Second-Difference Analysis of Bremsstrahlung Yield Curves

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The second-difference weighting function of the bremsstrahlung spectrum obtained from an empirical isochromat at 15.1 Mev and corrected for thick-target effects is shown to behave like a delta function of width approximately equal to the energy interval in analysis. This behavior greatly simplifies cross-section analysis of bremsstrahlung-induced activation curves when resonance phenomena are present. Application of the method to preliminary yield data for the (γ,n) reaction in nitrogen and oxygen gives good agreement with known level structure.

THE existence of sharp changes in curvature for bremsstrahlung-induced photoactivation yields in the case of light nuclei have been reported by several investigators.[1–3] These so-called "breaks" have been attributed to narrow isolated resonances in the photon absorption cross section. Extensive studies for carbon[1] and oxygen[2] reveal many "breaks" from threshold to 24 Mev. However, the lack of complete agreement between the energy assignments and the number of "breaks" with known levels obtained from other reactions has led to a confused situation and cast doubt upon the existence and physical significance of "breaks." This is partly due to errors in betatron energy scales and ambiguities inherent in the determination of "breaks" from integrated yield curves. For the purpose of nuclear spectroscopy, it becomes necessary to extract the (γ,n) cross section from the observed bremsstrahlung yield data. This problem is extremely difficult by conventional methods; detailed knowledge of the bremsstrahlung spectrum is required and analysis in energy intervals of 25–100 kev is essential in order to resolve the resonances. The following procedure, based on the second-difference yields, simplifies the calculations and produces promising results.

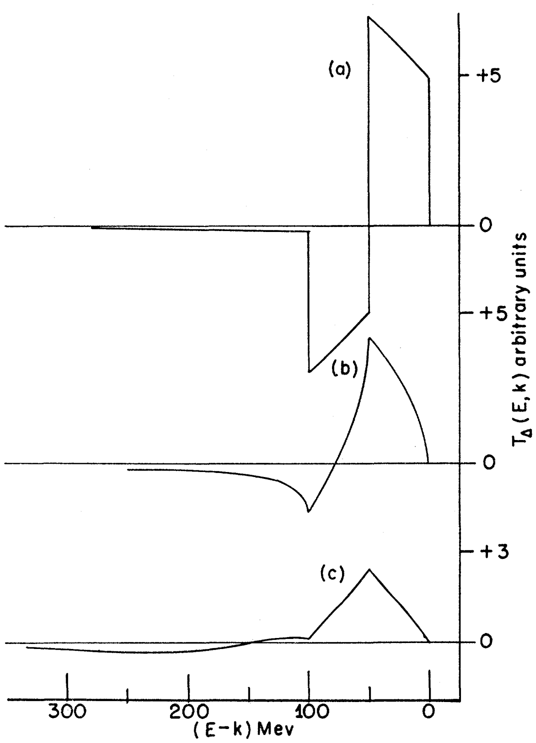

FIG. 1. Representative weighting functions, $T_\Delta(E,k)$ for (a) Schiff spectrum, (b) experimental isochromat, and (c) thick-target spectrum for $\Delta E=50$ kev.

In simplified form, the bremsstrahlung yield per unit monitor response is given by

$$Y(E)=\frac{g}{R(E)}\int_0^E N(E,k)\sigma(k)dk, \qquad (1)$$

where g is a normalization factor, $R(E)$ the monitor

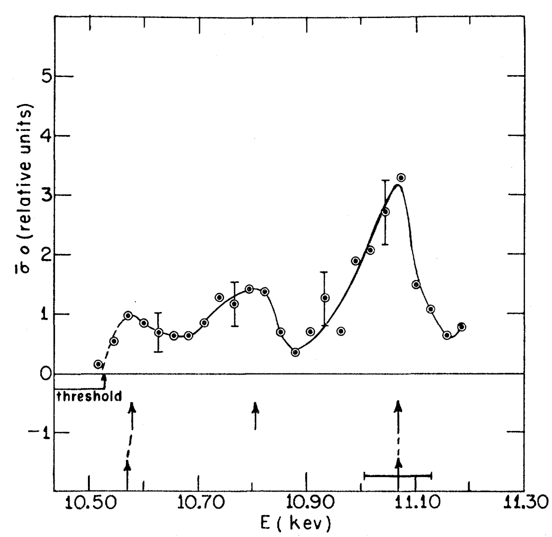

FIG. 2. Zero-order cross section for the reaction $N^{14}(\gamma,n)N^{13}$ near threshold. Baseline arrows indicate energy assignments of known levels and horizontal bars the level widths, where known. Correspondence between levels observed in the present experiment with known levels indicated by dashed line.

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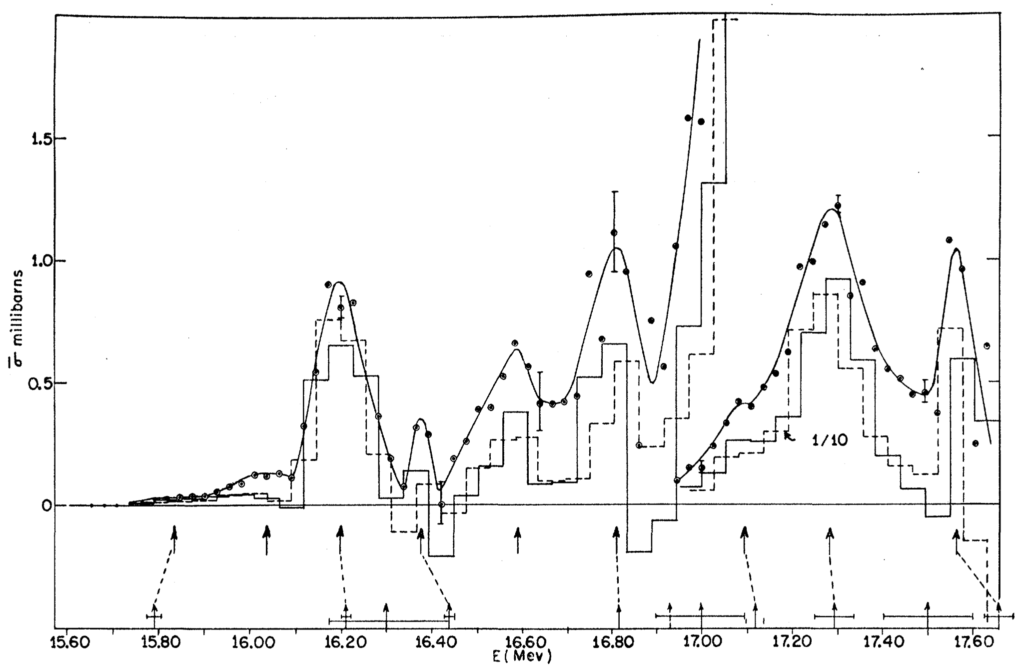

FIG. 3. Zero-order (bars) and final cross section (solid curve) for the $O^{16}(\gamma,n)O^{15}$ reaction from threshold to 17.6 Mev. Baseline arrows indicate energy assignments of known levels and horizontal bars the level widths, where known. Correspondence between levels observed in the present experiment with known levels indicated by dashed line.

response function which normalizes the spectrum to unit monitor response, $N(E,k)$ the incident photon spectrum at maximum energy E, and $\sigma(k)$ the cross section for the (γ,n) reaction. In practice, $Y(E)$ is known for a finite set of values separated by equal energy intervals ΔE. The second-difference of the reduced yields, defined as $y(E)=Y(E)R(E)$, is given by

$$y_2(E)=g\int_0^E T_\Delta(E,k)\sigma(k)dk, \qquad (2)$$

where

$$T_\Delta(E,k)\equiv N(E+\Delta,\, k)-2N(E,k)+N(E-\Delta,\, k), \quad (3)$$

and $N(E,k)=0$ for $k>E$.

Based on an empirical isochromat[4] at 15.12 Mev and correcting for thick target effects,[5] the weighting function, $T(E,k)$, is sharply peaked at $k=E$, of width $\sim\Delta E$, and slightly negative for $k<E$. Representative weighting functions corresponding to $\Delta E=50$ kev for Schiff spectrum, empirical isochromat, and thick target spectrum are shown in Fig. 1. The empirical isochromat and thick target spectrum correspond to an effective target thickness of 3 mil platinum (150 kev thick) and detector acceptance angles of 0.02 and 0.2 radian, respectively. The target thickness was determined from angular distribution measurements of the bremsstrahlung intensity. Thus, to zero order, the second-difference corresponds to the average cross section in the interval ΔE at $k=E$. Deviations from the true cross section are to be expected since $T_\Delta(E,k)$ is not a proper weighting function. The negative oscillation in T for $k<E$ causes the second-difference to go negative on passing a sharp resonance. However, resonant structure is well reproduced and reconstruction of the true cross section can be accomplished if $T_\Delta(E,k)$ is known.

This procedure has been applied to study the resonance structure of N^{14} near the neutron threshold at 10.53 Mev and of O^{16} from threshold at 15.65 Mev to 17.6 Mev. In this analysis, the $y_2(E)$ were obtained by fitting the measured yields in limited energy intervals to a smooth function of quartic form. This smoothing procedure, necessitated by the statistical fluctuations and reproducibility of preliminary yield data, broadens the weighting function, thereby reducing the effective resolution. Detailed discussion of the smoothing procedure will be given in a later publication.

The zero-order cross section for nitrogen is shown in Fig. 2. The yields were analyzed in intervals of 55 kev for two sets of data displaced 27 kev. Resonances observed at 10.58 Mev and 11.07 Mev are in excellent agreement with known levels reported at 10.57 Mev and 11.07 Mev.[6] The present data also indicates a level

[4] An isochromat represents the number of photons for a fixed energy k as the primary energy E is varied. To a good approximation, the isochromat represents the spectrum shape in the region 2 Mev removed from the high-energy tip for the range of primary energies here considered.

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[6] F. Ajzenberg-Selove and T. Lauritsen, Nuclear Phys. 11, 1 (1959).

TABLE I. Summary of levels observed in the present experiment and in other type reactions.

O^{16*a} (Mev)	O^{16*b} (Mev)	Γ^{b} (kev)	Reaction
15.83	15.79	30	(p,α)
16.04			
16.20	16.21	23	(p,n)
(16.38)	16.44	24	(p,α)
16.59			
16.81	16.75		(γ,α)
	16.82		(p,α)
17.10	17.12	41	(p,n)
	17.24[c]	280	(p,γ)
17.28	17.29	84	(p,n)
	17.30		(γ,α)

[a] Present experiment.
[b] F. Ajzenberg-Selove and T. Lauritsen, Nuclear Phys. **11**, 1 (1959).
[c] N. W. Tanner, G. C. Thomas, and W. E. Meyerhof, Nuovo cimento **14**, 257 (1959).

at 10.81 Mev which has previously not been observed. The uncertainties in the zero-order cross section indicated in Fig. 2 are due to statistical errors in the yields.

The oxygen zero-order cross section (bars) together with the final cross section (solid curve) are shown in Fig. 3. The latter curve was obtained by assuming $T(E,k)$ to be triangular with a small negative component for values of $k \geqslant E-10\Delta$. The results indicate the presence of a number of excited states in O^{16}. The energy assignments for these states are tabulated in Table I and compared with the known level scheme. The resonance observed at 16.38 Mev is indefinite and requires further investigation. The excited states at 16.04 Mev and 16.59 Mev have previously not been reported. The cross section presented in Fig. 3 is based on analysis in 55-kev intervals for two sets of data displaced 27 kev. The cross-section values are order of magnitude based on a rough estimate of the normalization factor g.

The above results were obtained from yield data taken without previous intentions of applying this type of analysis. Plans are now being implemented to acquire data specifically for this purpose.

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Evidence for the Statistical Model Obtained from (p,n) Spectra of the Odd-Even Elements V, Mn, Co, Nb, Rh, and In†

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Spectra of neutrons emitted from (p,n) reactions have been measured for the essentially monoisotopic odd-even elements V, Mn, Co, Nb, Rh, and In. The 7–8 Mev protons used were obtained from the Livermore 90-in. cyclotron. Neutron energy distributions were determined using time-of-flight techniques. Experimental results indicate agreement with the compound statistical model of the nucleus. Relative level densities obtained in this manner were found to increase with atomic weight as theoretically expected. Assuming a level density of the form $\exp[2(a\epsilon)^{\frac{1}{2}}]$, a values were obtained which varied as $a=A/13$.

INTRODUCTION

MEASUREMENTS of particle spectra emitted from nuclear reactions in the energy region below 30 Mev have been carried out by many investigators. Results of these measurements have been compared with theoretical calculations derived from the compound statistical model[1] of the nucleus, with various direct interaction models of the nucleus, and with various combinations of the two models.

Discrepancies between experiment and theoretical predictions of the statistical model have been observed.[2–8] Gugelot[2] measured inelastic scattering in various medium-atomic-weight targets using 18-Mev protons. He also measured neutron spectra arising from nuclear excitation produced by 16-Mev protons. Both of the experiments showed a considerable excess of high-energy particles over that expected according to the compound statistical model of the nucleus. Moreover, anisotropic behavior was observed; an excessive

† This work was done under the auspices of the U. S. Atomic Energy Commission.
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