ter charge distribution reduces to the Fermi two-parameter distribution [Eq. (1)] when w equals zero. The parameter z_2 is related to the surface thickness, but no simple relation can be given with the 90%–10% fall-off distance. We find a whole range of values of w for which we can reproduce the curve for the Fermi two-parameter case; however, we have not found a combination of parameters which will raise the cross sections by 35%. The Bi question, therefore, has not been resolved, and more analysis is necessary to determine a model to give a better fit to both the angular distribution and the absolute cross sections.

Figure 3 shows, in addition to the elastic scattering peak, peaks from inelastic scattering of electrons from In^{115} with excitation of nuclear levels at about 2.6 and 5.1 Mev. The form factors associated with inelastic scattering with excitation of these levels, and others in Ni^{58}, Ni^{60}, and Pb^{208}, is being studied; some of the results have been reported.[9]

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Effects of the Pion-Pion Resonance and the Three-Pion Resonance or Bound State on Neutral-Pion Decay*†

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We have applied the dispersion method to the problem of neutral-pion decay. It is shown that a pion-pion P-wave resonance can produce a large effect in the decay matrix element. The contribution is related to the semifundamental constant which determines the rate of photoproduction of pions from pions. The contribution of a strong three-pion state is also considered.

RECENTLY Frazer and Fulco[1] and Chew[2] have proposed that a two-pion P-wave resonance and a three-pion resonance or bound state may account for the isotopic vector and scalar components of nucleon electromagnetic structure, respectively. The purpose of this note is to investigate the effects of such resonances on neutral-pion decay.

The dispersion analysis of neutral-pion decay was first considered by Goldberger and Treiman,[3] but they assumed nucleon-antinucleon pairs to be the most important intermediate states and neglected multipion states. Here we adopt a different approach and consider the contributions of the least massive states. This can be done if we extend a photon variable q^2 into the complex plane instead of the meson variable p^2 used by Goldberger and Treiman.

Following the standard method, one has[4] (see Fig. 1)

$$\langle q(\mu),k(\nu)|T|p(3)\rangle = \frac{i(2\pi)^4\delta^4(p-q-k)F_\nu(-q^2;-k^2;-p^2)\epsilon_\nu'}{(8q_0k_0p_0)^{\frac{1}{2}}}, \quad (1)$$

where we have

$$F_\nu = (4p_0q_0)^{\frac{1}{2}}\langle q(\mu)|J_\nu(0)|p(3)\rangle,$$

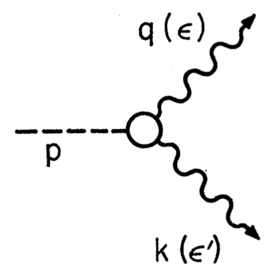

FIG. 1. Neutral-pion decay. Wavy lines are photons; broken line, pion.

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[4] We use the fundamental metric tensor such that $-p^2=M^2$. Units are used in which $\hbar=c=\mu=1$, where μ is the mass of the pion.

and p is the pion four-momentum. The indices μ and ν refer to the polarization of the photons of momenta q and k, respectively. The number "3" inside the matrix element represents a neutral pion in the initial state.

From general invariance arguments, the F function can be written in the form

$$F(-q^2;-k^2;-p^2) = \epsilon_{\alpha\beta\mu\nu}q_\alpha k_\beta \epsilon_\mu \epsilon_\nu' f(-q^2;-k^2;-p^2). \quad (2)$$

We assume that, with both p^2 and k^2 on the mass shell, the scalar function $f(-q^2)$ satisfies the following dispersion relation without subtraction:

$$f(-q^2)=\frac{1}{\pi}\int_4^\infty \frac{\mathrm{Im} f(t)dt}{t+q^2}. \quad (3)$$

The mean lifetime of π^0 is then given by

$$\tau=64\pi[f(0)]^{-2}. \quad (4)$$

Using the unitary condition, we can express the absorptive part of F as

$$\mathrm{Im}F=\pi\epsilon_\mu\epsilon_\nu'(2p_0)^{\frac{1}{2}}\sum_n \delta^4(q-p_n) \times\langle 0|J_\mu(0)|n\rangle\langle n|J_\nu(0)|p(3)\rangle.$$

Since our approach is to assume that the function is determined by nearby singularities, no intermediate state except the 2π and 3π states are considered here. Let us first investigate the effect from the two-pion P-wave resonance alone and leave the 3π contribution to be discussed later; then we can show that

$$\mathrm{Im}F(-q^2)=\frac{e_r}{48\pi}\epsilon_{\alpha\beta\mu\nu}q_\alpha k_\beta\epsilon_\mu\epsilon_\nu' \times\frac{(-q^2-4)^{\frac{3}{2}}}{(-q^2)^{\frac{1}{2}}}F_\pi^\dagger(-q^2)M_1(-q^2), \quad (5)$$

where $F_\pi^\dagger$ is the Hermitian conjugate of F_π, the pion form factor, and M_1 is the P-wave amplitude for photoproduction of pions from pions. It has been shown by the author that the M_1 function is linearly related to a real constant Λ and the F_π function.[5] Comparing Eqs. (2) and (5), we obtain

$$f_{2\pi}(-q^2)=\frac{e_r}{48\pi^2}\int_4^\infty \frac{(t-4)^{\frac{3}{2}}}{t^{\frac{1}{2}}(t+q^2)}F_\pi^\dagger(t)M_1(t)dt. \quad (6)$$

James Ball[6] has applied the Mandelstam representation to the $\gamma+N\rightarrow\pi+N$ problem, and finds that $|\Lambda|$ must be less than $1.8e$ in order to make his calculated cross section compatible with experimental data. If $|\Lambda|=1.8e$ and the Frazer-Fulco form factor is used for a resonance at $t_{2\pi}=10$, we find $\tau=2\times10^{-16}$ sec. Since the experimental upper limit for τ is 4×10^{-16} sec,[7] we see that the contribution of a two-pion resonance is capable of producing a large effect in the neutral-pion decay. If the resonance is sharp, it follows from (6) that $f_{2\pi}(-q^2)\sim(t_{2\pi}+q^2)^{-1}$.

We note that the f function in Eq. (6) depends on the virtual photon mass variable. Thus $-x\equiv q^2=(P_++P_-)^2$ represents the square of the total four-momentum of the electron-positron pair in the process $\pi^0\rightarrow\gamma+e^++e^-$. Since x is less than 1, we can write

$$f(x)=f_{2\pi}(0)(1+\alpha x)=f_{2\pi}(0)\left(1+\frac{x}{t_{2\pi}}\right), \quad (7)$$

for small x.[8] Thus, we see that α is always positive in this approximation.

The calculation so far is based on the assumption that only the 2π state contributes to the dispersion integral, but there is no good reason to expect the 3π contribution to be negligible. Since no one has succeeded in treating the matrix element $(\gamma\pi|3\pi)$, we are not able to handle this part. However, we may observe that if the three-pion $I=0$ and $J=1$ state is resonant or bound at energy $(t_{3\pi})^{\frac{1}{2}}$, as suggested by Chew, then

$$f_{3\pi}(-q^2)\approx\text{constant}/(q^2+t_{3\pi}),$$

so that

$$f(-q^2)=f_{2\pi}(-q^2)+f_{3\pi}(-q^2)\approx\frac{f_{2\pi}(0)}{1+(q^2/t_{2\pi})}+\frac{f_{3\pi}(0)}{1+(q^2/t_{3\pi})}.$$

Thus, if we define

$$\beta=f_{3\pi}(0)/f_{2\pi}(0),$$

then for small $x(\equiv-q^2)$,

$$f(x)\cong f(0)\left[1+\left(\frac{1}{1+\beta}\right)\left(\frac{1}{t_{2\pi}}+\frac{\beta}{t_{3\pi}}\right)x\right]. \quad (8)$$

Thus, if the 2π and 3π states make comparable contributions with opposite signs (as they do in the neutron charge structure) and $t_{2\pi}\neq t_{3\pi}$, it is possible that the parameter α defined in Eq. (7) may be negative.[9] An attempt is in progress to determine β from other experimental data involving the 3π state.

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[8] S. M. Berman and D. A. Geffen (to be published). These authors have also studied the f function from a different approach and obtained $\alpha\sim0.06$.

[9] The author is indebted to Dr. N. Samios of Brookhaven National Laboratory for advance communication of his preliminary experimental result that $\alpha=-0.24\pm0.16$.