where

$$T = 1 - \frac{i}{2\hbar}(u_\beta p_\beta + p_\beta u_\beta).$$

It is easy to show that this is the same T (to the first order in u_β) as was defined in (23), and it is clear from its form that it is an operator that translates the electron a distance $\mathbf{u}$. We also note that the electric field operator H_{elec}' (55) and p_α' (34) are obtained, to the first order in u_α, by application of T to H_{elec} and p_α.

Since in the body of this paper we neglect the last term in (C-4) (i.e., E^{III}) the only difference in the physical approximation made when using the ODB instead of the Bloch functions, is the use of (39) instead of the R.I. or D.I. models (i.e., except for E^{III} and the model used for the effective potential, H and H' are related by a similarity transformation and therefore are equivalent). We note further that since the third term in (C-4) involves a factor $(E_l - E_{l'})$, it is zero for matrix elements between states of the same total energy. Hence, the matrix elements of H, *to the first order in* $\mathbf{u}$, in the D.I. and R.I. models *and between states of the same energy*, are exactly the same in the Bloch and ODB representations. Since this property is true for both the R.I. and D.I. models, it is probably independent of the model used. These similarities probably do not extend to higher orders.[36]

[36] In second order perturbation theory, matrix elements between states of different energy are important and one may think that terms like the third term in (C-4) will become very large. The fact that these terms result from a change of representation rather than a different physical approximation suggests that they will have no profound effect on the observables. A situation somewhat similar to this was encountered by J. M. Ziman, Proc. Cambridge Phil. Soc. **51**, 707 (1955), and it became clear that these terms did not lead to an important physical effect. Bernard Goodman, Phys. Rev. **110**, 888 (1958); J. C. Taylor, Proc. Cambridge Phil. Soc. **52**, 693 (1956). In a situation like ultrasonic absorption where $\mathbf{u}$ is large, and the theory must be expanded in powers of $S^{\alpha\beta}$, there might be some real effect.

PHYSICAL REVIEW VOLUME 121, NUMBER 3 FEBRUARY 1, 1961

Decay of Excess Carriers in Semiconductors. II

K. C. Nomura and J. S. Blakemore
Honeywell Research Center, Hopkins, Minnesota
(Received August 22, 1960)

A physical interpretation is given of the nonlinear differential equations which govern the decay of excess carrier populations through recombination centers. No restrictions are placed on the magnitudes of the excess carrier densities or the center density. Criteria for trapping are presented; with semiconductors for which the trapping level lies in the opposite half of the intrinsic gap from the Fermi level, it is shown that trapping can be described as being of either a temporary or permanent nature. The variety of possible modes of decay are illustrated with the aid of numerical solutions and approximate analytic solutions.

I. INTRODUCTION

THE lifetime of an excess carrier population is for many semiconductors controlled by a process in which the charge of a recombination center changes by $+e$ and $-e$ alternately. For example, when a recombination center captures a hole of charge $+e$, the next process experienced by this center may be either the reliberation of the hole or the capture of a previously free electron. Either process will restore the center to its original charge state.

The details of the carrier dynamics will depend on a number of parameters and variables. As parameters we should list the absolute and relative magnitudes of the electron and hole capture cross sections, and the density and energy of recombination levels. Variables which enter into the problem comprise the equilibrium Fermi level (which characterizes the thermal-equilibrium carrier densities n_0 and p_0) and the concentrations of excess carriers. The excess free carrier densities $\Delta n = (n - n_0)$ and $\Delta p = (p - p_0)$ are not necessarily the same, since intermediate levels tend to indulge in trapping as well as recombination. For this reason, we shall use the terms "trap" and "recombination center" interchangeably in this paper.

The kinetics of the excess carriers during buildup, maintenance and decay may be described in terms of two coupled first order differential equations which are expressed in terms of the above mentioned parameters and variables. Solutions of these equations for the special case of steady state nonequilibrium have been presented by Shockley and Read[1] and by Hall.[2] Even then, these published solutions are valid only for the limiting conditions of vanishing trap density or vanishing excess carrier density. More recently the transient behavior of excess carrier populations confronted with arbitrary trap density has been discussed by us[3] in a paper to which we shall in future refer as NB1.

From the study of the general decay equations in

[1] W. Shockley and W. T. Read, Phys. Rev. **87**, 835 (1952).
[2] R. N. Hall, Phys. Rev. **87**, 387 (1952).
[3] K. C. Nomura and J. S. Blakemore, Phys. Rev. **112**, 1607 (1958).

NB1, it was concluded that the methods to be used in attempting solutions could be divided into two classes, Class I and Class II. Class I conditions exist when the equilibrium Fermi level lies in the same half of the intrinsic energy gap as the trap levels [i.e., n-type semiconductors with traps in the upper half of the gap, p-type semiconductors with traps in the lower half of the gap]. Class II conditions are deemed to hold when the traps and equilibrium Fermi level are in opposite halves of the intrinsic gap. In NB1 we concentrated on the forms of solution for Class I decay. In the present paper we wish to examine critically the conditions for trapping and to provide solutions for Class II decay. The differential equations are the same, but the relative influence of certain terms is drastically changed.

The differential equations under study are highly nonlinear; at best they can be reduced only to a general form of Abel's differential equation of the first kind.[4] Solutions are known only for certain restrictive values of the parameters since this equation defines new transcendental functions. Perturbation methods necessarily fail since all terms in the equation assume comparable importance in the range of interest. This does not, however, create an insuperable difficulty. A considerable amount of insight into the physical aspects of the decay process can be inferred from approximate solutions when the excess populations are supposed to be very large or very small. Since the equations are of a type easily handled by a digital computer, our conclusions are supported and augmented by solutions numerically evaluated with an IBM 650, a procedure we found helpful in NB1.

II. GENERAL DECAY EQUATIONS

The continuity equations for excess electron and hole densities customarily contain terms describing generation, recombination, diffusion, and drift. It is not necessary for our purposes to include the complications of spatially varying carrier densities. Thus, when excess generation of carriers in a homogeneous semiconductor has ceased, the continuity equations can be expressed as

$$-d\Delta n/dt=C_n[(n_0+\Delta n)(1-f_t)-n_1f_t], \qquad (1)$$

$$-d\Delta p/dt=C_p[(p_0+\Delta p)f_t-p_1(1-f_t)]. \qquad (2)$$

The notation is essentially that of Shockley and Read.[1] All the symbols used are defined in Table I, and correspond with the terminology employed in NB1.

The quantity f_t is the fraction of traps occupied by an electron. It is customary to eliminate this from (1) and (2) by using the charge neutrality condition. We then have two new equations for the time derivatives of Δn and Δp. At this point, however, it is convenient to go over to a dimensionless notation we employed in NB1, in which densities are renormalized by the equilibrium free hole density p_0 and time is renormalized by $\tau_{n0}=(C_n)^{-1}$. This renormalization procedure is adopted as being especially convenient for p-type semiconductors, but it will always be obvious how results would apply to n-type semiconductors.

[4] E. Kamke, *Differentialgleichungen Lösungsmethoden und Lösungen*, (Akademische Verlogsgesellschaft Geest and Portig K. G., Leipzig, 1956).

TABLE I. List of symbols.

n_0	thermal equilibrium electron density in conduction band
p_0	thermal equilibrium hole density in valence band
Δn	excess free electron density
Δp	excess free hole density
N_t	density of traps
n_1	thermal electron density when the Fermi level is at the energy of a nondegenerate trap
p_1	thermal hole density when the Fermi level is at the energy of a nondegenerate trap
$C_n=N_t\langle c_n\rangle=1/\tau_{n0}$	probability per unit time that an electron will be captured by any of a set of N_t available sites
$C_p=N_t\langle c_p\rangle=1/\tau_{p0}$	probability per unit time that a hole will be captured by any of a set of N_t available sites
f_t	fraction of traps occupied by electrons
$f_{t0}=(1+p_0/p_1)^{-1}=(1+n_1/n_0)^{-1}$	value of f_t in thermal equilibrium
$x=\Delta n/p_0$; $y=\Delta p/p_0$	
$a=n_1/p_0$; $b=p_1/p_0$	
$\gamma=C_p/C_n=\tau_{n0}/\tau_{p0}$; $N=N_t/p_0$	
$T=t/\tau_{n0}$	normalized time scale

In terms of the dimensionless parameters and variables, the continuity equations are:

$$-Nx'=(x-y)[x+a(1+b)]+Nx/(1+b), \qquad (3)$$

$$-(N/\gamma)y'=(y-x)[y+(1+b)]+Nby/(1+b). \qquad (4)$$

It will be noted that the dimensionless symbols used are all in Table I. A variety of approximate solutions for Eqs. (3) and (4) were given in NB1 for some limiting ranges of key parameters [e.g., N very small, N very large, etc.]. In NB1 such solutions were considered in the context of Class I decay, but they can equally well be applied to Class II decay by an appropriate change in the relative importance of a and b. It is not then necessary to explore this field of approximate solutions any further at the present. It is more important for us to consider the detailed physical picture of trapping and how this influences the general pattern of decay.

III. TRAPPING

One of the very useful quantities in the analysis of trapping phenomena is the ratio of excess electron to excess hole densities in the final stages of decay:

$$r=\lim_{x\to 0,y\to 0}\left(\frac{x}{y}\right)=\left.\frac{dx}{dy}\right|_{x=0,y=0}. \qquad (5)$$

As we have remarked in NB1, this ratio is rarely equal to unity; for contrary to popular belief, trapping almost always occurs when recombination centers control the decay. The deviation from unity of the ratio r estab-

lishes the kind of trapping which must occur. Electron trapping is the characteristic of a situation for which the parameter $r<1$, while hole trapping is always the rule if $r>1$.

The ratio r can be obtained as follows. First Eq. (3) is divided by Eq. (4). Numerator and denominator are divided by y. When the limit of this ratio is taken, a quadratic equation results for r with the solution

$$r=\frac{\{(1+b)^2(a+\gamma)^2+N(1-\gamma b)[2(a-\gamma)+N(1-\gamma b)(1+b)^{-2}]\}^{\frac{1}{2}}-[(1+b)(a-\gamma)+N(1-\gamma b)(1+b)^{-1}]}{2\gamma(1+b)}. \quad (6)$$

The criteria for trapping can be established by means of this seemingly cumbersome equation. For example, we know that—at any rate for vanishingly small excess populations—there is no trapping when $r=1$. This forces the condition [from Eq. (6)] that

$$N(1-\gamma b)=0 \quad \text{for} \quad r=1. \quad (7)$$

For an arbitrary value of γb, we obtain the obvious result that trapping can be absent ($r=1$) only if the trap density is very small. Actually, the upper limit on trap density consistent with $r\approx 1$ is that

$$N\ll\frac{(1+b)^2(a+\gamma)}{|1-\gamma b|} \quad \text{for} \quad r\approx 1, \quad (8)$$

as may be verified by solution of Eq. (6). When N is small enough to satisfy the condition (8), the decay of an excess population has the form

$$\frac{\tau}{\tau_{n0}}=\frac{(1-a)(1-\gamma b)}{\gamma(1+ab)}\ln\left[\frac{y_0+1+ab}{y+1+ab}\right]+\frac{(1+b)(a+\gamma)}{\gamma(1+ab)}\ln\left[\frac{y_0}{y}\right], \quad (9)$$

valid for any magnitude of the excess population $y=x$. This result was previously derived as Eq. (13) of NB1, though at that time the nature of the restriction (8) was not explicitly stated.

While we should not be surprised that trapping becomes imperceptible when N is very small, it is rather less obvious why Eq. (7) should require an absence of low-modulation trapping when $\gamma b=1$, irrespective of the magnitude of N. But as we showed in NB1, even for a very large trap density, there is no disparity between x and y for *any* degree of carrier modulation when the mutual relationship of the trap energy and the Fermi level is such that $\gamma b=1$. The solutions for this remarkable case are

$$x=y=x_0\exp[-T/(1+b)]. \quad (10)$$

If we now solve for N in Eq. (6), the remaining criteria for trapping may be determined. Thus

$$N=\frac{2(1+r^2)(a+\gamma r)}{r}\left[\frac{1-r}{1-\gamma b}\right]\geq 0, \quad (11)$$

so that for any finite trap density we require that

$$(1-r)/(1-\gamma b)>0. \quad (12)$$

When $\gamma b<1$, the condition (12) indicates that r is smaller than unity; the situation is one of electron trapping. Similarly, $\gamma b>1$ is the mark of a hole trapping situation. The relative magnitudes of γ and $(1/b)$ clearly establish the essential nature of the trapping.

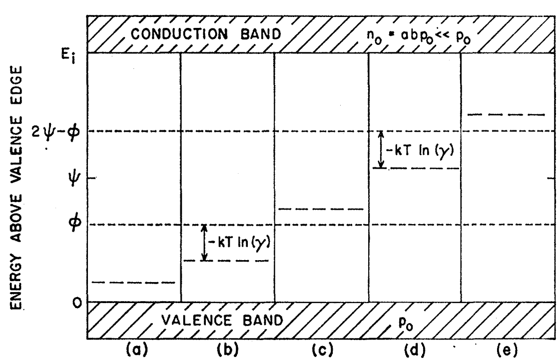

FIG. 1. Illustration of five possible positions for the trap energy E_t, and the kind of trapping corresponding with such positions when the Fermi level ϕ is in the lower half of the gap. This figure is drawn for donor-like traps, $\gamma=(C_p/C_n)<1$. (a) Majority hole trapping when $\gamma b>1$, so that $E_t<[\phi+kT\ln(\gamma)]$. (b) No trapping when $\gamma b=1$. (c) Minority electron trapping whenever $\gamma b<1$, so that $E_t>[\phi+kT\ln(\gamma)]$. This trapping is essentially permanent if $\gamma>a$ so that $E_t<[2\psi-\phi+kT\ln(\gamma)]$. (d) Trapping is "semi-permanent" when $\gamma=a$. (e) Trapping temporary when $\gamma<a$, $E_t>[2\psi-\phi+kT\ln(\gamma)]$.

We have attempted in Fig. 1 to illustrate the relative positions of Fermi level and trap energy for various kinds of trapping. This figure is an arbitrary choice based on a p-type semiconductor $n_0(=abp_0)\ll p_0$; then the Fermi level ϕ lies in the lower half of the intrinsic gap. The figure is drawn supposing donor-like traps ($\gamma<1$), but of course an analogous figure could equally well have been drawn for acceptor-type traps of $\gamma>1$.

Part (b) of Fig. 1 indicates the relationship of trap energy to Fermi level when trapping can not occur, $\gamma b=1$. The trap energy E_t is $kT\ln(\gamma)$ higher than the Fermi level ϕ [for donor traps, $\ln(\gamma)$ is a negative quantity]. When E_t is lower than this special position, as in part (a) of the figure, majority holes are trapped, while minority electron trapping must ensue if $E_t>\phi+kT\ln(\gamma)$.

Then parts (c), (d), and (e) of Fig. 1 all correspond with electron trapping situations. It will be shown in the next section that this classification should be divided again depending on whether trapped electrons tend to be re-excited or not.

IV. CLASS II DECAY

The general pattern of decay can easily be constructed from a study of the initial and final stages of the process. The electron and hole lifetimes, τ_n, τ_p, can be written down from the form of Eqs. (3) and (4):

$$\frac{\tau_n}{\tau_{n0}}=-\frac{x}{x'}=\left\{\frac{1}{1+b}-\frac{(y-x)[x+a(1+b)]}{Nx}\right\}^{-1}, \quad (13)$$

$$\frac{\tau_p}{\tau_{n0}}=-\frac{y}{y'}=\gamma^{-1}\left\{\frac{b}{1+b}+\frac{(y-x)(y+1+b)}{Ny}\right\}^{-1}. \quad (14)$$

We do not propose to consider intrinsic semiconductors here, but rather extrinsic p-type materials for which $ab=(n_0/p_0)\ll 1$. In NB1 we discussed Class I decay and now wish to concentrate on Class II processes for which in p-type material y will always be larger than x, and b will always be much smaller than unity. This enables us to simplify (13) and (14) to read

$$\tau_n\approx\tau_{n0}\left\{1-\left(\frac{y}{x}-1\right)\left(\frac{x+a}{N}\right)\right\}^{-1}, \quad \text{Class II,} \quad (15)$$

$$\tau_p\approx\tau_{p0}\left\{\left(\frac{y}{x}-1\right)\left(\frac{y+1}{N}\right)\right\}^{-1}, \quad \text{Class II.} \quad (16)$$

This section concentrates first on analytic and then on computed solutions.

(a) The Initial Stages of Decay

When the excess populations are equal, $x=y$ [as they are, for instance, in the initial moments of decay following a delta function burst of generation], τ_n tends to τ_{n0} while τ_p is very large. The physics behind this is easy to see. For in a Class II situation, the traps are well above the Fermi level (for a p-type semiconductor); they are completely denuded of electrons and are *all* available for electron capture as soon as excess carrier pairs are created. Hole capture can not begin to get under way *until* the traps have captured some electrons. This produces the early disparity of x and y which will be reflected all through the course of the subsequent decay.

Figure 2 illustrates an important point; that the initial values assumed for x and y are not important after a very short time. Curves of y and x diverging from the initial condition $y_0=x_0=1$ at time $t=44\ \tau_{n0}$ rapidly coalesce with the decay curves for a situation involving the same density and kind of traps but a larger initial carrier disturbance at an earlier time. Our principal interest lies in the dynamics of decay when the initial stages have been completed.

(b) The Final Stages of Decay

When Class II decay is pursued into the final stages of decay, the ratio (x/y) tends towards a constant value, the quantity r of Eq. (6). Since $b\ll 1$ in Class II cases this may be simplified to

$$r\approx(2\gamma)^{-1}\{[(a+N-\gamma)^2+4a\gamma]^{\frac{1}{2}}-(a+N-\gamma)\}, \quad \text{Class II.} \quad (17)$$

Such a statement of course carries the implication that the time constants for electron and hole decay must eventually become equal. This final lifetime may be found from the limit of (x/x') in Eq. (3) as $x\rightarrow 0$,

$$\frac{\tau_\infty}{\tau_{n0}}=\lim_{x\rightarrow 0}(-x/x')=\frac{rN}{rN-a(1-r)}. \quad (18)$$

By substituting r from Eq. (17), we have

$$\tau_\infty=\tfrac{1}{2}\tau_{p0}\{[(a+N+\gamma)^2-4\gamma N]^{\frac{1}{2}}+(a+N+\gamma)\}. \quad (19)$$

The same result could equally well have been obtained by dropping second order terms in (3) and (4), then solving the pair of equations simultaneously.

Evidently τ_∞ can not be smaller than τ_{n0} nor can it be larger than $[\tau_{n0}+\tau_{p0}(a+N)]$. Where it lies in between depends on the relative magnitudes of a, N, and γ. Whereas r depends primarily on the magnitude of the trap density, the functional dependence of the time constant is more crucially affected by the ratio

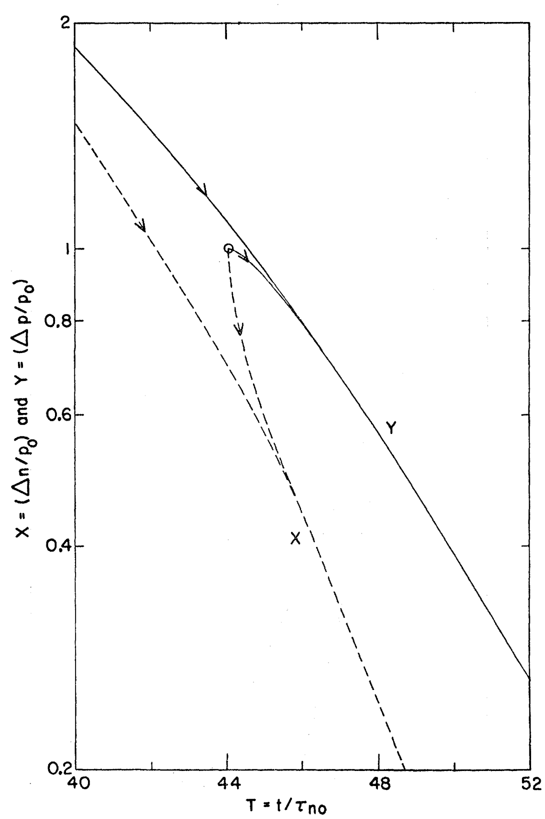

FIG. 2. Coalescence of decay curves for different initial conditions but the same trap parameters [$N=0.5$, $\gamma=0.1$, $a=0.02$, $ab\ll 1$]. Solid curves for $y=(\Delta p/p_0)$, broken curves for $x=(\Delta n/p_0)$.

a/γ. The value of that ratio is significant in differentiating between what might be called temporary and permanent trapping.

In NB1, when we discussed Class I situations, a trapped minority electron had virtually no chance of being re-excited to the conduction band. In this sense, trapping of an electron was always "permanent," the proceedings always being terminated by the subsequent capture of a hole. On the other hand, when the traps are much closer to the conduction band (as they are for Class II situations) capture of an electron can be followed by either of two processes; (a) capture of a hole, or (b) emission of an electron. We prefer to think of the trapping of an electron as being merely "temporary" when it has more than a 50% chance of re-excitation.

From Eqs. (1) and (2) in dimensionless form, the ratio of the rates of electron emission to hole capture is $a/\gamma(1+y)$. Except for very large excess hole densities, the border-line between temporary and permanent trapping occurs for $a=\gamma$. This borderline case of "semipermanent trapping" is indicated in part (d) of Fig. 1. Where ψ denotes the intrinsic position of the Fermi level, semi-permanent trapping occurs when the trap energy is $kt\ln(\gamma)$ higher than the position $(2\psi-\phi)$. This position for a trapping level of $[2\psi-\phi+kT\ln(\gamma)]$

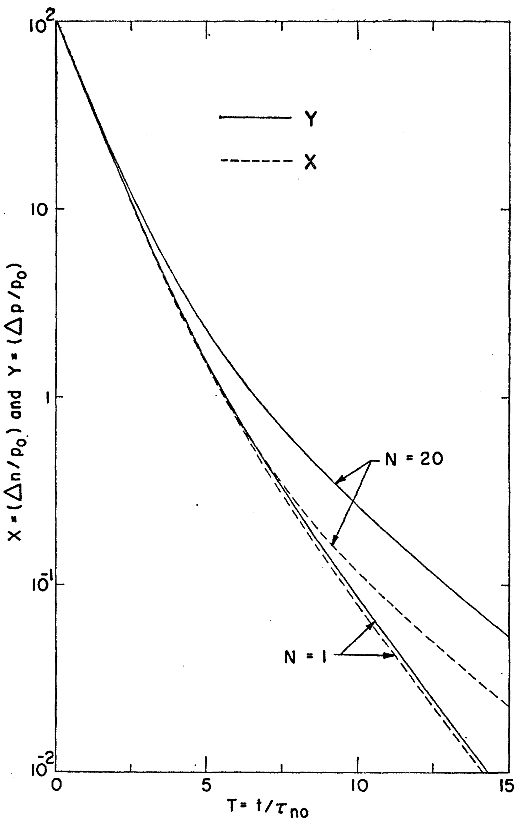

FIG. 4. Showing "semipermanent" decay $[ab\ll1,\ a=\gamma=10]$ for acceptor-like traps, indicating the slowing of the final decay (in terms of τ_{n0}) and the widening gulf between x and y when N becomes large.

has been called the "level of equality" by Shockley[5] and the "demarcation level" by Rose.[6]

We think of trapping as being a very temporary phenomenon when traps are closer to the conduction band than this "level of equality" to make $a>\gamma$. Under such conditions an electron is likely to be trapped and re-excited several times before a free hole can be procured with which to annihilate it. This is particularly prone to happen with very asymmetric traps for which $\gamma\ll1$, such as those reported in silicon by Hornbeck and Haynes.[7]

When a hole is captured, the probability of re-emission compared with the probability of electron capture is $[\gamma b/(ab+x)]$. For an extrinsic Class II situation this ratio is much smaller than unity and decreases with increasing excess electron density.

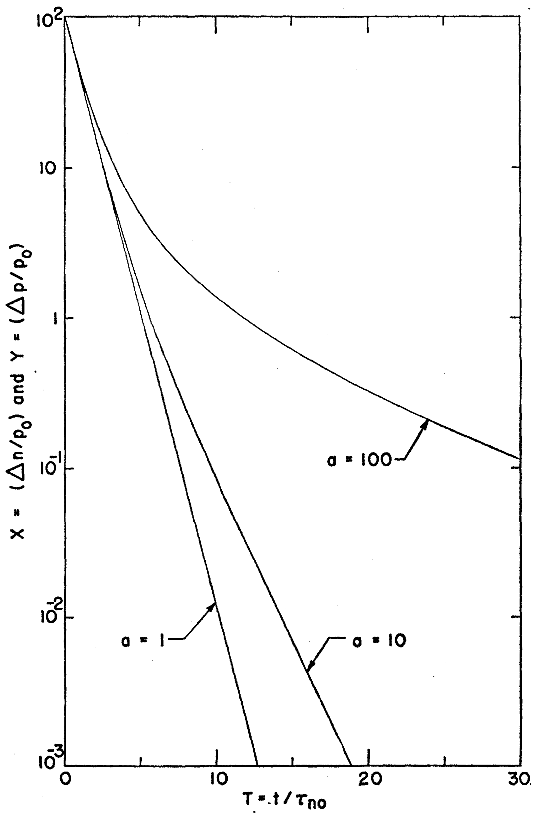

FIG. 3. Excess carrier decay for acceptor-like traps of low density $[N<0.1,\ \gamma=10,\ ab\ll1,\ a$ as indicated$]$.

(c) Decay Involving Acceptor-Like Traps

The general features of Class II decay via acceptor-like traps[8] are illustrated by the curves of Fig. 3. This

[5] W. Shockley, Proc. I. R. E. **46**, 973 (1958).
[6] A. Rose, *Progress in Semiconductors* (John Wiley & Sons, New York, 1957), Vol. 2, p. 111.
[7] J. A. Hornbeck and J. R. Haynes, Phys. Rev. **97**, 311 (1955).
[8] A trap is said to be "acceptor-like" if $\gamma=(C_p/C_n)>1$. An acceptor impurity is expected to have such a nature since Coulomb

particular figure is drawn supposing that $\gamma=10$, and a rather small trap density; thus the curves for $x=(\Delta n/p_0)$ and $y=(\Delta p/p_0)$ are indistinguishable. An initial condition $x_0=y_0=100$ has been supposed for our examples, so that the large modulation decay region can be seen. It is well known that the time constant in that region is $\tau_{n0}(1+1/\gamma)$, which for acceptor-like traps is not much larger than τ_{n0} itself. Trapping is decidedly permanent for the curve of $a=1$ and very temporary for $a=100$. The center curve is for traps at the "level of equality," when the time constant tends to $2\tau_{n0}$ if $N\ll(a+\gamma)$. For progressively larger trap densities, not only does the limiting carrier ratio r become less than unity; also the final time constant τ_∞ tends towards $[\tau_{n0}+\tau_{p0}(a+N)]$ in accordance with Eq. (19). These tendencies for the final stages of decay are illustrated by the curves of Fig. 4, which are for the same values of a, γ, etc., as the center curve of Fig. 3. Whereas for $N\leq 1$, the final time constant is $\tau_\infty\approx 2\tau_{n0}$ when traps are at the level of equality, a final time constant some 70% larger (in terms of τ_{n0}) results for $N=20$. This trap density also makes the final excess carrier ratio $r=(2^{\frac{1}{2}}-1)\approx 0.41$.

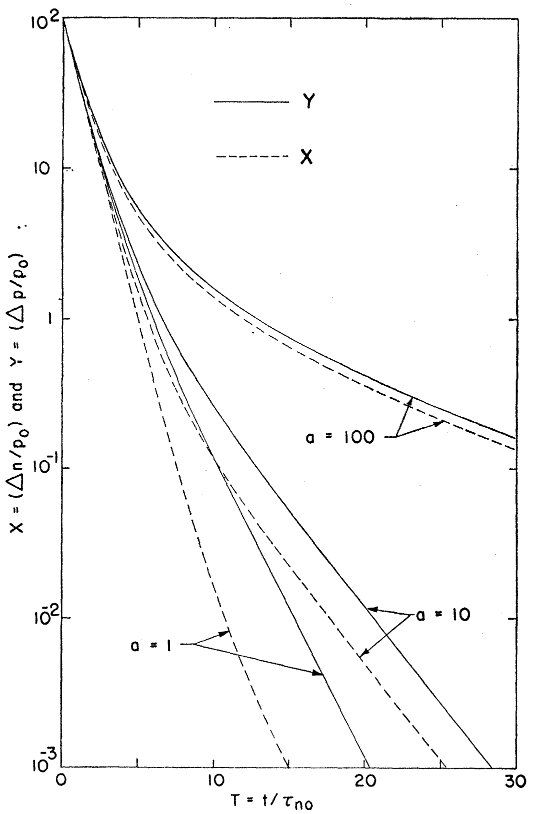

FIG. 5. The form of decay for acceptor-like traps of large density $[N=20,\ \gamma=10,\ ab\ll 1]$ with values of a ranging from permanent to temporary trapping.

attraction should assist in hole capture when ionized (negatively charged) but not in electron capture when the center is neutral. Similarly, a donor-like trap is one for which $\gamma<1$.

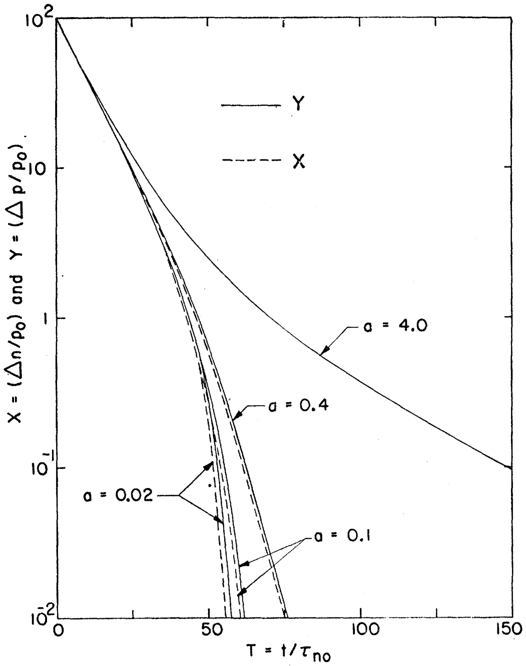

FIG. 6. The transformation from permanent to temporary trapping for a moderately small density of donor-like traps $[N=0.1,\ \gamma=0.1,\ ab\ll 1]$.

The increase of the parameter a in the three curves of Fig. 3 suggests the effect of increasing temperature. The three corresponding pairs of curves in Fig. 5 show that when there are many traps, an increase of temperature tends to decrease the proportion of trapped carriers at every stage of the decay and brings r closer to unity.

(d) Decay Involving Donor-Like Traps

The modes of decay of excess carriers via donor-like traps ($\gamma<1$) are illustrated in Fig. 6 for a moderately small trap density. Several interesting features may be noted in this figure. The first of these is the obvious transition from permanent to temporary trapping as the ratio a/γ is increased from a small value to one larger than unity. Secondly, we note that the electron to hole ratio approaches unity as a is increased. This has the physical interpretation that re-excitation of electrons is now highly likely and trapping becomes negligible.

Permanent trapping assumes a more prominent role when the trap density is increased, as shown in Fig. 7. When N is large, the limiting excess carrier ratio r diminishes rapidly, though this headlong collapse of (x/y) is eventually checked. Ultimately, electrons and holes assume the same time constant and (x/y) becomes fixed. This can be seen to occur when x falls below 10^{-2} in the curves of Fig. 7 for $N=0.5$. It can be seen from Eq. (17) that when traps are rather numerous [enough to make $N>\gamma$], $r\approx a/(N-\gamma)$, which is capable of

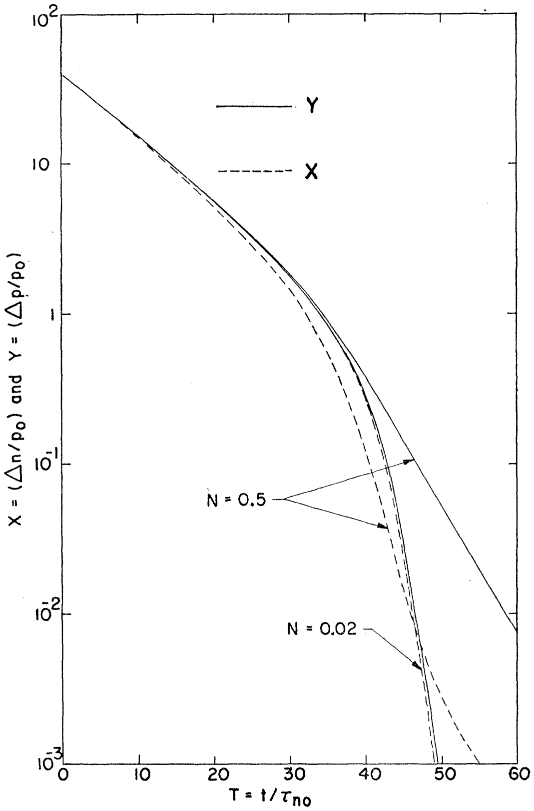

FIG. 7. Illustrating the pronounced influence of trapping on the decay for increasing values of N with donor-like traps if $a<\gamma<1$. [In this example, $\gamma=0.1$, $a=0.02$, $ab\ll 1$, as for one pair of curves in Fig. 6.]

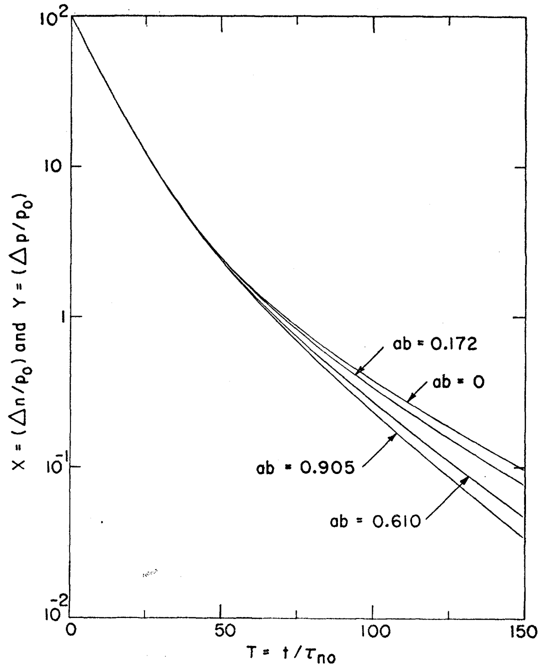

FIG. 8. The speeding up of the final decay when n_i is not necessarily negligible compared with $(N_a-N_d)_{\text{ion}}$. For $ab=0$, $n_i\ll(N_a-N_d)_{\text{ion}}$. For $ab=0.172$, $n_i=0.5(N_a-N_d)_{\text{ion}}$. For $ab=0.610$, $n_i=2(N_a-N_d)_{\text{ion}}$. For $ab=0.905$, $n_i=10(N_a-N_d)_{\text{ion}}$.

becoming very small compared with unity. This was a feature which we noted in NB1 for Class I decay when centers were rather close to the center of the gap: that the ratio (x/y) could tend towards an extremely small value in highly extrinsic semiconductors.

(e) The Approach of the Fermi Level to ψ

In the numerical examples we have illustrated up to the present, it has been assumed that the semiconductor was rather strongly extrinsic; $ab\ll 1$ and $(\psi-\phi)\gg kT$. The rate of carrier decay is in fact a little larger if the material is semi-intrinsic. This can be demonstrated graphically from the curves of Fig. 8. Of the four curves in this figure, the one with the slowest decay is identical with the curve for $a=4$ in Fig. 6. The three remaining curves show the speeding up of the final decay when n_i is *not* negligibly small compared with $(N_a-N_d)_{\text{ion}}$. Completely intrinsic material would correspond with $ab=1$, and this is almost true for the fastest of the four curves.

ACKNOWLEDGMENTS

As in our previous discussion of this subject, NB1, Mr. John A. Lindquist of Honeywell's Aeronautical Division devised and executed the program for obtaining numerical solutions to Eqs. (3) and (4) with an IBM-650 computer. It is a pleasure to thank Mr. Lindquist for his enthusiastic cooperation.