Possible Effect of $\bar{K}-N$ Reactions on $K-N$ Scattering*

RICHARD H. CAPPS†
Argonne National Laboratory, Argonne, Illinois, and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
(Received August 15, 1960)

Approximate P-wave dispersion relations for $K-N$ scattering are considered, under the assumption that the $K\Lambda N$ and $K\Sigma N$ interactions are of the odd intrinsic-parity, Yukawa type. If the crossing terms (which involve the P-wave $\bar{K}-N$ amplitudes) are neglected, the predictions of the equations are contradicted by the experimental evidence. However, the presence of virtual pion-hyperon states may enhance the $\bar{K}-N$ amplitudes sufficiently that they play an important role in the equations. It is shown that if the isotopic spin 1, $P_{\frac{3}{2}}$ $\bar{K}-N$ absorptive amplitude is particularly strong, the contradiction with experiment may be removed. The possibility of even-parity K-baryon reactions is discussed briefly.

THE experimental data concerning the scattering of 0–150 Mev K^+ particles from protons and neutrons suggests that the isotopic spin 1 scattering is nearly all in the S state, while at least one of the $I=0$, P-wave phase shifts is appreciable and positive.[1,2] It has been pointed out that this behavior is difficult to explain on the basis of either the scalar or pseudoscalar K-meson theories.[3]

In this letter we are concerned mainly with P-wave $K-N$ scattering in the pseudoscalar theory, in which the $K\Lambda N$ and $K\Sigma N$ interactions are both of the odd intrinsic-parity Yukawa type. The masses of the K meson, pion, nucleon, and hyperon are denoted respectively by μ_K, μ_π, m_N, and m_Y, (the $\Sigma-\Lambda$ mass difference is neglected, for simplicity). The magnitude of the particle momentum in the $K+N$ state in the center-of-mass system is denoted by k_K, and the energy variable ω is defined in terms of the total energy W in the center-of-mass system by the relation $\omega=W-m_N$. The constants $\hbar$ and c are set equal to unity. It is assumed that for the P-waves, unsubtracted dispersion relations are approximately valid, similar to those applied to pion-nucleon scattering by Chew, Goldberger, Low, and Nambu,[4] and to those applied to $\pi+Y$ production by the author.[5] In the small-momentum approximation of reference 5 (i.e., with k_K^2 neglected in comparison with ωm_N) the equations for the P-wave $K-N$ scattering amplitudes are

$$T_j=\frac{\mathcal{G}_j}{6m_N^2}\frac{C_j(\omega)}{\omega+m_Y-m_N}+\frac{1}{\pi}\int_{\mu_K}^{\infty}\frac{d\omega'\,\mathrm{Im}T_j(\omega')}{\omega'-\omega-i\epsilon}+\frac{1}{\pi}\sum_i A_{ji}\int_{\mu_\pi+m_Y-m_N}^{\infty}\frac{d\omega'\,\mathrm{Im}\bar{T}_i(\omega')}{\omega'+\omega}. \quad (1)$$

The subscript j denotes both the isotopic spin and angular momentum of the $K-N$ scattering amplitudes T_j, while i is a similar index for the $\bar{K}-N$ amplitudes $\bar{T}_i$. All amplitudes T are normalized in terms of elements of the unitary S matrix by the relation $2ik_K{}^3T=S-1$. The KYN interaction coefficients $\mathcal{G}_j$ and the crossing matrix A are given by

$$\begin{aligned}\mathcal{G}_{1,\frac{3}{2}}&=G_\Lambda{}^2+G_\Sigma{}^2, & \mathcal{G}_{0,\frac{3}{2}}&=3G_\Sigma{}^2-G_\Lambda{}^2,\\ \mathcal{G}_{1,\frac{1}{2}}&=-\tfrac{1}{2}(G_\Lambda{}^2+G_\Sigma{}^2), & \mathcal{G}_{0,\frac{1}{2}}&=\tfrac{1}{2}(G_\Lambda{}^2-3G_\Sigma{}^2),\end{aligned} \quad (2)$$

$$A=\frac{1}{6}\begin{array}{c}(1,\frac{3}{2})\ (1,\frac{1}{2})\ (0,\frac{3}{2})\ (0,\frac{1}{2})\\ \begin{bmatrix}1&2&1&2\\4&-1&4&-1\\3&6&-1&-2\\12&-3&-4&1\end{bmatrix}\end{array}, \quad (3)$$

where the $G_\Lambda{}^2$ and $G_\Sigma{}^2$ are normalized so that $G_Y{}^2\approx 14$ would correspond to the KYN and πNN interactions having equal strengths. The functions C_j are independent of isotopic spin, and are given by

$$C_{\frac{3}{2}}=4m_N{}^3/[(m_N+\omega)(m_N+m_Y-\omega)^2],$$
$$C_{\frac{1}{2}}=-2C_{\frac{3}{2}}+3m_N(3m_N-m_Y+\omega)/[(m_N+\omega)(m_N+m_Y-\omega)].$$

These functions both remain within 10% of the value 0.9 for $K-N$ scattering throughout the range of K kinetic energies from 0–200 Mev in the laboratory system. Hence we will set $C_{\frac{3}{2}}=C_{\frac{1}{2}}=0.9$.

One generally neglects the crossing terms ($\bar{T}$ terms) in considering these equations, in which case it is seen that if $G_\Lambda{}^2 \geqslant G_\Sigma{}^2$ the predicted P-wave amplitudes for isotopic spin 1 are expected to be as large or larger than the corresponding amplitudes for $I=0$, in contradiction to the experimental evidence. The purpose of this paper

* Supported by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.
† Present Address: Northwestern University, Evanston, Illinois.
[1] M. Grilli, L. Guerriero, M. Merlin, and G. A. Salandin, Nuovo cimento **10**, 205 (1958); D. Fournet Davis, N. Kwak, and M. F. Kaplon, Phys. Rev. **117**, 846 (1960).
[2] M. A. Melkanoff, D. J. Prowse, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters **5**, 108 (1960).
[3] C. Ceolin, V. DeSantis, and L. Taffara, Nuovo cimento **12**, 502 (1959); R. H. Dalitz, University of California Radiation Laboratory Report UCRL-8394 (unpublished); and *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferreti (CERN Scientific Information Service, Geneva, 1958).
[4] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).
[5] Richard H. Capps, Phys. Rev. **119**, 1753 (1960).

is to point out that this contradiction may be removed by considering the $\bar{T}$ terms. These crossing terms may be important for $K-N$ scattering because the $\bar{K}-N$ amplitudes are likely to be enhanced with respect to the $K-N$ amplitudes by the presence of virtual pion-hyperon states.

The effect of the $\bar{T}$ amplitudes may be especially large if a low-energy P-wave resonance exists for the states of strangeness -1. It has been shown[6,5] that the presumably strong pion-hyperon interactions are likely to lead to such a resonance in a state of isotopic spin 1 and angular momentum $\frac{3}{2}$. Such a resonance, although "driven" by the pion-hyperon interactions, can easily result in the sum of the $P_{\frac{3}{2}}$, $I=1$ cross sections for $\bar{K}+N\rightarrow\pi+Y$ and $\bar{K}+N\rightarrow\bar{K}+N$ being as large as 40–60 mb at the peak of the resonance.[5] The experimental data on these processes are scanty, but do indicate that the $P_{\frac{3}{2}}$ cross sections for the reactions $K^-+p\rightarrow\pi^\pm+\Sigma^\mp$ and $K^-+p\rightarrow K^-+p$ probably are large at a momentum of 400 Mev/c in the laboratory system (i.e., at $\omega=584$ Mev).[7] It is seen from the first column of Eq. (3) that a large$_1$, $\bar{T}_{\frac{3}{2}}$ would particularly enhance the $I=0$ $K-N$ amplitudes, and thus tend to remove the conflict with the experimental data.

In order to estimate the possible magnitude of this effect we neglect $\bar{T}_{1,\frac{1}{2}}$, $\bar{T}_{0,\frac{3}{2}}$, and $\bar{T}_{0,\frac{1}{2}}$ and assume that the major contribution from $\bar{T}_{1,\frac{3}{2}}$ comes from a sufficiently narrow energy region that the factor $(\omega'+\omega)^{-1}$ may be removed from the integral in Eq. (1). The relations for T_j then become

$$T_j=\frac{0.15\mathcal{G}_j}{m_N^2(\omega+m_Y-m_N)}+\frac{A_{j,(1,\frac{3}{2})}X}{\omega+\omega_r}+\frac{1}{\pi}\int_{\mu_K}^{\infty}\frac{d\omega'\,\mathrm{Im}T_j(\omega')}{\omega'-\omega-i\epsilon},\quad(4)$$

$$X=\pi^{-1}\int\mathrm{Im}\bar{T}_{1,\frac{3}{2}}(\omega')d\omega',$$

where ω_r is the energy at which $\bar{T}_{1,\frac{3}{2}}$ is large. The unitarity condition on $\mathrm{Im}\bar{T}_{1,\frac{3}{2}}$ at energies above the rest mass of $\bar{K}+N$ is $\mathrm{Im}\bar{T}_{1,\frac{3}{2}}=(8\pi k_K)^{-1}\bar{\sigma}_{1,\frac{3}{2}}$, where $\bar{\sigma}_{1\,\frac{3}{2}}$ is the sum of the elastic and inelastic cross sections for an initial $P_{\frac{3}{2}}\,\bar{K}+N$ state with $I=1$. In order to estimate the possible size of X, we assume that the energy dependence of $\bar{\sigma}_{1,\frac{3}{2}}$ is given by the simple resonance form $k_K^{-1}\bar{\sigma}_{1,\frac{3}{2}}=\alpha k_\pi^3\omega^{-2}/[(\omega_r-\omega)^2+k_\pi^6\omega^{-2}\gamma^2]$, where k_π is the momentum in the $\pi+Y$ state, and α and γ are constants. If the half-width $\gamma k_\pi^3(\omega_r)/\omega_r$ at resonance is taken as 85 Mev and α is chosen so that the maximum value of $\bar{\sigma}_{1,\frac{3}{2}}$ is 60 mb (corresponding to a cross section of 30 mb from an incident K^--p state), then $X\approx0.06$ fermi^{-2}. If we further assume $\omega_r=560$ Mev and $G_\Lambda^2=G_\Sigma^2=4$ and consider K-meson energies in the range 0–150 Mev in the laboratory system (i.e., 494 Mev$<\omega$ <590 Mev), the contributions of the Born terms and $A_{j,(1,\frac{3}{2})}$ terms of Eq. (4) to the various $K-N$ amplitudes are, respectively,[8]

$$\begin{aligned}&T_{1,\frac{3}{2}}\approx1.5+0.2, && T_{0,\frac{3}{2}}\approx1.5+0.55,\\ &T_{1,\frac{1}{2}}\approx-0.75+0.7, && T_{0,\frac{1}{2}}\approx-0.75+2.2,\end{aligned}\quad(5)$$

where the units are fermi^{-3}/100. Thus the crossing terms may be larger than the Born terms for some of the amplitudes.

Although the exact values in Eq. (5) cannot be taken seriously, it is seen that the assumption of a large $\bar{T}_{1,\frac{3}{2}}$ leads to positive, fairly large values of the $I=0$ $K-N$ amplitudes, a smaller but definitely positive value of $T_{1,\frac{3}{2}}$, and a small (positive or negative) value of $T_{1,\frac{1}{2}}$. The effect of the last term in Eq. (4) (the dispersion term) must be to further enhance the larger of the positive amplitudes. However, since the values of the $I=0$ amplitudes given in Eq. (5) are only about a tenth as large as the threshold value of the resonant $(\frac{3}{2},\frac{3}{2})$ pion-nucleon amplitude, it is not surprising if the enhancement is insufficient to produce a $K-N$ resonance.

The experimental data are not adequate to determine the signs and relative magnitudes of the four P-wave $K-N$ scattering amplitudes. The most easily measured amplitude for each isotopic spin is the nonspin-flip amplitude $T_N=2T_{\frac{3}{2}}+T_{\frac{1}{2}}$, since this amplitude interferes with the large S-wave amplitude to produce a $\cos\theta$ term in the angular distribution. The measured value of T_N is small for $I=1$, while $T_N\approx0.22$ fermi^{-3} for $I=0$.[2] This is about 4 times as large as the value of $T_{0,N}$ given in Eq. (5). In view of the various experimental and theoretical uncertainties, and the fact that the enhancement of the positive amplitudes by the dispersion term of Eq. (4) has been neglected, we conclude that the assumptions discussed above are consistent with experiment.

Now we consider briefly the possibility that the $K\Lambda N$ and $K\Sigma N$ interactions are of the scalar type. In this case the Born approximation terms for P-wave $K-N$ scattering are smaller. However, the crossing terms in the dispersion integrals involve the amplitudes for production of S-wave $\pi+Y$ states from $P_{\frac{1}{2}}$ $\bar{K}+N$ states, and these terms could enhance some of the P-wave $K-N$ scattering amplitudes in the manner discussed above. Thus, it is clear that the presence of an appreciable $\cos\theta$ term in the $K-N$ angular distributions is not strong evidence against the scalar K meson. It should be pointed out, however, that the large $\cos^2\theta$ terms in the cross sections for the processes $K^-+p\rightarrow\pi^\pm+\Sigma^\mp$ at 400 Mev/c are more easily explained in the pseudoscalar than in the scalar theory. It can easily

[6] Yukihisi Nogami, Progr. Theoret. Phys. 22, 25 (1959).

[7] L. W. Alvarez, Proceedings of the 1959 International Conference on Physics of High-Energy Particles at Kiev, July, 1959 (to be published).

[8] The actual values of G_Λ^2 and G_Σ^2 are not known and may be anywhere in the range from 1 to 8. If the K interactions are of the scalar type, the G_Λ^2 and G_Σ^2 that best fit the $K-N$ scattering data are only about a tenth of the corresponding values in the pseudoscalar theory. See M. M. Islam, Nuovo cimento 13, 224 (1959).

be shown that with either choice of the K parity the largest $\cos^2\theta$ terms arising in the Born approximation in the low-energy region are of the form $d\sigma \approx [k_\pi^3 k_K F^2 G^2/(\omega^2 m^4)]\cos^2\theta$, where F represents either the πNN, $\pi\Lambda\Sigma$, or $\pi\Sigma\Sigma$ (pseudoscalar) interaction constant, while G represents either the $K\Lambda N$ or $K\Sigma N$ interaction constant. If the K interactions are scalar, G^2 is of the order of 0.4 rather than 4, so these Born terms are small.[8] Since the crossing terms in the dispersion relations for $\pi+Y$ production are *not* expected to be particularly large, it is hard to see how a $\pi+Y$ production amplitude that is small in the Born approximation can actually be large. Thus, these large $\cos^2\theta$ terms are difficult to explain in the scalar theory.

It is concluded that the processes $\bar{K}+N\rightarrow\bar{K}+N$ and $\bar{K}+N\rightarrow\pi+Y$ may influence P-wave $K-N$ scattering appreciably through the crossing term in the $K-N$ dispersion relations. The experimental and theoretical understanding of these processes is insufficient to determine the parity of the K. However, if the K is pseudoscalar, and if the $I=1$, $P_{\frac{3}{2}}\,\bar{K}-N$ amplitudes are large, the existing $K-N$ data are in general agreement with the predictions of approximate P-wave dispersion relations.

PHYSICAL REVIEW VOLUME 121, NUMBER 1 JANUARY 1, 1961

Depolarization of Negative μ Mesons*

R. A. Mann† and M. E. Rose
Oak Ridge National Laboratory, Oak Ridge, Tennessee
(Received August 22, 1960)

The depolarization of negative μ mesons is discussed in terms of the processes occurring in the formation of μ-mesonic atoms and the subsequent cascade to the ground state. The initial distribution of μ mesons in capturing states of carbon is deduced. The depolarization due to the capture process is derived in a fashion free of essentially all approximations. The only assumption involved is that the scattering cross section is such as to randomize the direction of motion prior to capture and this assumption is shown to be well fulfilled. The effect of radiative transitions in producing depolarization is determined with all possible dipole transitions taken into account. Only nuclei with zero spin are treated in detail. The Auger process is included in a schematic fashion which is sufficient for the purpose in hand. It is shown that both radiative and Auger transitions must be included in the discussion of the depolarization processes in the cascade. The theoretical results are compared with experiment, and from the comparison it is concluded that the observed facts are well accounted for.

I. INTRODUCTION

SINCE the original experiments of Garwin, Lederman, and Weinrich it has been known that the observed asymmetry coefficients for the electron distribution in muon decay are considerably smaller for the negative μ meson than for the positive μ meson.[1] The current formulation of weak interaction theory predicts that the positive and negative μ mesons are created with complete and opposite polarization.[2] Further, the asymmetry coefficient is proportional to the magnitude of the polarization of the μ mesons when they decay, as was originally shown by Lee and Yang.[3] Consequently, the observed asymmetry coefficients are presumably to be interpreted in terms of a preferential depolarization of the negative μ mesons. Several authors have recognized that a mechanism for depolarization is provided by the ability of negative μ mesons to become captured into bound states and thus form μ-mesonic atoms.[4,5] Although the papers cited treat certain aspects of the depolarization associated with the formation of μ-mesonic atoms, there has been no comprehensive treatment of the problem, particularly as regards the depolarization occurring in the capturing event resulting in the formation of the μ-mesonic atom. In the followlng we present a quantitative account of the observed depolarization of the negative μ mesons. Since we attribute this depolarization, in part, to the processes involved in the formation of μ-mesonic atoms we present first a discussion of the events occurring before the μ-mesonic atoms are formed.

For the purpose of discussion we schematically divided the history of the μ mesons into four stages and consider qualitatively the depolarizing processes that may occur at each stage. Stage one begins when the μ mesons are created in π decay. Thus, depending on the

* Based, in part, on a dissertation submitted by R. A. Mann in partial fulfillment of the requirements for the degree of Doctor of Philosophy at the University of Alabama.

† Present address: Department of Physics, University of Alabama, University, Alabama.

[1] R. L. Garwin, L. M. Lederman, and M. Weinrich, Phys. Rev. **105**, 1415 (1957).

[2] A current review of weak-interaction theory is given by E. J. Konopinski, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, California, 1959), Vol. 9, p. 99.

[3] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).

[4] M. E. Rose, Bull. Am. Phys. Soc. 4, 80 (1959). This work was intended as a crude preliminary estimate of the part of the depolarization arising from the cascade process; see below.

[5] I. M. Schmuskevitch, Nuclear Phys. **11**, 419 (1959).