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Rf Hall Effect in a Superconductor

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The rf Hall field in a superconductor is calculated for the applied dc magnetic field and the rf electric field both parallel to the metal surface. A two-fluid model and specular reflection boundary conditions are used to solve the transport problem in the limits of the classical and the extreme anomalous skin effects. It is found in both limits that the rf Hall effect is nonzero. The magnitude of the Hall angle in superconducting tin is estimated.

INTRODUCTION

THE usual static experiment with crossed electric and magnetic fields can not give rise to a Hall field in a superconductor. The infinite static conductivity associated with the superconducting electrons implies that no macroscopic electric field exists inside the superconductor. However, since high-frequency electric fields can penetrate the superconductor, an rf Hall measurement may be possible.

An unsuccessful search for a low-frequency (80–800 cps) Hall field was made by Lewis.[1] He looked for the ac Hall voltage between the pole and equator of a superconducting vanadium prolate spheroid when an ac longitudinal magnetic field was applied. He concluded that the absence of a Hall effect indicated a failure of the two-fluid model of superconductivity.[1,2]

Although no measurements have been reported on a microwave Hall effect in superconductors, such an experiment on normal metals has been carried out by Cooke.[3,4] In the Cooke experiment the dc magnetic field is perpendicular to and the rf electric field is parallel to the metal surface. Because of demagnetization effects in a superconductor, the dc magnetic field at the surface is approximately parallel to the surface. Therefore, it is suggested that for the superconducting case, both the dc magnetic and the rf electric field be applied in the plane parallel to the surface. Then, if any Hall field exists, it would be directed normal to the surface.

An actual microwave experiment is limited to the study of the Hall field at the surface of the metal. The measurement consists of external excitation of one cavity mode which the Hall field then couples to an orthogonal mode.[5] The strength of the coupling can be related to the Hall angle θ_H defined by

$$\tan\theta_H = E_z(0)/E_x(0), \tag{1}$$

in which z is the direction into the superconductor, $E_z(0)$ is the Hall field evaluated at the surface ($z=0$) and $E_x(0)$ is the applied rf electric field at $z=0$. It is of interest to inquire whether the microwave Hall effect in a superconductor is large enough to be observed.

In this paper, the rf Hall field E_z is calculated for the applied dc magnetic field H_0 and the rf electric field E_x both parallel to the surface. A two-fluid model and specular reflection boundary conditions are used to solve the transport problem in the limits of the classical and extreme anomalous skin effects. For comparison, the result is given for a normal metal under conditions of the extreme anomalous limit. The calculation of E_z is limited to the term linear in H_0.

SUPERCONDUCTING HALL FIELD

The surface of the superconductor is considered to be a flat plate in the xy plane with the z direction normal to the surface and extending into the superconductor. The dc magnetic field H_0 is taken to be parallel to the y axis and to have a spatial variation $H_0(z)$. The rf electric field has components in the x and y directions.

A two-fluid model calculation for the surface impedance of a superconductor in a magnetic field with this geometry has been given previously.[6] The superconducting electron current is related to the fields by the London equations while the normal electron current is found by solution of the Boltzmann equation for a magnetic field with spatial z dependence. Since the solution of the Boltzmann equation involves an iterative technique, the presentation is limited to a Hall field linear in H_0.

It is found that for the rf electric field parallel to H_0, the magnetic field does not couple to components of E perpendicular to H_0. Thus a Hall field is found only for the E_x component. The Fourier transform for the Hall field is given in I by

$$E_{q,z} = \frac{2ic^2}{\pi}\frac{\omega_c\bar{\tau}E_x'(0)}{K_{zz}(i\bar{l}q)}\int_{-\infty}^{\infty}\frac{q'dq'}{q'^2+\lambda^{-2}} \times L_{xz}(i\bar{l}q,i\bar{l}q')\Gamma_x(q-q'), \tag{2}$$

in which $\omega_c = eH_0(m^*c)^{-1}$ and $\bar{\tau} = \bar{l}/v_F = \tau(1+i\omega\tau)^{-1}$, with m^*, v_F, and τ being the effective mass, the Fermi velocity and the relaxation time for the normal electrons in the superconducting state. The functions K_{zz}, L_{xz},

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[1] H. W. Lewis, Phys. Rev. **92**, 1149 (1953).
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Γ_x are given in I as

$$K_{zz}(t)=-\frac{2}{t^3}\left[2t-\ln\left(\frac{1+t}{1-t}\right)\right], \tag{3a}$$

$$L_{xz}(t,t')=\frac{2t}{t'}-\frac{t^2(1-t^2)}{2t'^2}K_{zz}(t) -\frac{(t-t')[2t'-t+t(t-t')^2]}{2t'^2}K_{zz}(t-t'), \tag{3b}$$

and

$$\Gamma_x(q)=\{-q^2c^2+\omega^2-\omega_s^2-\tfrac{3}{2}(i\omega\omega_n^2\bar{\tau}) \times[4-(1+\bar{l}^2q^2)K_{zz}(i\bar{l}q)]\}^{-1}. \tag{3c}$$

The parameters ω_s and ω_n are related to the superconducting penetration depth λ and to the normal electron concentration N by

$$\omega_s=c\lambda^{-1}, \tag{4a}$$

and

$$\omega_n^2=4\pi Ne^2m^{*-1}. \tag{4b}$$

The inverse Fourier transform of Eq. (2) gives the Hall field

$$E_z(z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}E_{q,z}e^{-iqz}dq, \tag{5}$$

or

$$E_z(z)=\frac{ic^2E_x'(0)\omega_c\bar{\tau}}{\pi^2}\int_{-\infty}^{\infty}dqe^{-iqz}\int_{-\infty}^{\infty}\frac{dp(q+p)\Gamma_x(p)}{(q+p)^2+\lambda^{-2}} \times L_{xz}(i\bar{l}q,i\bar{l}[p+q]). \tag{6}$$

Since, physically, the Hall field is odd in z, Eq. (5) implies that

$$E_{q,z}=-E_{-q,z}. \tag{7}$$

Application of this symmetry condition to Eq. (2) shows that $E_z(z)$ is either zero or discontinuous at $z=0$. The calculation presented here yields a discontinuous Hall field at the surface which implies the accumulation of a surface charge.

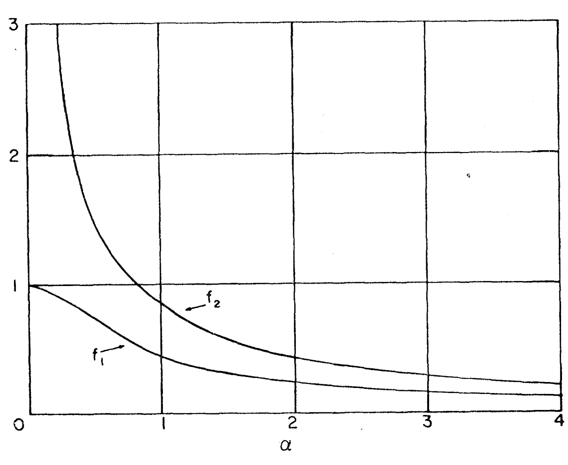

FIG. 1. Dimensionless plot of $f(\alpha)=f_1(\alpha)+if(\alpha)$ vs $\alpha=\lambda/\delta_n$.

The integration of Eq. (6) yields

$$E_z(z)=\frac{c^2E_x'(0)\omega_c\bar{\tau}}{\pi}e^{-z/\lambda} \times\int_{-\infty}^{\infty}dp\,\Gamma_x(p)e^{ipz}\frac{L_{xz}(-i\bar{l}p+\bar{l}/\lambda,\,\bar{l}/\lambda)}{K_{zz}(-i\bar{l}p+\bar{l}/\lambda)}, \tag{8}$$

which can be evaluated in the limits of the classical skin effect ($|\bar{l}p|$, $|\bar{l}/\lambda|\ll 1$) and of the extreme anomalous skin effect ($|\bar{l}p|$, $|\bar{l}/\lambda|$, $\gg 1$).

CLASSICAL LIMIT

The expansions of K_{zz}, L_{xz}, and Γ_x in Eq. (3) for $|\bar{l}p,|$, $|\bar{l}/\lambda|\ll 1$ are

$$K_{zz}(-i\bar{l}p+\bar{l}/\lambda)\cong\tfrac{4}{3},$$
$$L_{xz}(-i\bar{l}p+\bar{l}/\lambda,\,\bar{l}/\lambda)\cong\tfrac{4}{3}, \tag{9}$$
$$\Gamma_x(p)\cong-[c^2(p^2-k_c^2)]^{-1},$$

in which

$$k_c=(-\lambda^{-2}+\omega^2c^{-2}-i\omega\omega_n^2\bar{\tau}c^{-2})^{\frac{1}{2}}, \tag{10}$$

with $\mathrm{Re}(k_c)<0$ and $\mathrm{Im}(k_c)>0$. Substitution of Eqs. (9) into Eq. (8) yields upon integration

$$E_z(z)=-iE_x'(0)\omega_c\bar{\tau}k_c^{-1}e^{-z/\lambda}e^{ik_cz}. \tag{11}$$

In the classical limit, the electric field $E_x(z)$ is given in I as

$$E_x(z)=-iE_x'(0)k_c^{-1}e^{ik_cz}+\mathcal{O}(H_0^2), \tag{12}$$

which gives the result for the Hall angle

$$\tan\theta_H=\omega_c\tau(1+i\omega\tau)^{-1}. \tag{13}$$

The result for the rf Hall angle of a superconductor in the classical limit is the same as the static Hall angle for semiconductors.[7] This result is physically not unreasonable since the Hall field falls off faster than $E_x(z)$ by approximately the factor $e^{-z/\lambda}$. Thus the dc magnetic field is effectively constant over the region of the metal where the Hall field is appreciable. The Hall field increases with decreasing τ. However, in the limit of long relaxation time, the calculation must employ the anomalous skin effect.

EXTREME ANOMALOUS LIMIT

The expansions of K_{zz}, L_{xz}, and Γ_x in Eq. (3) for $|\bar{l}p|$, $|\bar{l}/\lambda|\gg 1$ are

$$K_{zz}(-i\bar{l}p+\bar{l}/\lambda)\simeq 4[\bar{l}(p+i/\lambda)]^{-2},$$
$$L_{xz}(-i\bar{l}p+\bar{l}/\lambda,\,\bar{l}/\lambda)\simeq\pm\pi[\bar{l}^3p^2(p+i/\lambda)]^{-1},$$
$$\Gamma_x(p)\simeq -pc^{-2}\sum_{i=1}^{3}C_i(p-p_i)^{-1}. \tag{14}$$

[7] W. Shockley, *Electrons and Holes in Semiconductors* (D. Van Nostrand Company, Inc., Princeton, New Jersey, 1950), Chap. 11.

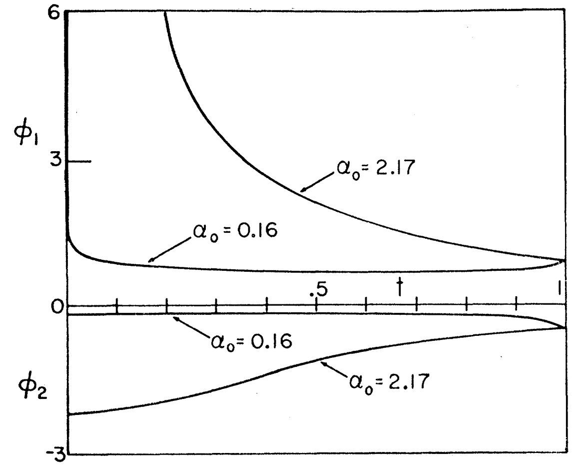

FIG. 2. Plot of $\phi(t)=\phi_1(t)+i\phi_2(t)$ vs $t=T/T_c$ for two values of the parameter α_0. Except for a scale factor (see text) the curves represent the tangent of the Hall angle vs t.

The symbol p_i is a root of the equation

$$p^3+p\lambda^{-2}-i\delta_n^{-3}=0, \tag{15}$$

in which δ_n is the skin depth in the extreme anomalous limit[8]

$$\delta_n=(\tfrac{3}{4}\pi\omega\omega_n^2c^{-2}v_F^{-1})^{-\frac{1}{3}}, \tag{16}$$

and

$$C_i=(p_i-p_j)^{-1}(p_i-p_k)^{-1}, \tag{17}$$

with $i=x$, y, z and $i\neq j\neq k\neq i$. The integration of Eq. (6) yields a nonvanishing Hall field at the surface

$$E_z(0)=\pm 2\lambda^2E_x'(0)\omega_c v_F^{-1}\sum_{i=1}^{3}\mathcal{C}_i\ln(-\epsilon_i), \tag{18}$$

in which the roots p_i are related to ϵ_i by

$$\begin{aligned}\epsilon_i&=i\lambda p_i,\\ \mathcal{C}_i&=-\lambda^{-2}C_i.\end{aligned} \tag{19}$$

The component of the electric field $E_x(z)$ at the surface is from I

$$E_x(0)=-(c^2/4\pi i\omega)Z_0E_x'(0)+\mathcal{O}(\omega_c^2), \tag{20}$$

where Z_0 is the zero-field surface impedance

$$Z_0=-8\omega\lambda c^{-2}\sum_{i=1}^{3}\mathcal{C}_i\epsilon_i\ln(-\epsilon_i). \tag{21}$$

The result for the Hall angle in the extreme anomalous limit is

$$\tan\theta_H=-\pi i\omega_c\lambda v_F^{-1}f(\alpha), \tag{22}$$

in which the function

$$\begin{aligned}f(\alpha)&=\sum_{i=1}^{3}\mathcal{C}_i\ln(-\epsilon_i)/\sum_{i=1}^{3}\mathcal{C}_i\epsilon_i\ln(-\epsilon_i)\\ &=f_1+if_2\end{aligned} \tag{23}$$

is plotted in Fig. 1 against the dimensionless parameter $\alpha=\lambda/\delta_n$. The sign of Eq. (22) is determined from the Hall angle in the normal state $(\alpha\rightarrow\infty)$,

$$\tan\theta_{HN}=(\sqrt{3}\pi/2)\omega_c\delta_n v_F^{-1}(1-i/\sqrt{3}). \tag{24}$$

The relation, $\lim_{\alpha\rightarrow\infty}f(\alpha)=(1+i\sqrt{3})/2\alpha$ has been used in deriving Eq. (24).

The result for the Hall angle as given by Eq. (22) is not valid near $\alpha=0$ (i.e., near $T=0°$K), since the expansions of Eq. (14) neglect important contributions near $p=0$ when one of the roots of Eq. (15) approaches zero. Physically, as $T\rightarrow 0°$K the normal electron concentration N approaches zero; thus, the skin depth δ_n becomes very large. In this limit, the conditions of the anomalous skin effect are no longer satisfied; the metal becomes more "classical," and the Hall angle increases (see Fig. 1). The Hall angle increases as the skin depth increases until the limiting value of $\delta_n=|\bar{l}|$ is reached. A cutoff is also obtained mathematically at $\alpha=\lambda/\bar{l}$, by treating the integral of Eq. (8) in two parts $0\rightarrow 1/\bar{l}$ and $1/\bar{l}\rightarrow\infty$ and by substituting appropriate expansions in the two regions.

In order to present the variation of the Hall angle with temperature, the Gorter-Casimir model is introduced. The parameter α is related to the reduced temperature t by

$$\alpha=\alpha_0t^{\frac{4}{3}}(1-t^4)^{-\frac{1}{3}}, \tag{25}$$

in which

$$\alpha_0=\lambda_0\left(\frac{3\pi^2\omega N_0e^2}{m^*v_Fc^2}\right)^{\frac{1}{3}}. \tag{26}$$

The temperature dependence of the Hall angle can then be written as

$$\tan\theta_H=\left(\frac{\pi e}{3\omega cN_0m^{*2}v_F^2}\right)^{\frac{1}{3}}H\phi(t), \tag{27}$$

in which the function

$$\phi(t)=\phi_1(t)+i\phi_2(t)=-\frac{i\alpha_0}{(1-t^4)^{\frac{1}{3}}}f[\alpha_0t^{4/3}(1-t^4)^{-\frac{1}{3}}] \tag{28}$$

is plotted in Fig. 2 vs t for $\alpha_0=0.16$ (free electron mass) and for $\alpha_0=2.17$ (light mass carrier). These values for α_0 were determined in studies of the field variation of the surface impedance of superconducting tin.[9] The numerical value for $\tan\theta_H$ in superconducting tin at 1 kMcps in a field of 100 gauss is $\sim 4\times 10^{-4}$ for $\alpha_0=0.16$ and ~ 2 for $\alpha_0=2.17$. Provided that the conditions of

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the extreme anomalous skin effect are satisfied, the magnitude of the Hall effect is not sensitive to frequency.

The two fluid model predicts an rf Hall effect to be observable in a superconductor. If a Hall field should be detected experimentally, Hall measurements would provide a useful tool for the study of the band structure of superconductors.

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Some Optical Properties of CdSe Single Crystals

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Optical transmission and reflection measurements were made on a single crystal of CdSe (crystal class C_{6v}) with light polarized parallel and perpendicular to the c axis. The absorption coefficients, $\mu_{\perp}$ and $\mu_{\parallel}$, were determined and are presented as functions of incident photon energy, at room temperature. The selection rule, $\mu_{\perp}>\mu_{\parallel}$, expected for all solids in the crystal classes C_{nv} and C_n, where $n=3$, 4, or 6, is very well obeyed in CdSe. The absorption coefficients are not simple functions of the photon energy, but are nearly exponential and almost follow Urbach's rule.

The fluorescence of the single crystal has been measured at 77°K. It is peaked at 920 mμ and it is slightly polarized perpendicular to the c axis. The results are similar to what has been obtained for hexagonal ZnS.

I. INTRODUCTION

RECENTLY, there have been extensive studies concerning the anisotropic behavior of the fundamental optical absorption processes in crystals possessing a unique direction. CdS has been studied by Dutton,[1] Thomas and Hopfield[2] and by Gross and Razbirin.[3] Piper, Marple, and Johnson,[4] Lempicki,[5] and Keller and Pettit[6] have studied ZnS. Thomas[7] has reported on ZnO and Casella and Keller[8] have reported on tetragonal $BaTiO_3$. For all of these materials it was found that at the absorption edge, light polarized perpendicular to the c axis is more strongly absorbed than light polarized parallel to the c axis. Hopfield[9] has given a theoretical treatment for ZnO and Wheeler[10] has treated CdS using the exciton picture, Birman[11] and Casella[12] independently proposed an explanation of the CdS and ZnS data by invoking interband transitions. The selection rule follows whether we assume an exciton model or direct interband transitions. There have also been attempts to apply the selection rules to fluorescent emissions resulting from transitions, involving deep lying states.[5,6,11,13]

Birman[11] has based his derivation of the selection rule, $\mu_{\perp}>\mu_{\parallel}$, where μ is the absorption coefficient, on explicit assumptions regarding the detailed symmetries of the valence and conduction bands of ZnS. Casella[12] has noted that, at the absorption edge, the selection rule follows independently of the symmetries of the bands, provided only that they be different. Both of these treatments are pertinent to and are borne out by the data of the previous workers. Birman's model is more amenable to a physical interpretation in that there is deduced from it two closely spaced valence bands derived from a spin-orbit interaction. The double valence band model has afforded an interpretation for measurements of the photoconductivity in CdS [14] and ZnS,[15] using polarized light.

The material CdSe has the wurtzite structure and it allows for the testing of the applicability of the above concepts to another compound in the crystal class C_{6v}. Transmission measurements were made and it has been determined that the selection rule is fairly well obeyed

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