Neutron-Proton Pairing Interaction*

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The neutron-proton pairing interaction λ between the last odd neutron and the last odd proton in the outermost neutron and proton shells of an odd-odd nucleus has been estimated from nuclear masses in the regions just beyond $Z=20$, $N=20$, and just beyond $Z=40$, $N=50$. Behavior of λ in these two regions and in the heavy element region $Z>82$, $N>126$, as estimated by Ghoshal and Saxena, is discussed. It is found that the behavior of λ may be understood in terms of a simple jj-coupling shell model. According to this model, λ arises from the spin-independent part of the two-body force and is proportional to $(2j_1+1-2z)(2j_2+1-2n)$, where z is the odd number of protons in the *outermost* proton shell j_1, and n is the odd number of neutrons in the outermost neutron shell j_2 of the odd-odd nucleus.

INTRODUCTION

THE pairing energy between the last odd proton and the last odd neutron has been estimated by Ghoshal and Saxena,[1] in the region of $Z>82$, $N>126$ using the experimental values of neutron and proton binding energies. In the present investigation, estimation of λ has been extended to the regions $Z>20$, $N>20$ and $Z>40$, $N>50$, and its behavior with changes in Z and N is discussed. The pairing energy associated with the "last" pair of neutrons in the region of $64<Z<82$, $92<N<126$ has been estimated by Johnson and Bhanot.[2]

ESTIMATION OF λ

Using the formalism of Ghoshal and Saxena,[1] we write for the nuclear mass formula for various even-odd combinations of protons and neutrons

$$\begin{aligned}
M(Z,N)&=f(Z,N)-2\sum_{j}^{Z/2}\pi_j-2\sum_{k}^{N/2}\nu_k \quad \text{(even-even)},\\
M(Z+1,\,N)&=f(Z+1,\,N)-2\sum_{j}^{Z/2}\pi_j-2\sum_{k}^{N/2}\nu_k \quad \text{(odd-even)},\\
M(Z,\,N+1)&=f(Z,\,N+1)-2\sum_{j}^{Z/2}\pi_j-2\sum_{k}^{N/2}\nu_k \quad \text{(even-odd)},\\
M(Z+1,\,N+1)&=f(Z+1,\,N+1)-2\sum_{j}^{Z/2}\pi_j-2\sum_{k}^{N/2}\nu_k-\lambda(Z+1,\,N+1) \quad \text{(odd-odd)},
\end{aligned} \tag{1}$$

where $f(Z,N)$ expresses the functional dependence of nuclear mass on Z and N; π_j and ν_k are the pairing energies per nucleon of proton and neutron, respectively, the suffixes j and k denoting the different proton and neutron pairs, respectively. The last odd proton or neutron does not contribute to the sum of the pairing energies of all the proton and neutron pairs considered in the above equations.[3] Ghoshal and Saxena[1] introduced a new term λ to take into account the effect of the n-p pairing interaction between the odd proton and the odd neutron in the outermost shells. According to this formalism, the "pairing energy level diagram" would be as shown in Fig. 1, for various types of nuclei.

From Eqs. (1), Ghoshal and Saxena[1] deduced the

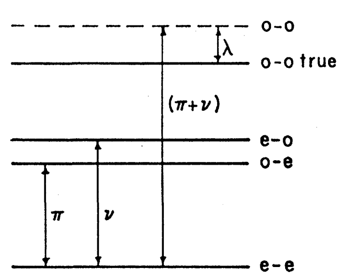

FIG. 1. "Pairing energy level diagram" for various types of nuclei. *e-e* nuclei (even Z, even N) are the most stable ones. *o-e* and *e-o* nuclei are raised above the *e-e* level by the pairing energies π and ν, respectively. *o-o* nuclei are raised above the *e-e* level not by an amount $(\pi+\nu)$ but instead are $(\pi+\nu-\lambda)$ higher, as shown by *o-o* true, due to n-p pairing interaction.

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[1] S. N. Ghoshal and A. N. Saxena, Proc. Phys. Soc. (London) **A69**, 293 (1956).

[2] W. H. Johnson, Jr. and V. B. Bhanot, Phys. Rev. **107**, 1669 (1957).

[3] M. G. Mayer, Phys. Rev. **78**, 22 (1950).

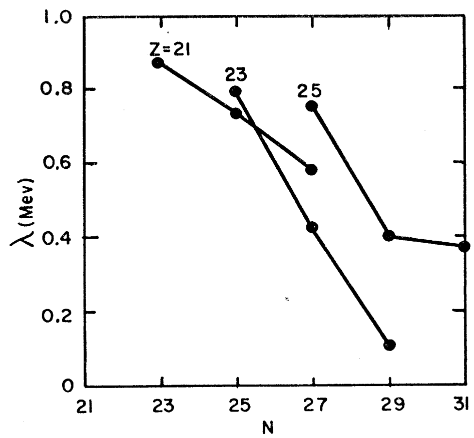

FIG. 2. Plot of λ against neutron number N for constant proton numbers Z.

following equations which give four independent evaluations of λ for a given odd-odd nucleus using the experimental values of neutron binding energies for a set of even and odd isotones and proton binding energies for a set of even and odd isotopes. (For details, see Ghoshal and Saxena.[1])

For even Z and N,

$$\begin{aligned}
\lambda(Z+1, N-1) &= \tfrac{1}{2}[B_n^{ee}(Z,N)+B_n^{ee}(Z+2, N) \\
&\qquad -2B_n^{oe}(Z+1, N)], \\
\lambda(Z+1, N-1) &= -\tfrac{1}{2}[B_n^{eo}(Z, N-1)+B_n^{eo}(Z+2, N-1) \\
&\qquad -2B_n^{oo}(Z+1, N-1)], \qquad (2) \\
\lambda(Z+1, N-1) &= \tfrac{1}{2}[B_p^{ee}(Z+2, N-2)+B_p^{ee}(Z+2, N) \\
&\qquad -2B_p^{eo}(Z+2, N-1)], \\
\lambda(Z+1, N-1) &= -\tfrac{1}{2}[B_p^{oe}(Z+1, N-2)+B_p^{oe}(Z+1, N) \\
&\qquad -2B_p^{oo}(Z+1, N-1)],
\end{aligned}$$

where B_n and B_p are the last neutron and last proton

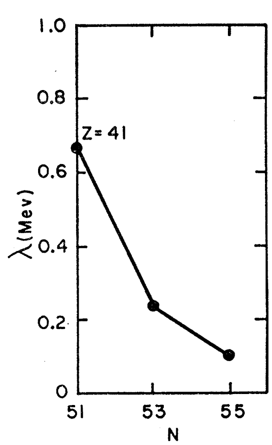

FIG. 3. Plot of λ against neutron number N for constant proton number Z.

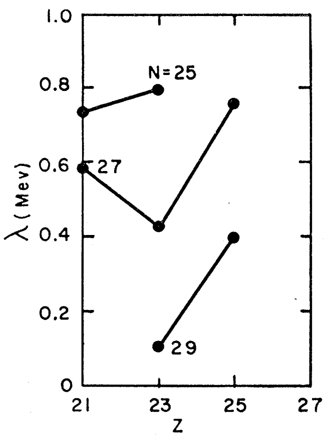

FIG. 4. Plot of λ against proton number Z for constant neutron numbers N.

binding energies in nuclei of various even-odd combinations. The first superscript denotes the even-odd nature of Z and second that of N.

The neutron and proton binding energies used to evaluate λ were calculated from the masses tabulated by Wapstra[4] and nuclear data cards.[5] In a few cases the binding energies were deduced from the β-decay energies.[6,7] The compilation of neutron and proton binding energies by Feather[8] was found quite useful in checking the over-all behavior of the binding energies calculated from the new data.

BEHAVIOR OF λ AND ITS INTERPRETATION

Figures 2, 3, 4, 5, 6, and 7 show λ for various isotopic and isotonic series as evaluated from the experimental values of the neutron and proton binding energies. For

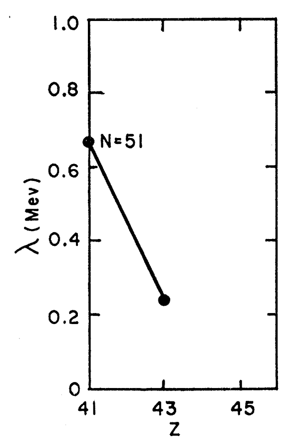

FIG. 5. Plot of λ against proton number Z for constant neutron number N.

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[4] A. H. Wapstra, Physica **21**, 367 and 385 (1955).

[5] K. Way, G. Andersson, G. H. Fuller, N. B. Gove, D. N. Kundu, J. B. Marion, C. L. McGinnis, M. K. Ramaswamy, and M. Yamada, *Nuclear Data Cards and Sheets*, National Academy of Science, National Research Council (U. S. Government Printing Office, Washington, D. C., 1957, 1958).

[6] D. Strominger, J. M. Hollander, and G. T. Seaborg, Revs. Modern Phys. **30**, 585 (1958).

[7] S. N. Ghoshal and A. N. Saxena, Indian J. Phys. **29**, 81 (1955); A. N. Saxena, Indian J. Phys. **29**, 501 (1955).

[8] N. Feather, *Advances in Physics*, edited by N. F. Mott (Taylor and Francis, Ltd., London, 1953), Vol. 2, p. 141.

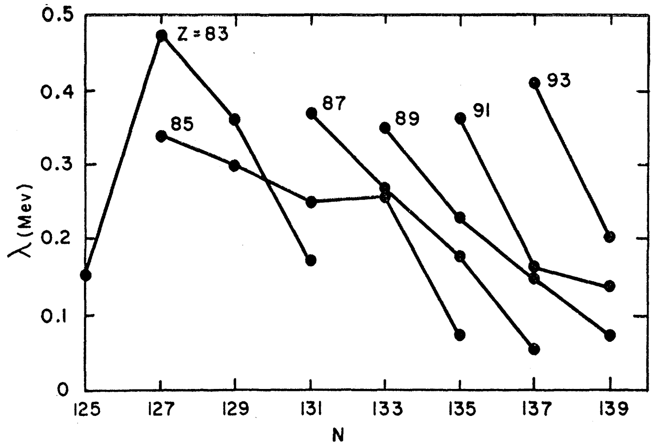

FIG. 6. Plot of λ against neutron number N for constant proton numbers Z.

a given isotopic series λ decreases with increase in neutron number. This is true for all the three regions considered here except at the shell crossings $Z=82+1$ and $N=126+1$. When a neutron is added to the hole at $N=125$ for the $Z=83$ series, λ increases. Variation of λ with changes in Z is slightly more complicated. For isotonic series near the neutron magic number, λ is observed to decrease with increase in proton number near the proton magic number. For the isotonic series far from the neutron magic number, λ increases with increase in proton number. Behavior of λ for $N=27$ is an interesting example of both of these cases. $N=27$ means a "hole" in the neutron shell $f_{7/2}$, therefore addition of a pair of protons at $Z=21$ causes λ to decrease at first, but further addition of a pair of protons causes it to increase again.

The n-p pairing interaction can be assumed to arise from an attractive interaction between the neutrons and protons. Let us consider z protons in the j_1 orbit and n neutrons in the j_2 orbit outside their respective closed shells, where z and n are both odd numbers. The wave function of the combined configuration with total spin J and magnetic quantum number M is given by

$$\Psi[j_1^z(j_1)j_2^n(j_2),JM] = \sum_{m_1m_2} \psi_1[j_1^z(j_1)m_1]\psi_2[j_2^n(j_2)m_2] \times\langle j_1m_1j_2m_2|j_1j_2JM\rangle. \quad (3)$$

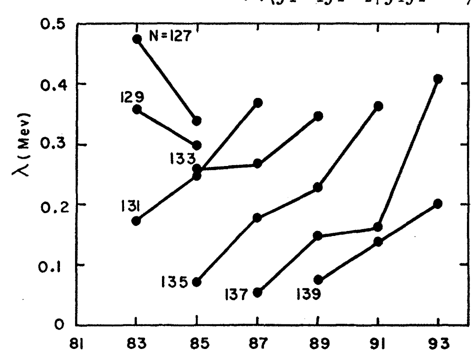

FIG. 7. Plot of λ against proton number Z for constant neutron numbers N.

In the absence of interaction between the neutrons and protons, they form a group of levels which are degenerate with all spins from $J=|j_1-j_2|$ to $J=j_1+j_2$. If we introduce an attractive interaction V_{np} between the neutrons and protons, these states are split up in energy. Each state of given J receives an energy E_J given by[9]

$$E_J=\langle j_1^z(j_1)j_2^n(j_2)JM|V_{np}|j_1^z(j_1)j_2^n(j_2)JM\rangle. \quad (4)$$

V_{np} can be expressed as the sum of spin-independent and spin-dependent forces. If α is the mixing constant of the two forces, then we can write V_{np} as

$$V_{np}=[1-\alpha+\alpha(\boldsymbol{\sigma}_n\cdot\boldsymbol{\sigma}_p)]V_0(r_{np}). \quad (5)$$

Hence E_J will receive a contribution from the spin-independent force, E_0, as well as from the spin-dependent force, E_σ:

$$E_J=E_0+E_\sigma. \quad (6)$$

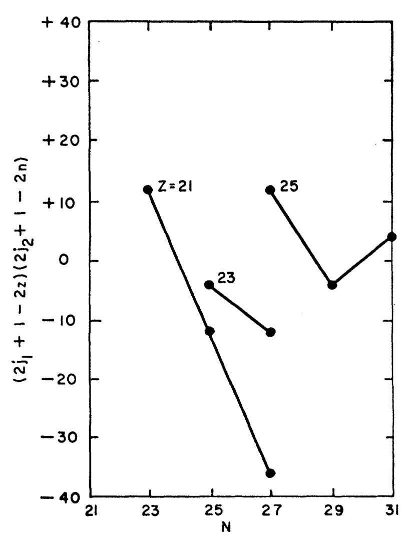

FIG. 8. Plot of $(2j_1+1-2z)(2j_2+1-2n)$ vs neutron number N for constant proton numbers Z.

[9] A. de-Shalit, Phys. Rev. 91, 1479 (1953).

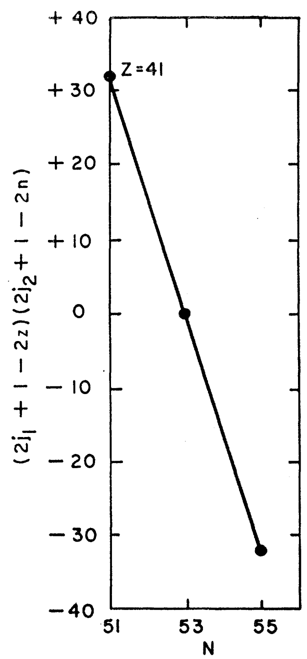

FIG. 9. Plot of $(2j_1+1-2z)(2j_2+1-2n)$ vs neutron number N for constant proton number Z.

It has been shown[10,11] that E_σ is independent of z and n, and that E_0 is given by[12]

$$E_0[j_1^z(j_1)j_2^n(j_2),J] = \left(\frac{2j_1+1-2z}{2j_1-1}\right)\left(\frac{2j_2+1-2n}{2j_2-1}\right) \times[E_0(j_1j_2,J)-(1-\alpha)F^0(l_1l_2)] +(1-\alpha)znF^0(l_1l_2). \quad (7)$$

Thus we can write for the interaction energy between the odd protons and the odd neutrons as

$$E_J=(2j_1+1-2z)(2j_2+1-2n)f'(j_1j_2J;\alpha;l_1l_2) +znf''(\alpha,l_1l_2)+f'''(j_1j_2,J), \quad (8)$$

where f' and f'' are the functions defined in Eq. (7) and f''' is the contribution from the spin-dependent forces. If either z or n or both are even and J_p or $J_n=0$, then only the second term appears in Eq. (7).[12] Therefore, second and third terms in Eq. (8) are cancelled out in the evaluation of λ from Eqs. (2). Thus λ is proportional to the first term in Eq. (8). If j_1, j_2, and J (more rigorously l_1, l_2, and α also) remain constant for an isotopic or isotonic series, then the behavior of λ with changes in z and n is determined by the term

[10] C. Schwartz, Phys. Rev. **94**, 95 (1954).

[11] For definitions of various functions, see original literature referred to above or see the review article by J. P. Elliott and A. M. Lane, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 241.

[12] A. de-Shalit, Phys. Rev. **105**, 1528 (1957).

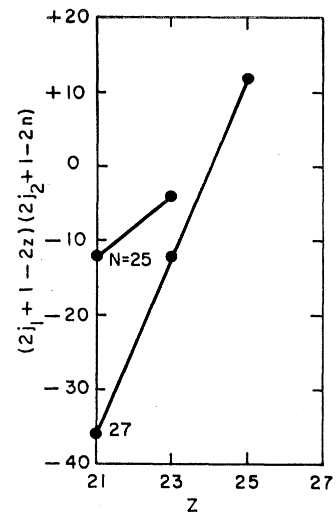

FIG. 10. Plot of $(2j_1+1-2z)(2j_2+1-2n)$ vs proton number Z for constant neutron numbers N.

$(2j_1+1-2z)(2j_2+1-2n)$. It should be borne in mind that the absolute value of λ cannot be compared with this term because we do not know the values of the other functions involved.

Calculation of $(2j_1+1-2z)(2j_2+1-2n)$ and its comparison with λ plotted in Figs. 2, 3, 4, 5, 6, and 7 becomes unsatisfactory in some of the regions where the shell assignments are not rigorous or when they change in a given isotopic and isotonic series.[13,14] Therefore $(2j_1+1-2z)(2j_2+1-2n)$ has been plotted in Figs. 8, 9, 10, 11, 12, and 13 for only a few cases where the shell assignments are relatively better known. Behavior of λ and $(2j_1+1-2z)(2j_2+1-2n)$ are quite similar.

It has also been found in the present investigation that λ is a constant for a given pair of magic numbers for a given $(n-z)$. Figure 14 shows the plot of λ vs $(n-z)$. At each point are listed various nuclei whose λ values were taken in calculating the average value plotted. λ decreases with increase in $(n-z)$ for a given

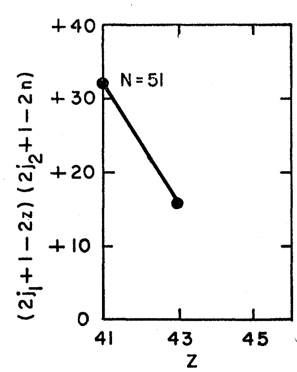

FIG. 11. Plot of $(2j_1+1-2z)(2j_2+1-2n)$ vs proton number Z for constant neutron number N.

[13] B. Oquidam and B. Jancovici, Nuovo cimento **11**, 578 (1959).

[14] A. de-Shalit and J. D. Walecka, Phys. Rev. **120**, 1790 (1960).

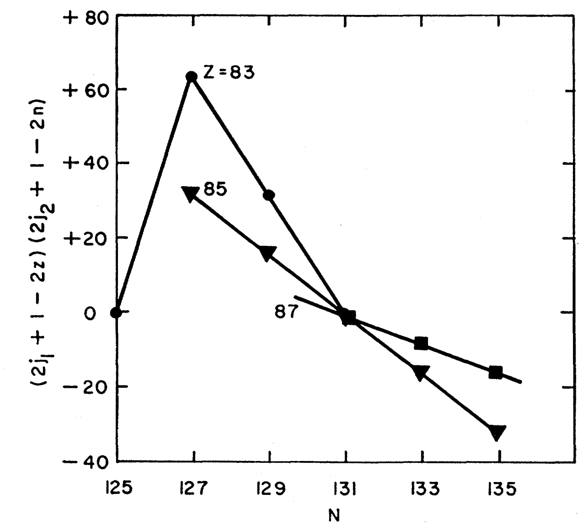

FIG. 12. Plot of $(2j_1+1-2z)(2j_2+1-2n)$ vs neutron number N for constant proton numbers Z.

pair of neutron and proton magic numbers. No theoretical explanation of this behavior of λ has been found yet. Nevertheless, one can explain qualitatively the decrease of λ with increase in $(n-z)$ as follows.

Suppose we have one proton in the outermost proton shell and one neutron in the outermost neutron shell. They can have their l's (orbital angular momenta)

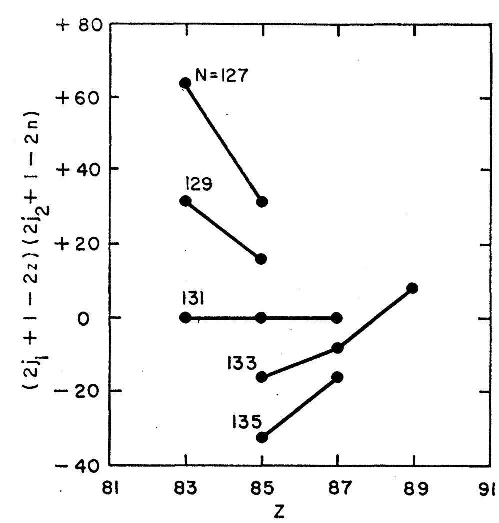

FIG. 13. Plot of $(2j_1+1-2z)(2j_2+1-2n)$ vs proton number Z for constant neutron numbers N.

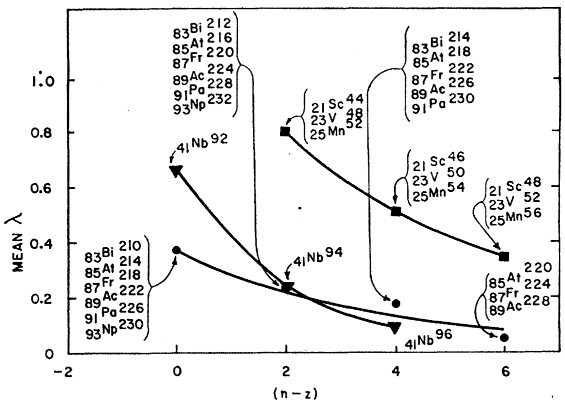

FIG. 14. Plot of λ vs $(n-z)$. At each point are listed various nuclei whose values were taken in calculating the average value plotted. For a given pair of neutron and proton magic numbers, λ is found to be a constant for a given $(n-z)$ and it decreases with increase in $(n-z)$.

antiparallel to each other 100% of the time. When we add a pair of neutrons to the neutron shell, then the effective time during which the l's of the proton and the neutrons can be antiparallel would be reduced. Hence the strength of the attractive interaction will decrease which will decrease the value of λ. If we add another pair of neutrons, this effective time will be further reduced, though by a smaller amount. Similar decrease in the value of λ will be obtained by further addition of neutron pairs. Therefore, this explains the rapid decrease of λ when $(n-z)$ increases from 0 to 2 and its slow decrease as $(n-z)$ becomes higher.

Neutron-proton pairing energy in ${}_{41}Nb^{90}$ has been found to be 2.32 Mev. Both the odd proton and the odd neutron are in the $g_{9/2}$ shell. Proton pairing energy in the $g_{9/2}$ shell has been calculated to be 2.17 Mev by Talmi and Unna.[15]

ACKNOWLEDGMENTS

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[15] I. Talmi and I. Unna (to be published). The author is indebted to Professor A. de-Shalit for communicating the results contained in this preprint.