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Comparison of Inelastic Scattering from Sm^{152} with Coulomb Excitation Theory

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In order to check the theory of electric quadrupole Coulomb excitation, accurate measurements have been made of differential cross sections for inelastic scattering from the first excited state of Sm^{152}. Protons, deuterons, and alpha particles of energies in the region of 4 Mev were used as the bombarding particles. In addition to the angular distribution of inelastically scattered deuterons, measurements were also made with protons at two scattering angles and at a backward angle with deuterons and alpha particles of different incident energies. The data are found to be in excellent agreement with the semiclassical description of the Coulomb excitation process.

I. INTRODUCTION

A PREVIOUS measurement of the angular distribution of inelastically scattered particles following Coulomb excitation was made by Elbek and Bockelman.[1] The results, however, do not represent a critical test of the theory due to the rather large experimental uncertainties. Later, more accurate measurements of differential Coulomb excitation cross sections have been made by Elbek, Olesen *et al.*[2–6] Although, in these later experiments, both protons and deuterons of different bombarding energies were used, only a rather limited comparison with the theory can be made, especially since these measurements were confined to a single scattering angle (145°).

In order to make a critical comparison between experimental data and Coulomb excitation theory, it is necessary that the bombarding energy be low compared with the Coulomb barrier so that the projectiles do not penetrate into the nucleus and produce proper nuclear reactions. This condition is well satisfied in the present experiment since, in the worst case (4.3-Mev protons), the classical distance of closest approach in a head-on collision is larger than the nuclear radius by more than a factor of two. Furthermore, the total cross section for compound nucleus formation for 4.3-Mev protons on Sm^{152} is about a factor of 35 lower than the total Coulomb excitation cross section.

Wisseman and Williamson[7] have made measurements of the inelastic scattering of 3.5- to 4.0-Mev protons and deuterons from Ge^{74} and Ge^{76}. While some of their results can be explained in terms of Coulomb excitation, there are also indications that nuclear force processes are taking place for 4.0-Mev deuterons.

In the present experiment, accurate measurements of differential cross sections for electric quadrupole Coulomb excitation of the first excited state of Sm^{152} have been made. In addition to the angular distribution of inelastically scattered deuterons, measurements were also made with protons at two scattering angles and at a backward angle with deuterons and alpha particles of different incident energies. The particular bombarding conditions were chosen in such a manner that they represent a wide range of the parameters entering the theoretical description of the process.

II. THEORY

The spin of the first excited state of Sm^{152} at 122 kev has been assigned[8] as 2^+ in agreement with the systematics of deformed even-even nuclei. It is therefore expected that Coulomb excitation of this level must be pure electric quadrupole.[9] The differential cross section in the center-of-mass system for $E2$ Coulomb excitation can be expressed[10] in the following form:

$$\frac{d\sigma}{d\Omega}=4819\left(1+\frac{A_1}{A_2}\right)^{-2}\frac{A_1}{Z_2{}^2}(E_i-\Delta E')\times B(E2)df(\eta_i,\xi)\frac{\text{millibarns}}{\text{steradian}},$$

$$\xi=\frac{Z_1Z_2A_1{}^{\frac{1}{2}}\Delta E'}{12.65(E_i-\frac{1}{2}\Delta E')^{\frac{3}{2}}},\quad \eta_i=\frac{Z_1Z_2}{2}\left(\frac{A_1}{10.08E_i}\right)^{\frac{1}{2}},$$

$$\Delta E'=\left(1+\frac{A_1}{A_2}\right)\Delta E,$$

where Z_1 and Z_2 are the atomic numbers of the projectile and the target nucleus, respectively, A_1 is the projectile mass in units of the proton mass, A_2 is the target mass number, E_i and ΔE are the laboratory bombarding energy and the excitation energy expressed in Mev, ϑ

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[1] B. Elbek and C. K. Bockelman, Phys. Rev. **105**, 657 (1957).
[2] V. Ramšak, M. C. Olesen, and B. Elbek, Nuclear Phys. **6**, 451 (1958).
[3] B. Elbek, K. O. Nielsen, and M. C. Olesen, Phys. Rev. **108**, 406 (1957).
[4] B. Elbek, M. C. Olesen, and O. Skilbreid, Nuclear Phys. **10**, 294 (1959).
[5] B. Elbek, M. C. Olesen, and O. Skilbreid, Nuclear Phys. **19**, 523 (1960).
[6] M. C. Olesen and B. Elbek, Nuclear Phys. **15**, 134 (1960).
[7] W. R. Wisseman and R. M. Williamson, Nuclear Phys. 21, 688 (1960).
[8] O. Nathan and M. A. Waggoner, Nuclear Phys. **2**, 548 (1957).
[9] The $E2$ character of the excitation is also demonstrated in the present experiment. See Sec. III.
[10] K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Revs. Modern Phys. **28**, 432 (1956).

is the scattering angle in the center-of-mass system, and $B(E2)$ is the reduced electric quadrupole transition probability for excitation[11] expressed in units of $e^2 \times 10^{-48}$ cm^4. Alder and Winther[12] have calculated the function $df(\vartheta,\eta_i,\xi)$ in the limit $\eta_i=\infty$ which corresponds to a classical description of the projectile path. This semiclassical calculation leads to a total cross section which differs by about 2% from the exact quantal calculation[10] for $\eta_i=5$. However, it was found that the angular distribution of the deexcitation gamma rays was not given correctly by the semiclassical calculation, even for rather large values of η_i. Therefore, one might suspect that the quantum corrections to the differential cross section would be much larger than the corrections to the total cross section. Recently, Bang[13] has made quantum mechanical calculations of the function $df(\eta_i,\xi)$ in the limiting case of no energy loss ($\xi=0$). The results of these latter calculations show that, at least for $\xi=0$, the semiclassical and quantal calculations of the differential cross section differ by less than 2% for $\eta_i \geqslant 4$.

The values of ξ and η_i for the measurements reported here range from $\xi=0.07$, $\eta_i=4.7$ for protons to $\xi=0.34$, $\eta_i=20$ for alpha particles.

III. EXPERIMENTAL PROCEDURE

The experimental techniques are essentially the same as those that were used in the previous measurements[2–6] of inelastically scattered particles made at this Institute.

Protons, deuterons, and alpha particles from the Institute's 4.5-Mev electrostatic accelerator were used as bombarding particles. The spectrum of scattered particles was measured by means of a broad-range magnetic spectrograph, utilizing Ilford *C*2 photographic emulsions of 25 or 50μ thickness as a detector. The 50μ plates were used in the deuteron exposures in order to distinguish deuterons from protons of the same momentum originating in (d,p) reactions in the target backing.

Two targets prepared at different times in the Institute's electromagnetic isotope separator were used for the measurements reported here. The target material, enriched to better than 99% in Sm^{152}, was deposited over an area of about 5×5 mm on carbon foil backings of about 75 μg/cm^2 thickness. A detailed description of the method of target preparation is given in reference 5.

The targets were rather uniform over the area of the beam spot (0.3 mm high$\times$3 mm wide). However, there was some variation in the thickness of the target material from the top to the bottom. Therefore, it was possible to bombard a region on the targets corresponding to a thickness of about 12 μg/cm^2 for the alpha-particle measurements, whereas the proton and deuteron measurements were made with a target thickness corresponding to about 45 μg/cm^2.

A thinner target is necessary for good energy resolution with alpha particles, since they have much larger energy loss than protons and deuterons.

The inelastic cross sections were obtained from the ratio of the inelastic to the elastic yield, assuming that the elastic cross section is given by the Rutherford scattering law. The Coulomb excitation cross section per unit solid angle in the center-of-mass system is given by the product of the measured ratio[14] and the Rutherford cross section per unit solid angle evaluated in the center-of-mass system. Due to the large ratio of the elastic to the inelastic scattering, it was not possible to count the particle tracks corresponding to the elastic group on the same exposure which gave a reasonable inelastic yield. Therefore, a short exposure of a few microcoulombs was made before and after the long exposure and all three were recorded on adjacent zones of the same plate. The three exposures were normalized by means of a current integrator.

The intensities and positions of the elastic peaks on the short exposures gave a check that there were no significant changes in the target or bombarding conditions during the long exposure.

For the angular distribution measurements the input aperture of the spectrograph was adjusted so that the angular spread of the detected particles was approximately $\pm1°$ for angles smaller than 90° and approximately $\pm2°$ for larger angles.

The spectrograph is constructed in such a way that the accepted particles always leave the horizontal plane containing the beam at an angle of approximately 35°. Thus, the maximum obtainable scattering angle in the laboratory system is approximately 145° while the minimum angle is approximately 35°. The absolute scattering angles were determined by comparing the energy loss of the elastically scattered particles from Sm^{152} with the energy loss of the elastically scattered particles from several light element impurities. These results were consistent with the above-mentioned geometry. The measured angle between the horizontal plane and the direction of the accepted particles was 35.5 ± 0.8 degrees.

The problem of contaminants in the target and the backing proved to be the main factor limiting the accuracy and extent of the angular distribution measurements. Due to the rapid increase of the elastic yield compared to the inelastic yield as the scattering angle is decreased, contaminants of one or two parts per thousand produce peaks of the same order of magnitude as the inelastic peak at the forward angles. This is illustrated in Fig. 1, where a spectrum obtained at 60.9°

[11] Throughout this paper, $B(E2)$ always refers to the transition probability for excitation and not decay.

[12] K. Alder and A. Winther, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **31**, No. 1 (1956); see also reference 10.

[13] J. Bang, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **32**, No. 5 (1960).

[14] Even for alpha particles in the present case the ratio of the inelastic to the elastic differential cross section is the same in both systems to an accuracy of better than one part per thousand.

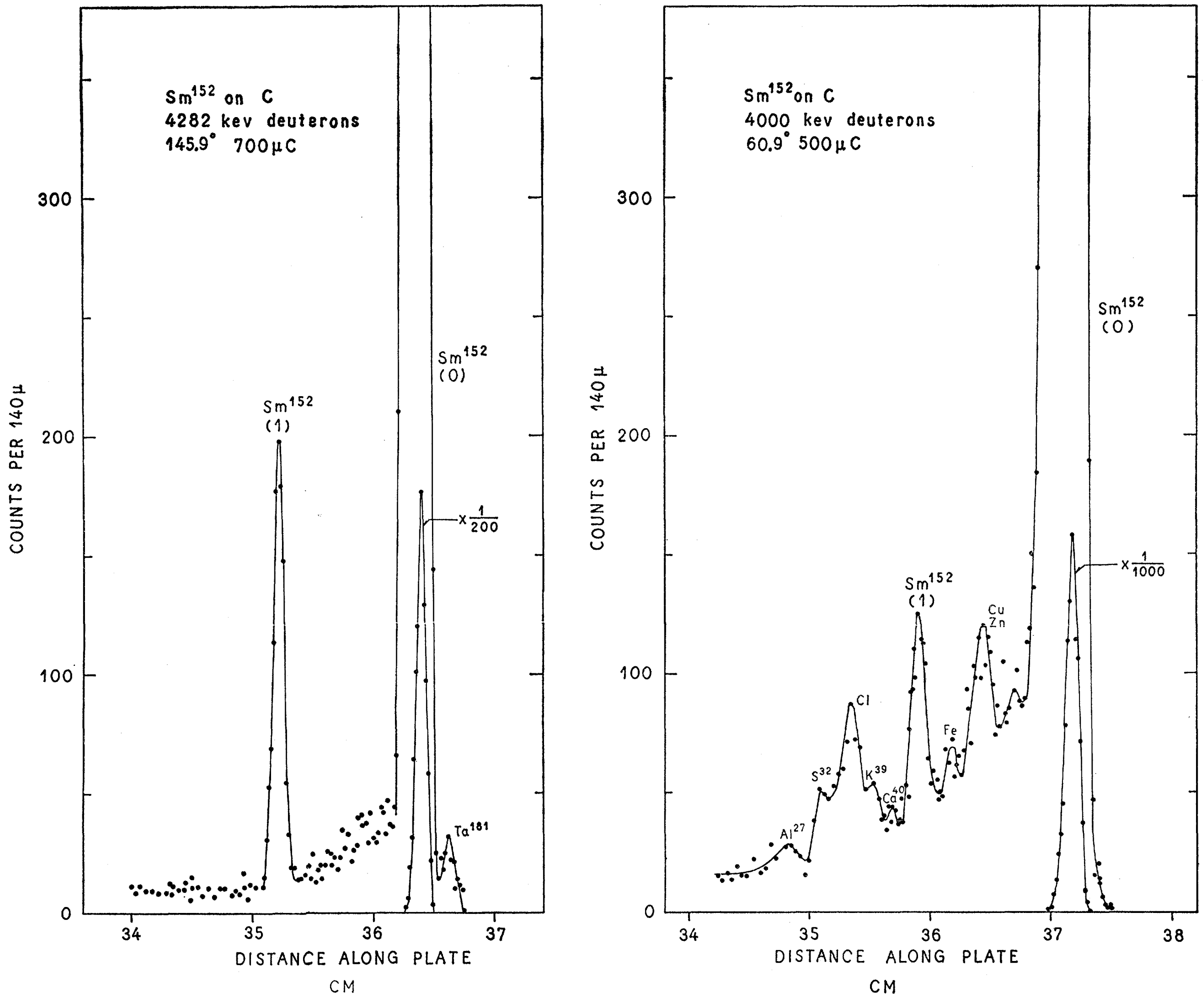

FIG. 1. Spectra of deuterons scattered from Sm^{152}. The elastic and inelastic groups from Sm^{152} are labeled $Sm^{152}(0)$ and $Sm^{152}(1)$, respectively. The most intense impurity peaks in the 60.9° spectrum correspond to contaminations of a few parts per thousand.

is compared with one obtained at the back angle. Also, in the forward direction it is difficult to find a "gap" for the inelastic peak since the contaminant peaks move closer together as the scattering angle is decreased.

For deuteron measurements on Sm^{152} the inelastic peak at about 60° falls between the elastic peaks of Ca and Fe (see Fig. 1). As there are no common contaminants between these two elements, it was thought that accurate data could be obtained at that angle. The absence of contaminants between Ca and Fe was established from an exposure made at 48° expressly for this purpose[15] and from the exposure made at 91°. In three cases, the inelastic peak was not completely resolved from contaminant peaks and therefore the errors of these measurements are larger than the others. The worst case was at 40° where the Sm^{152} inelastic peak was sitting on the low-energy side of the elastic peak from Al. The other cases were the deuteron measurement at 91° and the proton measurement at 146°. In the latter the subtraction was very small (1.5%) and therefore did not appreciably affect the accuracy.

IV. RESULTS

The average value for the energy of the first excited state of Sm^{152} determined from the position of the inelastic peak is 121.7 ± 0.7 kev. This is in excellent agreement with the accurate value of 121.85 kev determined from bent-crystal gamma-ray measurements.[16]

The results of the differential cross-section measurements are summarized in Table I. In most cases, two exposures were made at different times with the same bombarding conditions. The listed values are averages of the two determinations. The errors quoted correspond to standard deviations which include, in addition to

[15] A cross-section determination was impossible at this angle since the inelastic peak was completely unresolved from the elastic peak from Cl.

[16] E. L. Chupp, J. W. M. DuMond, F. J. Gordon, R. C. Jopson, and Hans Mark, Phys. Rev. 112, 518 (1958).

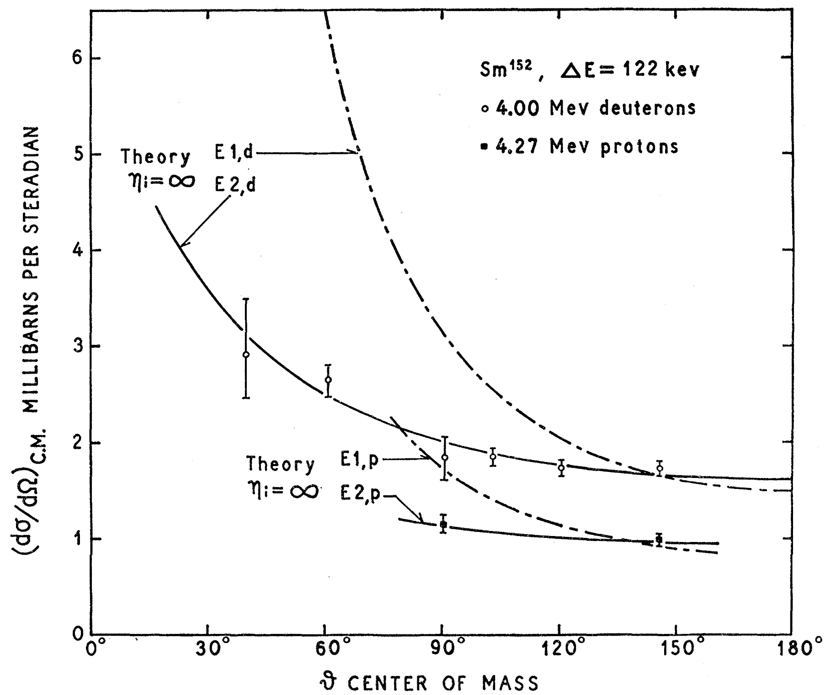

FIG. 2. Angular distributions of 4.00-Mev deuterons and 4.27-Mev protons inelastically scattered from Sm^{152}. The theoretical $E2$ deuteron curve is a least-squares fit to the experimental points. The theoretical $E2$ proton curve is an absolute prediction of the theory calculated from the deuteron results. To illustrate the sensitiveness of the angular distribution to the multipole order of the excitation, the theoretical $E1$ curves are also shown. These are both normalized to the deuteron cross section at the back angle.

the statistical error, the uncertainties in background subtraction, scattering angle, and bombarding energy.

In Fig. 2, the angular distribution measurements are compared with the theory. The semiclassical theoretical curve ($\eta_i=\infty$) for $E2$ excitation with 4.00-Mev deuterons is a least-squares fit to the experimental data and corresponds to a $B(E2)$ value of 3.52. The semiclassical curve for $E2$ excitation with 4.27-Mev protons is an absolute prediction of the theory calculated from the deuteron results. To illustrate the sensitiveness of the angular distribution to the multipole order of the excitation, the theoretical curves for $E1$ excitation are also shown. Both $E1$ curves are normalized to the deuteron cross section at the back angle. The theoretical curves for other multipolarities ($E3$, $M1$, and $M2$) deviate even more from the experimental points than the $E1$ curves.

The measurements at the back angle are compared with the theory in Figs. 3 and 4. In Fig. 3 values of $df(\vartheta,\eta_i,\xi)$ calculated from the measured cross sections are plotted as a function of ξ along with the semiclassical curve. The experimental values of df are based on a

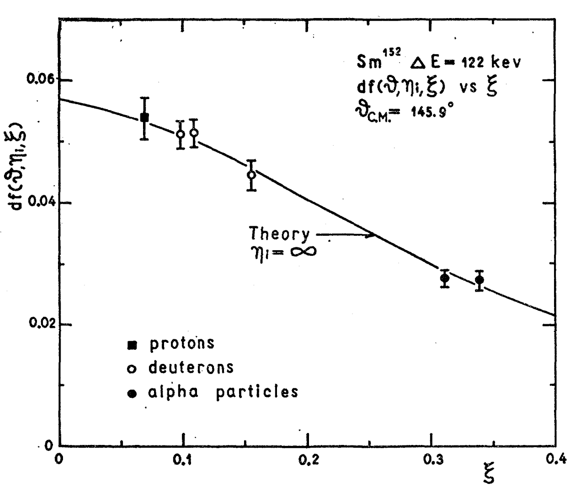

FIG. 3. Experimental values of df compared with the semiclassical theory. The experimental points, calculated from the measured cross sections at 145.9°, correspond to a $B(E2)$ value of 3.56 $e^2\times10^{-48}$ cm^4, which represents a least-squares fit to the theory.

TABLE I. Differential cross sections in the center-of-mass system for inelastic scattering from the 122-kev level of Sm^{152}. The quoted errors correspond to standard deviations which include the statistical uncertainty and the uncertainties in the background subtraction, the scattering angle, and the bombarding energy. The parameters ξ and η_i entering the theoretical description of the Coulomb excitation process are given in columns 4 and 5, respectively. The $B(E2)$ values given in column 6 are calculated from the measured cross sections using the semiclassical theory.

Energy in kev and particle	$\vartheta_{c.m.}$ in degrees	$(d\sigma/d\Omega)_{c.m.}$ in millibarns per steradian	ξ	η_i	$B(E2)$ in $e^2\times10^{-48}$ cm^4
4275 p	90.4	1.15 ±0.09	0.070	4.74	3.58±0.29
4275 p	145.9	0.987±0.06	0.070	4.74	3.63±0.22
3177 d	145.9	1.18 ±0.06	0.156	7.77	3.46±0.19
4000 d	39.9	2.90 $^{+0.59}_{-0.44}$	0.110	6.93	3.26 $^{+0.66}_{-0.50}$
4000 d	60.9	2.64 ±0.16	0.110	6.93	3.73±0.23
4000 d	90.8	1.83 ±0.22	0.110	6.93	3.21±0.38
4000 d	103.0	1.84 ±0.09	0.110	6.93	3.44±0.17
4000 d	120.7	1.73 ±0.08	0.110	6.93	3.44±0.16
4000 d	145.9	1.73 ±0.08	0.110	6.93	3.65±0.16
4282 d	145.9	1.84 ±0.08	0.099	6.70	3.56±0.16
3805 α	145.9	1.69 ±0.10	0.339	20.1	3.68±0.22
4023 α	145.9	1.81 ±0.09	0.311	19.5	3.41±0.17

least-squares fit to the theory and correspond to a $B(E2)$ value of 3.56. This is in excellent agreement with the $B(E2)$ value of 3.52 obtained from the deuteron angular distribution measurements. These same data are shown in Fig. 4 where the $B(E2)$ values calculated using the semiclassical theory are plotted as a function of η_i. From this representation it can be seen that the calculated $B(E2)$ value is constant within the experimental uncertainty of a few percent over a range of η_i of about a factor of 4.

V. DISCUSSION

It has been shown by Bohr[17] that $\eta_i \gg 1$ (η_i is defined in Sec. II) is a necessary and sufficient condition for describing the motion of a charged particle in the Coulomb field of a nucleus in a classical manner. Although in the present experiment the maximum η_i value is still finite (≈ 20), it is expected that the Coulomb excitation process for such a large value will be very well described by the semiclassical theory. This is substantiated by the fact that for $\eta_i = 20$ the semiclassical calculation of the total Coulomb excitation cross section differs by less than a tenth of a percent from the exact quantum mechanical calculation. As was pointed out in Sec. II, it might be suspected that the quantum corrections to the differential cross sections would be much larger than the corrections to the total cross sections. If this were the case, deviations would have been observed in the measurements reported here, since the correction to the total cross section for $\eta_i = 4.7$ is at least a factor of 20 larger than for $\eta_i = 20$. However, it can be seen that the experimental results are in excellent agreement with the semiclassical calculations.

This is also in agreement with the quantum mechanical calculations[13] of the differential cross sections for the case $\xi = 0$ which indicated that the quantum effects were only of the order of 2% for η_i values as low as 4.

It is also of interest to compare the $B(E2)$ values obtained from Coulomb excitation measurements with the values obtained from direct lifetime measurements.

TABLE II. Comparison of $B(E2)$ values for the 122-kev transition in Sm^{152} obtained from Coulomb excitation with those calculated from direct lifetime measurements. The total conversion coefficient of 1.17 and the transition energy of 121.85 kev were used in the calculations of $B(E2)$ from the lifetimes.

Mean life τ in 10^{-9} sec	$B(E2)_\tau$ in $e^2\times10^{-48}$ cm^4	$B(E2)_{\text{C.E.}}$ in $e^2\times10^{-48}$ cm^4	Reference
2.02±0.15	3.46±0.26	...	Sunyar[a]
2.09±0.08	3.34±0.13	...	Birk, Goldring, and Wolfson[b]
...	...	3.40±0.15	Elbek, Olesen, and Skilbreid[c]
...	...	3.53±0.10	Present experiment

[a] A. W. Sunyar, Phys. Rev. 98, 653 (1955).
[b] M. Birk, G. Goldring, and Y. Wolfson, Phys. Rev. 116, 730 (1959).
[c] See reference 5.

[17] N. Bohr, Kgl. Danske Videnskab. Selskab, 18, No. 8 (1948).

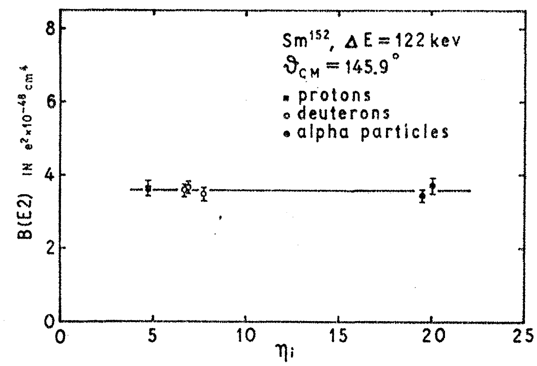

FIG. 4. $B(E2)$ values calculated, using the semiclassical theory from the measured cross sections at 145.9°, plotted as a function of η_i. The horizontal line corresponds to a $B(E2)$ value of 3.56 $e^2\times10^{-48}$ cm^4 which is the weighted average of the data shown.

This comparison is made in Table II. In order to calculate the $B(E2)$ value from the measured lifetime, it is necessary to have an accurate knowledge of the total conversion coefficient and the energy of the transition. It should be noted that, while there are at present two different calculations[18,19] of internal conversion coefficients, the agreement in the present case is within 2%, which is the accuracy of the necessary interpolation. The total conversion coefficient of 1.17 was obtained from the published theoretical K and L values by including 0.3 times the L-shell coefficient to account for the contribution from the higher shells. The accurate value of the transition energy[16] of 121.85 kev has been used in the calculations. The errors given in Table II for the $B(E2)$ values calculated from the lifetime measurements only include the uncertainty quoted for the measured lifetimes.

There is good agreement within the quoted errors between the various determinations of the $B(E2)$ value.

In conclusion, the data presented here indicate that the semiclassical calculations of the differential cross sections for electric quadrupole Coulomb excitation are accurate to at least a few percent for $\eta_i \gtrsim 4.7$.

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[18] M. E. Rose, *Internal Conversion Coefficients* (North Holland Publishing Company, Amsterdam, 1958).

[19] L. A. Sliv and I. M. Band, *Internal Conversion Coefficient Tables*, Leningrad Physico-Technical Institute Report, 1957 and 1958 [translation: Reports 571CC *K*1 and 581CC *L*1, issued by Physics Department, University of Illinois, Urbana, Illinois (unpublished)].