be shown that with either choice of the K parity the largest $\cos^2\theta$ terms arising in the Born approximation in the low-energy region are of the form $d\sigma \approx [k_\pi^3 k_K F^2 G^2/(\omega^2 m^4)]\cos^2\theta$, where F represents either the πNN, $\pi\Lambda\Sigma$, or $\pi\Sigma\Sigma$ (pseudoscalar) interaction constant, while G represents either the $K\Lambda N$ or $K\Sigma N$ interaction constant. If the K interactions are scalar, G^2 is of the order of 0.4 rather than 4, so these Born terms are small.[8] Since the crossing terms in the dispersion relations for $\pi+Y$ production are *not* expected to be particularly large, it is hard to see how a $\pi+Y$ production amplitude that is small in the Born approximation can actually be large. Thus, these large $\cos^2\theta$ terms are difficult to explain in the scalar theory.

It is concluded that the processes $\bar{K}+N\rightarrow\bar{K}+N$ and $\bar{K}+N\rightarrow\pi+Y$ may influence P-wave $K-N$ scattering appreciably through the crossing term in the $K-N$ dispersion relations. The experimental and theoretical understanding of these processes is insufficient to determine the parity of the K. However, if the K is pseudoscalar, and if the $I=1$, $P_{\frac{3}{2}}$ $\bar{K}-N$ amplitudes are large, the existing $K-N$ data are in general agreement with the predictions of approximate P-wave dispersion relations.

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Depolarization of Negative μ Mesons*

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The depolarization of negative μ mesons is discussed in terms of the processes occurring in the formation of μ-mesonic atoms and the subsequent cascade to the ground state. The initial distribution of μ mesons in capturing states of carbon is deduced. The depolarization due to the capture process is derived in a fashion free of essentially all approximations. The only assumption involved is that the scattering cross section is such as to randomize the direction of motion prior to capture and this assumption is shown to be well fulfilled. The effect of radiative transitions in producing depolarization is determined with all possible dipole transitions taken into account. Only nuclei with zero spin are treated in detail. The Auger process is included in a schematic fashion which is sufficient for the purpose in hand. It is shown that both radiative and Auger transitions must be included in the discussion of the depolarization processes in the cascade. The theoretical results are compared with experiment, and from the comparison it is concluded that the observed facts are well accounted for.

I. INTRODUCTION

SINCE the original experiments of Garwin, Lederman, and Weinrich it has been known that the observed asymmetry coefficients for the electron distribution in muon decay are considerably smaller for the negative μ meson than for the positive μ meson.[1] The current formulation of weak interaction theory predicts that the positive and negative μ mesons are created with complete and opposite polarization.[2] Further, the asymmetry coefficient is proportional to the magnitude of the polarization of the μ mesons when they decay, as was originally shown by Lee and Yang.[3] Consequently, the observed asymmetry coefficients are presumably to be interpreted in terms of a preferential depolarization of the negative μ mesons. Several authors have recognized that a mechanism for depolarization is provided by the ability of negative μ mesons to become captured into bound states and thus form μ-mesonic atoms.[4,5] Although the papers cited treat certain aspects of the depolarization associated with the formation of μ-mesonic atoms, there has been no comprehensive treatment of the problem, particularly as regards the depolarization occurring in the capturing event resulting in the formation of the μ-mesonic atom. In the followlng we present a quantitative account of the observed depolarization of the negative μ mesons. Since we attribute this depolarization, in part, to the processes involved in the formation of μ-mesonic atoms we present first a discussion of the events occurring before the μ-mesonic atoms are formed.

For the purpose of discussion we schematically divided the history of the μ mesons into four stages and consider qualitatively the depolarizing processes that may occur at each stage. Stage one begins when the μ mesons are created in π decay. Thus, depending on the

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[1] R. L. Garwin, L. M. Lederman, and M. Weinrich, Phys. Rev. **105**, 1415 (1957).

[2] A current review of weak-interaction theory is given by E. J. Konopinski, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, California, 1959), Vol. 9, p. 99.

[3] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).

[4] M. E. Rose, Bull. Am. Phys. Soc. 4, 80 (1959). This work was intended as a crude preliminary estimate of the part of the depolarization arising from the cascade process; see below.

[5] I. M. Schmuskevitch, Nuclear Phys. **11**, 419 (1959).

kinetic energy of the π meson, the μ mesons initially have energies of the order of several Mev to 100 Mev. Further they may be created within the region of an accelerator fringing field. The depolarization of Dirac particles by a magnetic field has been treated by Case.[6] We take the depolarizing effect of the fringing field to be negligible in accordance with his result; since, there are excellent reasons to believe that the μ mesons are normal Dirac particles.[7] The effect of the kinetic energy of the π meson on the μ polarization has been treated by Jensen and Øverås.[8] The effect is dependent on the geometry of each experiment and should normally be small; it does not distinguish between positive and negative μ mesons. Thus, at the end of stage one the μ mesons are removed from the accelerator region and the two species have equal and opposite polarizations.

Stage two consists of the slowing-down process. This is essentially ionization by which the μ mesons lose most of their kinetic energy and small-angle elastic scattering which also contributes to energy loss in the laboratory frame. The end of stage two occurs when the μ mesons have energies of the order of a few kev. The difference in the scattering of positive and negative μ mesons in the energy range of stage two is negligible. The depolarization of Dirac particles due to such small angle scattering and the cumulative effects of multiple scattering have been discussed in the literature and found to be quite small.[9,10] Thus at the end of stage two the μ mesons have energies of order 10 kev and the magnitudes of the polarization of the two charge species are equal. The time taken for the muons to reach the end of stage two is of the order of 10^{-9} sec, and the additional time for the negative muons to reach the ground state of a μ-mesonic atom is much less.[11] Thus, essentially none of the muons will have decayed till the end of stage four.

Stage three begins with the μ mesons having energies of several kev so that ionization losses are negligible but elastic scattering is still an important process. At such low energies the scattering is spin independent and so neither species is depolarized by this mechanism. Nevertheless, as we show in Sec. 3 below, this scattering is important in determining the final polarization of the negative μ mesons. Stage three ends with the negative μ mesons being captured to form μ-mesonic atoms and with the positive μ mesons either forming muonium or undergoing some other reaction of a chemical nature.[12] Thus at the end of stage three the negative μ mesons have been captured into excited bound states and therefore may be preferentially depolarized.

Stage four is not defined for the positive μ mesons; for the negative μ mesons stage four consists of the various transitions that occur in the cascade from the highly excited capturing state to the ground state of the μ-mesonic atom. Additional depolarization occurs in this cascade. The end of stage four occurs when the μ-meson decays. We have no reason to consider the competition of nuclear capture.

When the μ mesons, either positive or negative, decay, and if the entire electron spectrum is observed, the angular distribution of the decay electrons is given by

$$I(\theta)=1+a\cos\theta, \tag{1}$$

where the asymmetry coefficient, a, is

$$a=-|P|/3, \tag{2}$$

and θ is the angle between the electron direction and the original meson beam direction, which we shall take to be the axis of quantization. $|P|$ is the magnitude of the μ-meson polarization just before decay and is the quantity that we determine. Garwin, Lederman, and Weinrich report that the magnitude of the asymmetry coefficient observed for positive μ mesons stopping in carbon is 0.33 ± 0.03, and for negative μ mesons stopping in carbon they find $|a|\approx0.05$.[1] We mention other experimental data in connection with our final results; the point here is that the positive μ mesons retain essentially complete polarization when stopped in carbon. We are not concerned with the fact that the low-energy interactions of positive μ mesons cause considerable depolarization in some materials.[13] Such interactions would occur at meson energies lower than those of interest in the formation of μ-mesonic atoms and as we have pointed out above there is no preferential depolarization of either species of meson before the mesonic atoms are formed. Thus, we take the negative μ mesons to have complete polarization until they undergo the capture process. The first step in determining the depolarization due to the formation of μ-mesonic atoms is to deduce the initial distribution of the μ mesons among the bound states of the capturing atom.

II. INITIAL DISTRIBUTION OF μ MESONS IN BOUND STATES

The negative μ mesons are captured by the process of Auger capture. This means that a μ-meson incident on an atom ejects an atomic electron and becomes bound. One may show that the alternate possibility of radiative capture has a cross section too small to be competitive with Auger capture. Our procedure is to determine first the partial cross sections for Auger capture into the μ-mesonic states n, l, j; where n is the principle quantum number, l the orbital angular momentum, and j the total angular momentum. Since we find these partial

[6] K. M. Case, Phys. Rev. **106**, 173 (1957). Aside from the evidence for spin $\frac{1}{2}$, there is the measurement of the magnetic moment.

[7] R. L. Garwin, D. P. Hutchinson, S. Penman, and G. Shapiro, Phys. Rev. **118**, 271 (1960). The value of the meson magnetic moment is relevant to the Dirac nature of the muon. See also their remarks concerning the asymmetry coefficient for positive muons.

[8] S. H. P. Jensen and H. Øverås, Kgl. Norske Videnskab. Selskabs, Forh. **31**, 34 (1958).

[9] L. Wolfenstein, Phys. Rev. **75**, 1664 (1949).

[10] B. Mühlschlegel and H. Koppe, Z. Physik **150**, 496 (1958).

[11] E. Fermi and E. Teller, Phys. Rev. **72**, 399 (1947).

[12] V. W. Hughes, D. W. McColm, K. Ziock, and R. Prepost, Phys. Rev. Letters **5**, 63 (1960).

[13] D. H. Wilkinson, Nuovo cimento **6**, 516 (1957).

cross sections to be quite large and to fluctuate rapidly with the incident meson energy, the cross sections alone are not sufficient to determine the distribution in initial states. To determine the distribution we must introduce a mechanism for slowing down the μ mesons and consider the capture rate as a function of the incident meson energy. This is discussed in detail below.

To determine the partial cross sections we proceed as follows. We take the incident μ meson and the ejected electron to be plane waves; the initial electron wave function is that appropriate for the K shell of a hydrogen-like atom; the final meson wave function is that of a μ meson in the bound hydrogen-like state n, l, j. We define

$$\phi=\phi(e)_{\text{free}}\phi(\mu)_{\text{bound}}, \tag{3a}$$

$$\psi=\psi(\mu)_{\text{free}}\psi(e)_{\text{bound}}. \tag{3b}$$

Dropping the designations "free" and "bound" which will be self-evident, we have

$$\phi(e)=4\pi\sum_{l_2,m_2} i^{l_2}Y_{l_2m_2}{}^*(\hat{k}_2)Y_{l_2m_2}(\hat{r}_2)j_{l_2}(k_2r_2)\chi_{\frac{1}{2}}{}^{\tau_2}, \tag{4a}$$

$$\phi(\mu)=\sum_{\tau} C(l\tfrac{1}{2}j;m-\tau,\tau)Y_{l,m-\tau}(\hat{r}_1)R_{n,l}(r_1)\chi_{\frac{1}{2}}{}^{\tau}, \tag{4b}$$

$$\psi(\mu)=\sum_{l_1}[4\pi(2l_1+1)]^{\frac{1}{2}}i^{l_1}Y_{l_10}(\hat{r}_1)j_{l_1}(k_1r_1)\chi_{\frac{1}{2}}{}^{\tau_1}, \tag{5a}$$

$$\psi(e)=\frac{1}{(4\pi)^{\frac{1}{2}}}R(r_2)\chi_{\frac{1}{2}}{}^{\tau_e}. \tag{5b}$$

Then the cross section is

$$\sigma=2\frac{v_2}{v_1}\left(\frac{1}{2\pi a_e}\right)^2\int\left|\left[\phi,\left(\frac{1}{r_{12}}-\frac{Z}{r_2}\right)\psi\right]\right|^2 d\Omega(\hat{k}_2), \tag{6}$$

where v_2 is the ejected electron velocity, v_1 is the incident meson velocity, and a_e is the electron Bohr radius. The occurrence of Z/r_2 arises from the fact that the zero-order wave functions are eigenfunctions of different Hamiltonians. In Eq. (6) there is an implicit sum over all unobserved quantities and the factor two has been inserted to account for the two K-shell electrons. The notation and conventions used for the spherical harmonics and the Clebsch-Gordan coefficients are those of Rose.[14] We define

$$b_1=nk_1a_\mu/Z, \tag{7a}$$

$$b_2=k_2a_e/Z, \tag{7b}$$

$$b_0=na_\mu/2a_e, \tag{7c}$$

$$x=2Zr_1/na_\mu, \tag{7d}$$

$$y=Zr_2/a_e, \tag{7e}$$

where a_μ is the meson Bohr radius with $m_\mu=207m_e$ and a_e is the electron Bohr radius.

[14] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, New York, 1957).

The partial cross section is then evaluated as

$$\frac{\sigma}{\pi a_e{}^2}=\frac{b_2}{2b_1}\left(\frac{n}{Z}\right)^4\left(\frac{1}{207}\right)^3\frac{(2l_1+1)(2j+1)}{(2l+1)(2l_2+1)} \times[C(l_1l_2l;00)]^2[I-ZF\delta_{l_20}]^2. \tag{8}$$

The quantities F and I are radial integrals and are

$$F=\tfrac{1}{2}\left\{\frac{(n-l-1)!(n+l)!\pi}{n}\right\}^{\frac{1}{2}}\frac{1}{1+b_2{}^2}\left(\frac{b_1}{2}\right)^l\left(\frac{1}{b_1{}^2+\frac{1}{4}}\right)^{l+2} \times\frac{1}{\Gamma(l+\frac{3}{2})}\sum_{k=0}^{n-l-1}(-\tfrac{1}{2})^k\frac{(2l+k+2)}{(n-l-1-k)!k!} \times\left(\frac{1}{b_1{}^2+\frac{1}{4}}\right)^k {}_2F_1(-\tfrac{1}{2}k-\tfrac{1}{2},-\tfrac{1}{2}k;l+\tfrac{3}{2};-4b_1{}^2), \tag{9a}$$

$$I=\int_0^\infty j_{l_1}(b_1x)X(x)[K_1(x)+K_2(x)]dx, \tag{9b}$$

where K_1 and K_2 are defined below. The quantities occurring in (9b) are

$$X(x)=-2\left\{\frac{(n-l-1)!}{n[(n+l)!]^3}\right\}^{\frac{1}{2}}e^{-\frac{1}{2}x}x^lL_{n+l}{}^{2l+1}(x), \tag{10a}$$

where $L_{n+l}{}^{2l+1}(x)$ is the associated Laguerre polynomial, and

$$K_1(x)=x^{1-l_2}b_0{}^{-l_2-1}\int_0^{b_0x}y^{l_2+2}e^{-y}j_{l_2}(b_2y)dy, \tag{10b}$$

$$K_2(x)=x^{2+l_2}b_0{}^{l_2}\int_{b_0x}^\infty y^{1-l_2}e^{-y}j_{l_2}(b_2y)dy. \tag{10c}$$

Energy conservation requires

$$b_2{}^2=(4m_\mu b_1{}^2/m_en^2)+(m_\mu/m_en^2)-1. \tag{11}$$

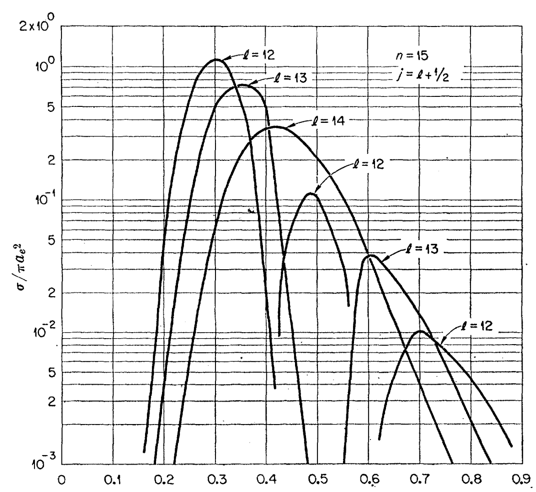

FIG. 1. Some partial cross sections for Auger capture of μ mesons by carbon.

The integrals in (9b) may be done analytically; however, the results involve multiple sums and for computational purposes it was found desirable to do the integrations numerically. It was found that the term ZF in (8) is predominant for $Z>2$ and that the significant contributions to the capture cross section occur for s-wave electron emission ($l_2=0$); we use these facts in all that follows. Representative partial cross sections are shown in Fig. 1.

It should be emphasized at this point that the evaluation of the cross sections need not be made with great accuracy. The numerical results, it will be seen, serve two purposes. First, they are used to show that the scattering is so large by the time capture occurs that the directions of motion are almost completely randomized before capture. Second, the cross sections are used, as will be explained in the following paragraph, to determine the relative populations of initial states from which the subsequent cascade begins.

To obtain the relative populations of states into which capture occurs we must consider the competition between slowing down and Auger capture. Each state nlj is characterized by a capture cross section and the efficacy of this partial capture process is determined by the number of mesons surviving in the unbound state at each energy. For slowing down we consider the elastic scattering, as previously mentioned. Then we must consider a problem somewhat like resonance escape in that we have a number of resonance-like cross section peaks for capture and the question to be answered is: After all mesons are captured, how many will disappear from the initial state into each nlj state?

We therefore consider the capture rate as a function of the meson energy. If $N(E)$ is the number of free μ mesons at energy, E, then removal of mesons from energy E occurs by the energy loss process.

$$\frac{dN(E)}{dE}=\frac{dN(E)}{dx}\frac{dx}{dE}, \tag{12}$$

where dx is the path increment. Here fluctuations in energy loss are neglected. Of course, the stopping power is

$$dE/dx=-\langle\Delta E\rangle n_s\sigma_T(E), \tag{13}$$

where n_s is the number of scattering centers per unit volume, $\langle\Delta E\rangle$ is the average energy loss in a collision between a μ meson and scattering atom (mass M) at energy E and $\sigma_T(E)$ is the total cross section for the scattering event. The energy loss at scattering angle θ is

$$\Delta E=\kappa E(1-\cos\theta), \tag{14a}$$

where

$$\kappa=2m_\mu M/(M+m_\mu)^2 \tag{14b}$$

and we find $\langle\Delta E\rangle$ as the average of (14a) over the scattering angles. Thus,

$$\langle\Delta E\rangle=\kappa E\sigma_{\mathrm{Tr}}(E)/\sigma_T(E), \tag{15}$$

where $\sigma_{\mathrm{Tr}}(E)$ is the transport cross section. Further we have that

$$dN/dx=-n_sN(E)\sigma_{\mathrm{A}}(E), \tag{16}$$

where $\sigma_{\mathrm{A}}(E)$ is the cross section for Auger capture at energy E summed over all n, l, j. We now have

$$\frac{dN(E)}{dE}=\frac{N(E)\sigma_{\mathrm{A}}(E)}{\kappa E\sigma_{\mathrm{Tr}}(E)}, \tag{17}$$

which is integrated as

$$N(E)=N(E_0)\exp\left[-\frac{1}{\kappa}\int_E^{E_0}\frac{\sigma_{\mathrm{A}}(E)}{\sigma_{\mathrm{Tr}}(E)}\frac{dE}{E}\right];\quad E_0\geq E. \tag{18}$$

At this point it is necessary to consider specific capturing atoms as the integration of (18) must be done numerically. Carbon was selected as the capturing element since there is more experimental data available for carbon and also because there is no complication due to a hyperfine interaction. The transport cross section for μ mesons incident on carbon atoms was calculated using hydrogen-like wave functions to obtain the electron form factor. Contributions of all six electrons in C were taken into account.

The integration of (18) was done using an electronic computer. A suitable upper limit, E_0, was determined by trial such that very few μ mesons were captured above this energy. After the number of μ mesons captured in each energy interval was found the relative number going into each state n, l was determined by finding the quantities

$$N_{nl}(E)=n_s\int_{E_0}^{E}\frac{dE'\,\sigma_{nl}(E')}{dE'/dx}\times\exp\left[-n_s\int_{E'}^{E_0}\frac{dE''\,\sigma_A(E'')}{dE''/dx}\right]. \tag{19}$$

To obtain the initial distribution, this quantity is then evaluated at the final energy where $N(E)\ll 1$. The initial distribution of μ mesons captured into states n, l of carbon atoms is given in Table I. The two states j belonging to a state l are populated according to their statistical weights. It was found unnecessary to consider states for which $n>16$. The μ mesons are captured when their kinetic energy is around 8 kev, very few are captured when this energy is above 12 kev and essentially none remain free after their energy is reduced to 2.5 kev.

From Fig. 2, one notes that the states $n\approx 7$ capture the most mesons. This is in strong contrast to the conjecture that $n\approx 14$ is the state into which most capture occurs.[15,16] There is a preference for the circular

[15] G. R. Burbidge and A. H. de Borde, Phys. Rev. **87**, 189 (1953); see also A. H. de Borde, Proc. Phys. Soc. (London) **A67**, 57 (1954).

[16] M. Demuer, Nuclear Phys. **1**, 516 (1956).

TABLE I. Initial distribution of μ mesons among the states of μ-mesonic carbon. The table lists the number of mesons, in 10 000, captured into the atomic state n, l in carbon.

n \ $n-l$	Circular 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1															
2	35	13														
3	232	38	21													
4	499	54	101	12												
5	525	256	82	87	15											
6	338	449	134	102	64	16										
7	151	401	302	98	93	48	15									
8	50	227	346	204	82	77	38	13								
9	13	91	236	273	145	70	62	31	11							
10	3	27	110	215	212	108	59	49	26	9						
11	...	6	37	113	186	165	84	50	40	21	7					
12	...	1	10	42	107	157	130	67	41	32	17	6				
13	...	...	2	12	44	97	132	104	54	35	27	14	5			
14	...	...	...	3	13	42	87	110	85	44	29	22	12	4		
15	...	...	...	1	3	14	40	77	93	69	37	25	19	10	3	
16	...	...	...	...	1	4	14	37	67	79	58	31	21	16	9	3
Total	1847	1563	1381	1152	965	798	661	538	417	289	175	98	57	30	12	3

Total number shown: 9986

orbits as may be seen from the bottom line of Table I. This result has also been conjectured in the literature but the reason for the preference for maximum l is not as simple as the statement that these states have largest statistical weight would imply.

III. THE DEPOLARIZATION DUE TO CAPTURE

We now determine the depolarization of the μ mesons which occurs in the Auger capture. The solution of this problem depends on the physical situation that prevails at the instant that the μ meson is captured; specifically, the depolarization on capture depends on the amount of scattering the mesons have suffered before capture. The procedure is as follows.

Since the μ mesons have low energy when they are captured, we can represent the polarization $\boldsymbol{P}$ vector by

$$\boldsymbol{P}=(\psi,\boldsymbol{\sigma}\psi)/(\psi,\psi). \tag{20}$$

As we have indicated previously the μ mesons can be taken to be completely polarized along the initial beam direction which we take to be the axis of quantization. Thus the quantity we determine is the expectation value of σ_z after capture. Since we finally determine the polarization in the $1s$ state and since

$$\langle\sigma_z\rangle=\langle j_z\rangle/j,\qquad j=l+\tfrac{1}{2}, \tag{21a}$$

$$\langle\sigma_z\rangle=-\langle j_z\rangle/(j+1),\quad j=l-\tfrac{1}{2}, \tag{21b}$$

we choose to calculate in terms of $\langle j_z\rangle/j$. Then

$$\langle j_z\rangle/j=\textstyle\sum_m m\,\mathrm{pop}(m)/\sum_m \mathrm{pop}(m), \tag{22}$$

where the (unnormalized) population of the states, pop(m) is taken as

$$\mathrm{pop}(m)=\int |H_{fi}|^2 d\Omega, \tag{23}$$

where $d\Omega$ is the angle element for the ejected electron. If the previous formulation is used; then H_{fi} is the matrix element in Eq. (6) and τ_1 takes the single value, $\frac{1}{2}$, in Eq. (5a). Then if one considers only the ejection of s wave electrons,

$$\langle j_z\rangle/j=1/2j \tag{24}$$

in the initial capturing state. This represents a case where the μ mesons are captured from a beam. Presently, we show that this is not the proper description of the mesons when they are captured.[17]

We now show, that if the μ mesons have random directions when they are captured then one may obtain a description of the depolarization due to capture independent of the physical model. Retaining the definitions (3), we use

$$\psi(\mu)=4\pi\chi_{\frac{1}{2}}^{\tau_1}\sum_{l_1,m_1} i^{l_1}\exp(i\delta_{l_1})Y_{l_1,m_1}{}^*(\hat{k}_1)\times Y_{l_1,m_1}(\hat{r}_1)f_{l_1}(k_1r_1), \tag{25a}$$

$$\psi(e)=\sum_{\tau_3} C(l_3\tfrac{1}{2}j_3;\,m_3-\tau_3,\,\tau_3)\times Y_{l_3,m_3-\tau_3}(\hat{r}_2)\chi_{\frac{1}{2}}^{\tau_3}R_e(r_2), \tag{25b}$$

$$\phi(e)=4\pi\chi_{\frac{1}{2}}^{\tau_2}\sum_{l_2,m_2} i^{l_2}\exp(i\delta_{l_2}{}')f_{l_2}(k_2r_2)\times Y_{l_2,m_2}{}^*(\hat{k}_2)Y_{l_2,m_2}(\hat{r}_2), \tag{25c}$$

$$\phi(\mu)=\sum_{\tau} C(l\tfrac{1}{2}j;\,m-\tau,\,\tau)Y_{l,m-\tau}(\hat{r}_1)R_\mu(r_1)\chi_{\frac{1}{2}}^{\tau}, \tag{25d}$$

where all of the radial wave functions may be defined to be the correct radial functions; for example, one might

[17] Incidently, the value $1/2j$ is easily understood from conservation of the z component of angular momentum for the total $\mu-e$ system and the fact that the electron, and therefore the muon, does not change its z projection of angular momentum in a process where the interaction is spin-independent.

approximate the f_l by Coulomb wave functions with appropriate phase factors. The interaction will be

$$V_{\text{int}}=\sum_{\lambda,M_\lambda}\frac{4\pi}{2\lambda+1}Y_{\lambda,M_\lambda}^*(\hat{r}_2)Y_{\lambda,M_\lambda}(\hat{r}_1)F_\lambda(r_1,r_2), \quad (26)$$

where the F_λ are determined by the decomposition of the total Hamiltonian to extract the perturbation. One notes that in (25a) the incident meson has direction $\hat{k}_1$; when we deal with mesons having random direction with respect to the axis of quantization then $\hat{k}_1$ is to be averaged over after the squares of the matrix elements have been formed. Thus we define

$$Q=\sum|(\phi,V_{\text{int}}\psi)|^2 d\Omega(\hat{k}_2)d\Omega(\hat{k}_1)(1/4\pi), \quad (27)$$

with the stipulation that the sum is over all unobserved quantities. Thus, there is no sum over m. Then Q is found to be

$$\begin{aligned}Q=(4\pi)^3\sum&[C(l\tfrac{1}{2}j;m-\tau,\tau)C(l_3\tfrac{1}{2}j_3;m_3-\tau_3,\tau_3)]^2\\&\times C(l_1\lambda l;00)C(l_1\lambda' l;00)C(l_1\lambda l;m_1,m-\tau-m_1)\\&\times C(l_1\lambda' l;m_1,m-\tau-m_1)C(l_3\lambda l_2;m_3-\tau_3,m_2-m_3+\tau_3)\\&\times C(l_3\lambda' l_2;m_3-\tau_3,m_2-m_3+\tau_3)C(l_3\lambda l_2;00)\\&\times C(l_3\lambda' l_2;00)\frac{(2l_3+1)(2l_1+1)}{(2l_2+1)(2l+1)}I^{*\prime}I, \quad (28)\end{aligned}$$

where the sum is over l_2, l_1, λ, λ', m_1, m_2, m_3, and τ_3, and where

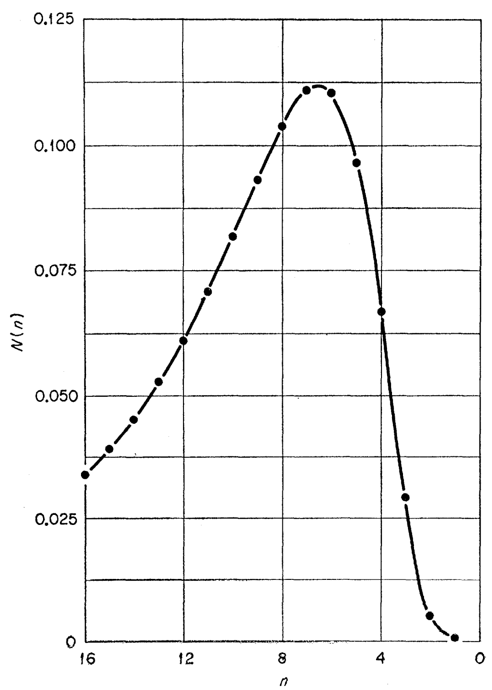

FIG. 2. Distribution in states, n, of μ mesons captured by carbon atoms.

$$I=\int_0^\infty\int_0^\infty r_1^2r_2^2dr_1dr_2F_\lambda(r_1,r_2)R_\mu(r_1)\times R_e(r_2)f_{l_2}^*(k_2r_2)f_{l_1}(k_1r_1), \quad (29)$$

and the prime on I^* means that λ is replaced by λ' in the definition of I. The standard techniques of Racah algebra may be used to perform the sums over the magnetic quantum numbers.[14] This leads to $\lambda=\lambda'$ and

$$Q=(4\pi)^3[C(l\tfrac{1}{2}j;m-\tau,\tau)]^2\sum_{l_2l_1\lambda}\frac{(2l_1+1)(2j_3+1)}{(2l+1)(2\lambda+1)}\times[C(l_1\lambda l;00)C(l_3\lambda l_2,00)]^2|I|^2. \quad (30)$$

Thus pop$(m)\propto Q$, and when we form $\langle j_3\rangle/j$ we obtain

$$\frac{\langle j_z\rangle}{j}=\frac{\sum_m m[C(l\tfrac{1}{2}j;m-\tau,\tau)]^2}{\sum_m[C(l\tfrac{1}{2}j;m-\tau,\tau)]^2}, \quad (31)$$

and all other quantities have cancelled. Performing the sums in (31) yields

$$\frac{\langle j_z\rangle}{j}=\frac{2\tau}{3j}[j(j+1)+\tfrac{3}{4}-l(l+1)]. \quad (32)$$

We take $\tau=+\frac{1}{2}$ and obtain the results[18]

$$\langle j_z\rangle/j=\tfrac{1}{3}(1+1/j);\quad j=l+\tfrac{1}{2}, \quad (33a)$$

$$\langle j_z\rangle/j=-\tfrac{1}{3};\quad j=l-\tfrac{1}{2}. \quad (33b)$$

Thus, if the μ mesons have random direction, the polarization after capture is determined by (33). We explicitly point out that the results, (33), are independent of any assumptions concerning radial wave functions, that they apply to the ejection of an electron from any initial state and that they apply to the capture of the meson in any state l. We must now justify the assumption of random direction for the mesons.

The quantity of interest is the average value of the projection of the mesons momentum direction after some scattering from its original direction. Thus we determine $\langle\hat{k}_E\cdot\hat{k}_{E0}\rangle$, where $\hat{k}_{E0}$ is the unit propagation vector in the original beam direction and $\hat{k}_E$ is the corresponding vector for the direction of the meson when it has energy E. We define

$$s=\cos\theta=\frac{\hat{k}_z(E')}{\hat{k}_z(E)}, \quad (34a)$$

$$s'=\cos\theta'=\frac{\hat{k}_z(E'-\Delta E)}{\hat{k}_z(E)}, \quad (34b)$$

where ΔE is the energy loss suffered in the collision occurring at energy E' and $\hat{k}_z(E)$ is the z component of $\hat{k}_E$. Now Δs is

$$\Delta s=s'-s=\cos\Theta\cos\theta+\sin\Theta\sin\theta\cos\phi-\cos\theta, \quad (35)$$

[18] Of course, these results can be obtained by simple considerations of vector coupling and the three-dimensionality of space.

where Θ is the scattering angle for the collision $E' \rightarrow E' - \Delta E$. We average over scattering events to obtain

$$\frac{\Delta\langle s\rangle}{\Delta x} = n_s \int \sigma(\theta)\Delta s d\Omega, \tag{36}$$

where n_s is the number of scattering centers per unit volume, $\sigma(\theta)$ is the differential scattering cross section, and $d\Omega$ is again the angle element. Substituting (35) in (36) and noticing that there is no contribution from the second term on the right of (35) yields

$$d\langle s\rangle/dx = -n_s\langle s\rangle\sigma_{\mathrm{Tr}}, \tag{37}$$

where σ_{Tr} is again the transport cross section. Integration of (37) yields

$$\langle s\rangle = \exp\left(-n_s\int_0^x \sigma_{\mathrm{Tr}} dx\right), \tag{38}$$

and we now use Eqs. (13) and (14) to find

$$\langle s\rangle = \exp\left[-\frac{1}{\kappa}\int_E^{E_0}\frac{dE}{E}\right], \tag{39}$$

which integrates immediately to give

$$\langle \hat{k}_E \cdot \hat{k}_{E0}\rangle = (E/E_0)^{1/\kappa}. \tag{40}$$

In (40), E_0 is taken to be the same as E_0 in (18) for the purpose of comparing the rate at which the muons lose their "memory" of their initial direction with the capture rate. The comparison is shown in Fig. 3. From this data we conclude that the μ mesons have very nearly random direction when they are captured. Thus the depolarization on capture is obtained by using the simple results of Eq. (33). We now see that the cross sections play a subordinate role in the calculation in that they serve only to determine the initial distribution of bound states but not the polarization in these states. The final results are relatively insensitive to the initial distribution as will be readily understood from the following.

IV. DEPOLARIZATION IN THE CASCADE

We have determined the distribution of the μ mesons among the capturing states and the depolarization due to capture. We now discuss the depolarization that occurs in the cascade of transitions by which the μ mesons reach the ground state of the μ-mesonic atom. The first step in such a discussion is the determination of the depolarization in a specific transition. This is done as follows.

We consider a radiative transition from an initial state n', l', j' to a final state n, l, j and define p_m as the meson population of a state m belonging to j and $p_{m'}$ as the population of a state m' belonging to j'. The transition probability from state m' to m is

$$\lambda_{m'm} = \sum_P |(\psi_{j,l}{}^m, \mathbf{V}\cdot\mathbf{A}_{LM}\psi_{j',l'}{}^{m'})|^2\rho(E), \tag{41}$$

where $\mathbf{V}$ is the current operator and $\mathbf{A}_{LM}$ is the vector potential for the radiation field,[14]

$$\mathbf{A}_{LM} = (2\pi)^{\frac{1}{2}} \sum_{L,M} i^L(2L+1)^{\frac{1}{2}} D_{MP}{}^L(\varphi,\theta,0) \times[\mathbf{A}_{LM}(\mathrm{mag}) + iP\mathbf{A}_{LM}(\mathrm{el})], \tag{42}$$

where $D_{MP}{}^L(\varphi,\theta,0)$ is a rotation matrix element. For our purpose we consider only electric multipole radiation (eventually $L=1$ only) and we need only retain the factors in (41) that depend on the magnetic quantum numbers. Using the properties of the rotation matrices, the Wigner-Eckart theorem, and integrating over the final photon direction, we find

$$\lambda_{m'm} \propto [C(j'Lj;m',m-m')]^2. \tag{43}$$

Now the population of state m is given by

$$p_m = \sum_{m'} p_{m'}\lambda_{m'm}, \tag{44a}$$

and $p_{m'}$ may be expanded as

$$p_{m'} = \sum_n a_n C(j'nj';m,0), \tag{44b}$$

with the restriction that $n \leq 2j_{\mathrm{min}}' \leq 1$.

The ratio of the meson polarizations in the transition $j' \rightarrow j$ is given by

$$\frac{P}{P'} = \frac{\sum_m m p_m / j \sum_m p_m}{\sum_{m'} m' p_{m'} / j' \sum_{m'} p_{m'}}; \tag{45}$$

and by use of (44) and the Racah algebra, this may be evaluated as

$$\frac{P}{P'} = \frac{j'(j'+1)+j(j+1)-L(L+1)}{2j(j'+1)}. \tag{46}$$

Since we are concerned with electric dipole transitions, $L=1$, and we have for the three possible transitions, $\Delta j = 0, \pm 1$,

$$P/P' = 1; \qquad \Delta j = -1, \tag{47a}$$

$$P/P' = 1 - 1/(j(j+1)); \qquad \Delta j = 0, \tag{47b}$$

$$P/P' = 1 - 1/j^2; \qquad \Delta j = +1. \tag{47c}$$

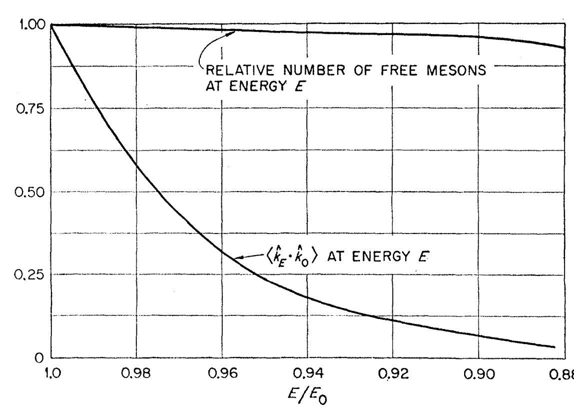

FIG. 3. Comparison of capture rate and memory of initial direction for μ mesons in carbon.

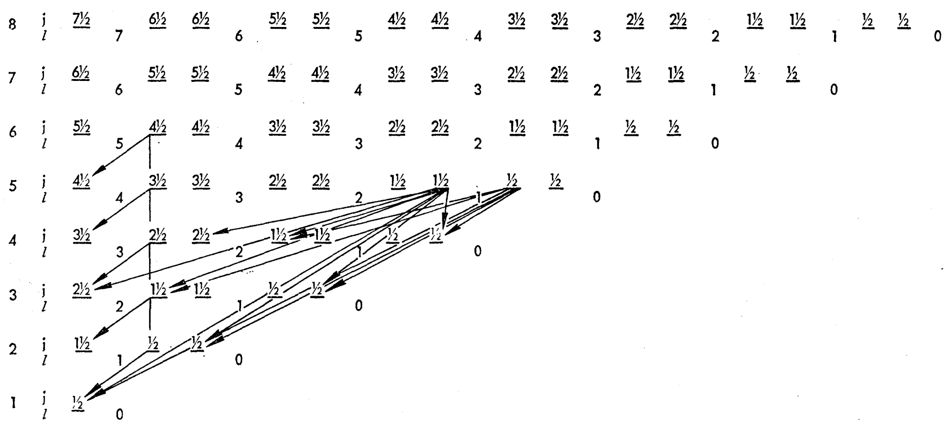

FIG. 4. Some states involved in the cascade, showing typical branching.

Further, if we take $L=0$, $\Delta j=0$, (46) yields

$$P/P'=1. \qquad (48)$$

This will be relevant for the ordinary Auger processes.

At this point we discuss the cascade under the assumption that only *radiative* transitions occur (except for the $2s\rightarrow 1s$ transition). The purpose in making this restriction is to emphasize the need for considering the ordinary μ Auger transitions and this is done in subsequent paragraphs. It will be noted that in reference 4 only radiative transitions were taken into account. We use the distribution given in Table I and the initial polarization in these states as determined by Eqs. (33) together with Eqs. (47) and the known theory of radiative transitions.[19] The relative number of μ mesons making each possible transition was determined (with the cascade originating in each possible initial state) and the final polarizations in the $2s$ and $1s$ states were determined. The treatment of the mesons which go to $2s$ states is deferred for later discussion. We emphasize that the appropriate values of all the radial integrals in the radiative transitions as well as all the $\Delta l=\pm 1$ transitions and all possibilities for Δn were included. In obtaining the final polarizations each possible cascade was given a weight appropriate to its over-all branching ratio and an average over the entire ensemble of starting points of the cascade was taken using the populations calculated previously as weight factors. The computation was done with an electronic computer. Some typical transitions are shown in Fig. 4. A study of this figure is quite informative since one may show that when the states are arranged in this fashion the only possible transitions are those straight down or to the left.

We note that the Eq. (46) is not restricted to radiative transitions since (46) is determined by (42) which is a consequence of the Wigner-Eckart theorem. If we assume that the $2s\rightarrow 1s$ transition proceeds through an ordinary Auger process we see by Eq. (48), that this last step of the cascade does not cause further depolarization.[20] Then the final polarization in the $1s$ state of μ-mesonic carbon is $P=0.24$ and therefore the asymmetry coefficient would be $|a|=0.08$, if only radiative transitions occur. However, it is known that Auger transitions compete with the radiative transitions and dominate the cascade in its initial stages. We include these as follows.

It has been shown by Burbidge and de Borde that the Auger transitions obey dipole selection rules to a good approximation and that they favor $\Delta n=-1$.[15] This is to be contrasted with the radiative transitions where $|\Delta n|=$maximum is preferred. Further, Demuer has shown that the branching ratios for the dipole Auger transitions with $\Delta n=-1$ are the same as those for the equivalent radiative transitions.[16] We use the results of Burbidge and de Borde for the Auger to radiative branching as given by Rainwater.[21] From this one may determine at what state n the Auger transitions cease to be predominant. This branching ratio varies extremely rapidly with the principal quantum number so that it is a fairly good approximation to assume that only Auger transitions occur when n is larger than some critical value and only radiative transitions occur thereafter. Thus, for carbon the final Auger transition is taken as the transition starting from the state with $n=4$ into the state $n=3$. Thus, the Auger transitions are taken into

[19] For computing the branching ratios in the radiative cascade we use the formulation for the radiative lifetime as given by M. E. Rose, *Multipole Fields* (John Wiley & Sons, New York, 1955), Chap. 6. The expression, given by H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Academic Press, New York, 1957), p. 262, is used to evaluate the radial integrals. The details of this and all the other calculations mentioned in this paper are given by R. A. Mann, Oak Ridge National Laboratory Report ORNL-2990 (unpublished).

[20] We could treat the $2s\rightarrow 1s$ transitions in the manner described by M. A. Ruderman, Phys. Rev. **118**, 1632 (1960). This would introduce unnecessary complications since we find that only 1.6% of the μ mesons enter the $2s$ state when we consider pure radiative transitions. When we also include the Auger transition, as will be explained in the text, then for the case of carbon only 5.4% of the μ mesons enter the $2s$ state. In this connection we point out that since we have treated the Auger transitions in a schematic fashion, our results should be very insensitive to any new features brought to light by resolution of the questions raised by the data of M. B. Stearns and M. Stearns, Phys. Rev. **105**, 1573 (1957).

[21] J. Rainwater, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, California, 1957), Vol. 7, p. 1.

account by instructing the computer program for radiative transitions to allow only $\Delta n = -1$ for n greater than some integer (3 in the case of carbon). This will result in a greater depolarization than was found in the purely radiative cascade because fewer states are bypassed.

By use of the procedure just outlined we find that the final polarization of the μ mesons in the $1s$ state of μ-mesonic carbon is

$$P = 0.133. \tag{49}$$

V. DISCUSSION OF THE RESULTS

From the results of the last section we obtain a computed asymmetry coefficient for the decay of negative μ mesons in carbon:

$$|P|/3 = |a| = 0.044. \tag{50}$$

Two experimental values have been reported in the literature; these are $|a| = 0.054 \pm 0.006$ and $|a| = 0.04 \pm 0.005$.[22–24] It is clear that our calculated value is in good agreement with the measured results. There is, of course, some uncertainty connected with the schematic treatment of the Auger-radiative branching ratio. For example, if we had taken the final n to be 4 instead of 3, we would have obtained $P = 0.153$ corresponding to $|a| = 0.051$ which is also consistent with the observations. A rough estimate of the uncertainty in the theoretical result is that the spread in the calculated values corresponds almost precisely to the spread in measured values. It is, of course, possible to make the calculations more accurately but it appears pointless to do so until better experimental results are available. All the machinery for improving the theoretical results is available and this can be done quite straightforwardly. Our primary purpose in this paper has been to obtain an understanding of the depolarization mechanisms with spin-zero nuclei in the capturing atoms, and from the comparison given above it is fair to say that this has been achieved.

If we turn our attention to other atoms, we must first recognize that when the nucleus has a spin the depolarization will be much more severe. First, the random magnetic field associated with the hyperfine coupling will produce a "loss of memory" so far as the direction of the μ-meson spin is concerned and some depolarization will occur in every intermediate state in the cascade. The lifetime in these intermediate states is, moreover, usually large compared with the period of the hyperfine precession. Second, the hyperfine coupling will produce a small splitting of the ground state and in some of the substates of the hyperfine multiplet a strong depolarization of the μ meson will occur. For instance, when the nuclear spin is $\frac{1}{2}$ the states in which m, the z component of the total angular momentum, F, is zero will be characterized by vanishing μ-meson polarization. Consequently, the final asymmetry factor is expected to be much smaller in these cases than one would deduce from a consideration of the mechanisms discussed above. Excluding the effect of collisions which could induce transitions between states in the hyperfine multiplet, it is estimated that the polarization for nuclear spin $\frac{1}{2}$ is reduced by a factor 3 because of these hyperfine effects.[4] If we adopt this factor 3 for the odd-mass isotopes of Cd, for example, and assume the same distribution over the initial states of the cascade as was found for C, we can obtain an estimate for the expected asymmetry coefficient for that element. Since the Auger transitions should play a negligible role for $Z = 48$, we use our result $P = 0.24$ for the purely radiative cascade. This has to be multiplied by 0.75 to take into account the odd-mass isotopes with a total abundance of about 25%. Consequently, we would obtain $|a| \approx 0.06$. The reported value is $|a| = 0.055 \pm 0.012$.[23] This agreement is very satisfactory.

[22] R. Prepost, V. W. Hughes, S. Penman, D. McColm, and K. Ziock, Bull. Am. Phys. Soc. **5**, 75 (1960).

[23] A. E. Ignatenko, L. B. Egorov, B. Khalupa, and D. Chultem, J. Exptl. Theoret. Phys. U.S.S.R. **35**, 1131 (1959) [translation: Soviet Phys.-JETP **35**(8), 792 (1959)].

[24] A third value, $|a| = 0.045$ was communicated from the floor at the 1960 Washington meeting of the American Physical Society. We have been unable to ascertain any further information regarding this matter.

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