (Kyoto) **23**, 433 (1960).]

[30] The replacement of the bare potential by the appropriate $K$-matrix element (see reference 9) is meant.

[31] N. Tzoar and A. Klein, Bull. Am. Phys. Soc. **5**, 275 (1960).

[32] B. D. Silverman and P. R. Weiss, Phys. Rev. **114**, 989 (1959).

[33] In the case of the diamagnetism of a dense electron gas, the solution $f(\mathbf{k}+\mathbf{r}_>, \mathbf{k}_<)=-k_y/(\epsilon_{\mathbf{k}+\mathbf{r}}-\epsilon_k)$, of the integral equation,

$$k_y+f(\mathbf{k}+\mathbf{r}_>, \mathbf{k}_<)(\epsilon_{\mathbf{k}+\mathbf{r}}-\epsilon_{\mathbf{k}})+2\sum_{\mathbf{k}'} f(\mathbf{k}'+\mathbf{r}_>, \mathbf{k}_<')v(r)=0,$$

corresponds to the pair approximation of Wentzel[25] which omits exchange.[29]