Lifetime of the Neutral Pion*

V. GLASER
CERN, Geneva, Switzerland

AND

R. A. FERRELL†
University of Maryland, College Park, Maryland and CERN, Geneva, Switzerland

(Received September 6, 1960)

As Primakoff has noted, the phenomenological coupling constant of the neutral pion with the electromagnetic field can be investigated by considering the photoproduction of neutral pions in an external Coulomb field. This is the inverse of the usual two-photon decay (one of the photons being provided by the external field). The relationship between the cross section and the free lifetime of the π^0 is derived. Although the total cross section is small, it is found at high energy that the differential cross section is strongly peaked near the forward direction. The peak cross section is proportional to the fourth power of the photon energy. It is this feature which makes possible an experimental determination of the lifetime by the photoproduction method to an accuracy of about ten percent. A minimum photon energy of one Gev is required to avoid uncertainties in the nuclear form factor. A higher photon energy would be necessary only if the π^0 mean life is greater than 5×10^{-17} sec. The backgrounds to be expected from nuclear photoproduction are estimated and found to be sufficiently small. In particular, the interference between the coherent nuclear π^0 photoproduction and the Primakoff process is not excessive.

I. INTRODUCTION

THE decay of the neutral pion into two photons is understood to proceed through the intermediate stage of a nucleon-antinucleon pair, which then undergoes 2γ annihilation as, for example, in the case of the positron-electron pair of positronium. The mean life for the neutral pion has been calculated from meson theory by Steinberger,[1] in which perturbation theory was necessarily made use of, and was predicted to be of the order of 5×10^{-17} sec. Despite the obvious shortcomings of perturbation theory, it would nevertheless be interesting to compare this prediction of meson theory with experiment. Furthermore, the exact value of the lifetime is of importance for proton Compton scattering.[2] The predicted lifetime is too short to be easily measured by the time-of-flight method, which has established an upper limit[3] of about 10^{-15} sec, and hence it is useful to adopt an indirect approach. Such a possibility is provided by the observation of Primakoff,[4] who has noted that the same interaction which produces the decay can also be studied in the inverse process. This is simply the production of the π^0 by the scattering of light on light, or more practically, by the scattering of a photon on a second, virtual photon provided by the fixed Coulomb field of a heavy nucleus. The purpose of this paper is to examine in detail this indirect method of determining the neutral pion lifetime.[5] Section II establishes the quantitative relationship between the cross section for the photoprocess and the lifetime. Section III deals with the experimental details of the method and establishes that the experiment should be feasible. Section IV constitutes a discussion and summary. The chance of exciting the nucleus in the Primakoff process is estimated in Appendix I.

II. ELECTROMAGNETIC INTERACTION OF THE π^0

The coupling of the neutral pion with the electromagnetic field can be described phenomenologically by the perturbing term in the Hamiltonian for the interacting fields of

$$H'=\lambda\int_V \mathbf{E}(\mathbf{x})\cdot\mathbf{H}(\mathbf{x})\psi(\mathbf{x})d^3x, \tag{1}$$

where the integration is carried out over the volume of quantization V. $\mathbf{E}(\mathbf{x})$ and $\mathbf{H}(\mathbf{x})$ are the electric and magnetic fields, and $\psi(\mathbf{x})$ represents the pion field. The coupling constant λ has been calculated from perturbation theory by Steinberger,[1] but here it is sufficient to regard it as a free parameter related to the lifetime. We now proceed to determine this relationship, but we note in passing that the local coupling expressed by Eq. (1) is necessarily an approximation to the true

* Publication supported by the U. S. Air Force Office of Scientific Research.

† National Science Foundation Senior Postdoctoral Fellow.

[1] J. Steinberger, Phys. Rev. **76**, 1180 (1949). For more recent work, see M. Goldberger and S. Treiman, Nuovo cimento **9**, 451 (1958), and other references given there.

[2] T. Yamagata *et al.*, Bull. Am. Phys. Soc. **1**, 350 (1956); M. Jacob and J. Mathews, Phys. Rev. **117**, 854 (1960); G. Bernardini *et al.* (to be published).

[3] G. Harris, J. Orear, and S. Taylor, Phys. Rev. **106**, 327 (1957). *Note added in proof.* As reported at the Tenth Annual High Energy Physics Conference, Glasser, Seeman, and Stiller have obtained by this method the value for the mean life of $(2.3\pm0.8)\times10^{-16}$ sec. Similarly, R. F. Blackie, A. Engler, and J. H. Mulvey [Phys. Rev. Letters **5**, 384 (1960)] report the value $(3.2\pm1)\times10^{-16}$ sec.

[4] H. Primakoff, Phys. Rev. **81**, 899 (1951).

[5] We understand that such an experiment is now in progress [A. Tollestrup (private communication)]. *Note added in proof.* A. V. Tollestrup has reported the detection of the Primakoff effect at the Tenth Annual High Energy Physics Conference. The most recent Caltech value for the π^0 lifetime is $(1.7\pm1.4)\times10^{-16}$ sec [M. A. Ruderman *et al.*, Bull. Am. Phys. Soc. **5**, 508 (1960)], which is consistent with the time-of-flight measurements of footnote 3.

nonlocal interaction of the fields. The accuracy to be expected from this approximation will be estimated in Sec. IV.

The quantized field operators are represented by the Fourier expansions

$$\mathbf{E}(\mathbf{x})=-i\left(\frac{2\pi}{V}\right)^{\frac{1}{2}}\sum_{\mathbf{k},\nu}\boldsymbol{\varepsilon}_{\mathbf{k},\nu}k^{\frac{1}{2}}(a_{\mathbf{k},\nu}{}^{\dagger}e^{-i\mathbf{k}\cdot\mathbf{x}}-a_{\mathbf{k},\nu}e^{i\mathbf{k}\cdot\mathbf{x}}),\quad(2)$$

and

$$\psi(\mathbf{x})=(2V)^{-\frac{1}{2}}\sum_{\mathbf{k}}k_0{}^{-\frac{1}{2}}(c_{\mathbf{k}}{}^{\dagger}e^{-i\mathbf{k}\cdot\mathbf{x}}+c_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{x}}),\quad(3)$$

where $\boldsymbol{\varepsilon}_{\mathbf{k},\nu}$ is a unit polarization vector, and $a_{\mathbf{k},\nu}{}^{\dagger}$ and $a_{\mathbf{k},\nu}$ are the creation and annihilation operators for a photon of momentum $\mathbf{k}$ and polarization ν. $c_{\mathbf{k}}{}^{\dagger}$ and $c_{\mathbf{k}}$ are the corresponding operators for the meson field. $\mathbf{H}(\mathbf{x})$ has the same Fourier expansion as the electric field, except that the polarization vector is replaced by $\hat{k}\times\boldsymbol{\varepsilon}_{\mathbf{k},\nu}$, where $\hat{k}\equiv\mathbf{k}/k$. Conforming to standard usage, we designate with a zero subscript the time-like component of a relativistic four-vector. Thus k_0 denotes the energy of a free π^0 and

$$k_0=(\mu^2+k^2)^{\frac{1}{2}},\quad(4)$$

where the mass μ is equal to 265 electron masses or to 135 Mev. (Throughout we employ units in which c, the velocity of light, and $\hbar$, the reduced Planck's constant, are unity.)

The matrix element for the decay of a π^0 at rest into two γ rays, each of energy $\mu/2$ but of opposite directions and polarization is given by

$$\langle H'\rangle=\lambda\pi(2\mu/V)^{\frac{1}{2}}.\quad(5)$$

(Note that this matrix element is the coherent sum of two terms, since either the electric or the magnetic field can produce either of the two photons.) The density of the two-photon states with respect to their energy $2k=\mu$ is found by integrating over one hemisphere and including the double polarization degeneracy. Thus we find

$$\rho(2k)=\frac{4\pi Vk^2dk}{8\pi^3d(2k)}=\frac{V\mu^2}{16\pi^2}.\quad(6)$$

From the standard formula of time-dependent perturbation theory the transition rate from the discrete pion state to the continuum is given by

$$\begin{aligned}\tau^{-1}&=2\pi\rho(2k)|\langle H'\rangle|^2\\&=\tfrac{1}{4}\pi\lambda^2\mu^3.\end{aligned}\quad(7)$$

Thus, if the mean life of the π^0 is considered to be the basic parameter which characterizes its coupling with the electromagnetic field, the coupling constant of Eq. (1) can be expressed in terms of it by

$$\lambda=(4/\pi\mu^3\tau)^{\frac{1}{2}}.\quad(8)$$

Now, as Primakoff[4] has pointed out, any phenomenon which depends on the interaction of Eq. (1) can be used to determine the lifetime. Such a phenomenon is the inverse of the normal decay, or the production of neutral pions by the collision of light with light. Of course, it is more practical to supply one of the photons by the Coulomb field of the nucleus. Therefore, we now proceed to calculate the cross section for the photoproduction of the neutral pion in the Coulomb field, which we shall call the "Primakoff process." Let the angle of the final π^0 momentum $\mathbf{k}'$ with respect to $\mathbf{k}$ (the photon momentum) be θ. Then the matrix element of H' for the creation of the π^0 and the absorption of the photon in the Coulomb field is found to be

$$\langle H'\rangle=i\lambda\frac{\pi^{\frac{1}{2}}}{V}(\hat{k}\times\boldsymbol{\varepsilon})\cdot\int\mathbf{E}(\mathbf{x})e^{i\mathbf{K}\cdot\mathbf{x}}d^3x.\quad(9)$$

Energy conservation requires $k=k_0'$. Since the mass changes, some momentum transfer to the nucleus must take place, of amount

$$\mathbf{K}=\mathbf{k}-\mathbf{k}',\quad(10)$$

and of magnitude given by

$$\begin{aligned}K^2&=k^2+k'^2-2kk'\cos\theta\\&=4kk'[\tfrac{1}{4}\Delta^2+\sin^2(\theta/2)],\end{aligned}\quad(11)$$

where

$$\Delta=\beta^{-\frac{1}{2}}-\beta^{\frac{1}{2}}.\quad(12)$$

β is the velocity of the final pion. The electric field of the nucleus which occurs inside the integral in Eq. (9) is given by

$$\mathbf{E}(\mathbf{x})=-\mathrm{grad}\phi(\mathbf{x}),\quad(13)$$

where the electrostatic potential $\varphi(\mathbf{x})$ must satisfy Poisson's equation relating it to the charge density $\rho(\mathbf{x})$:

$$\nabla^2\varphi(\mathbf{x})=-4\pi\rho(\mathbf{x}).\quad(14)$$

The Fourier transform is

$$\int\phi(\mathbf{x})e^{i\mathbf{K}\cdot\mathbf{x}}d^3x=\frac{4\pi Ze}{K^2}F(K),\quad(15)$$

where the form factor is given by the expression

$$F(K)=(Ze)^{-1}\int\rho(\mathbf{x})e^{i\mathbf{K}\cdot\mathbf{x}}d^3x.\quad(16)$$

Z is the atomic number of the nucleus and e is the electron charge. Integrating by parts now gives

$$\int\mathbf{E}(\mathbf{x})e^{i\mathbf{K}\cdot\mathbf{x}}d^3x=\mathbf{K}\frac{4\pi iZe}{K^2}F(K).\quad(17)$$

If we denote the angle between the plane of polarization and the plane of reaction by ϕ, we find the following expression for the matrix element

$$\langle H'\rangle=\lambda\frac{4\pi^{\frac{3}{2}}Zek'}{VK^2}\sin\theta\sin\phi F(K).\quad(18)$$

Substitution from Eq. (11) gives

$$\langle H'\rangle=\lambda\frac{\pi^{\frac{3}{2}}Ze}{Vk}\frac{\sin\theta\sin\phi}{\frac{1}{4}\Delta^2+\sin^2(\theta/2)}F(K). \qquad (19)$$

We can now calculate the differential cross section for the production of a π^0 in a differential solid angle $d\Omega$ from perturbation theory by multiplying the square of the matrix element by 2π times the state density of $(2\pi)^{-3}Vk'^2\beta^{-1}d\Omega$, and by dividing by the incident photon flux density of V^{-1}. Elimination of the coupling constant by substitution from Eq. (8) finally gives for the differential cross section

$$\frac{d\sigma(\theta,\phi)}{d\Omega}=\frac{Z^2e^2\beta}{\mu^3\tau}\frac{\sin^2\theta\sin^2\phi}{[\frac{1}{4}\Delta^2+\sin^2(\theta/2)]^2}|F(K)|^2. \qquad (20)$$

For unpolarized incident radiation, $\sin^2\phi$ is to be replaced by $\frac{1}{2}$. For convenience, in studying the consequences of this formula, we shall neglect for the moment the extension of the nucleus and set the form factor equal to unity. The error incurred in this approximation will be estimated further below. We should further mention that we ignore completely here any incoherent production by the individual protons since this contribution to the production of π^0's is proportional to the first power of the atomic number, and is significantly smaller than the coherent cross section for the large atomic numbers of interest. We also neglect nuclear excitation to excited bound states. (See Appendix I.) The total cross section is easily obtained by integration over Eq. (20):

$$\sigma_t=16\pi\frac{Z^2e^2\beta}{\mu^3\tau}\left[\left(\frac{1}{2}+\frac{\Delta^2}{4}\right)\ln\left(1+\frac{4}{\Delta^2}\right)-1\right]. \qquad (21)$$

It is clearly advantageous for the photon energy to be well above the threshold, and therefore in our subsequent work we shall make approximations justified in the relativistic limit of $k\gg\mu$, or $\beta\approx 1$. Thus we obtain

$$\Delta=\mu^2/2k^2, \qquad (22)$$

and

$$\sigma_t=4\pi C_Z[\ln(2k/\mu)-\tfrac{1}{2}], \qquad (23)$$

where the Z-dependent coefficient is

$$C_Z=8Z^2e^2/\mu^3\tau. \qquad (24)$$

Primakoff[4] has given a similar formula for the total cross section, but his expression must be divided by π and multiplied by the quantity in brackets in Eq. (23) in order to make it agree with our expression. Our result agrees, however, with some more recent work[6] of Morpurgo and of Berman. It should be emphasized that Eq. (23) applies only to the ideal case of a point nucleus. The effect of finite nuclear size is discussed in the following section.

III. PHOTOPRODUCTION IN THE COULOMB FIELD

It is evident that an experimental measurement of the cross section for the Primakoff process would lead to a numerical value for the coefficient C_Z which in turn, by means of Eq. (24), would yield a value for the pion lifetime. We therefore investigate in this section whether or not such an experiment would be feasible, by estimating the order of magnitude to be expected for this coefficient. Let us assume a specific value for the lifetime, and as a reasonable choice take

$$\tau=10^7\mu^{-1}=4.7\times10^{-17}\text{ sec}. \qquad (25)$$

This is actually a quite conservative choice for the lifetime, since values greater than this by as much as an order of magnitude are excluded by the time-of-flight measurements.[3] Furthermore, the limits $10^{-18}\text{ sec}\lesssim\tau\lesssim10^{-16}$ sec have been deduced from the proton Compton effect.[2] The value which we are choosing here is the same as that given by Steinberger's perturbation calculation.[1] With this choice and taking as the most favorable choice of target material lead, with $Z=82$ we find

$$\begin{aligned}C_Z&=1.24\times10^{-34}Z^2\text{ cm}^2\\&=0.84\ \mu\text{b}.\end{aligned} \qquad (26)$$

At a photon energy of 1 Gev this would yield a total cross section according to Eq. (23) of 23 μb (1 $\mu\text{b}=10^{-30}$ cm^2). This result can be compared with the background cross section which arises from the ordinary photonuclear process. According to Berkelman and Waggoner,[7] at 950 Mev, the differential cross section for the photoproduction of π^0's on protons in the forward direction is of the order of $\sigma_H=1$ μb/sr. At other angles it is not more than a factor of two or three times this. Let us adopt this number also as a rough estimate for the neutron cross section. Because of the short mean free path of the π^0's at high energy, only the nucleons on the half of the nuclear surface away from the incident photon beam will be effective. π^0's produced inside the nucleus or at the front surface will be absorbed before leaving the nucleus. The number of nucleons on one hemisphere is given roughly by $2A^{\frac{2}{3}}$ or about 77 for lead, with $A=208$. Thus, we should expect a background differential cross section in lead of roughly $77\sigma_H=77$ μb. The ratio of the total cross sections for the electromagnetic process and the nuclear process is roughly given by the ratio of the coefficient C_Z to the background differential cross section that we have just calculated, or

$$C_Z/2A^{\frac{2}{3}}\sigma_H=0.01. \qquad (27)$$

In other words, the nuclear process can be expected to be two orders of magnitude stronger than the electromagnetic process of interest. Thus the total cross section is clearly too small to lend itself to a measurement of the π^0 lifetime. The logarithmic energy dependence in Eq.

[6] G. Morpurgo, Suppl. Nuovo cimento 16, 445 (1960); S. Berman, California Institute of Technology Report (unpublished).

[7] K. Berkelman and J. A. Waggoner, Phys. Rev. 117, 1364 (1960).

(23) is too weak to alter this conclusion even at very high energy.

However, the differential cross sections for the two different processes have strikingly different angular distributions which should make it possible to separate them experimentally. When the high-energy and small-angle approximations are made in Eq. (20) we find for unpolarized photons the following expression:

$$d\sigma(\theta)/d\Omega = C_Z\theta^2/(\Delta^2+\theta^2)^2. \tag{28}$$

But before considering this formula in detail it is desirable to study first other sources of background in addition to that estimated in the preceding paragraph. First is the interference between the Primakoff process and the coherent non-spin-flip nuclear photoproduction of π^0's. The amplitude for the π^0 wave arising from the former is equal to $C_Z^{\frac{1}{2}}\theta/(\Delta^2+\theta^2)$. This interferes coherently with the non-spin-flip amplitude arising from each nucleon, which vanishes for $\theta=0$ and can be estimated roughly as $2A^{\frac{2}{3}}\sigma_{\mathrm{H}}^{\frac{1}{2}}\theta$. Thus the cross product becomes independent of θ for $\theta>\Delta$ and is simply a constant addition to the ordinary incoherent background. (Actually, inclusion of a form factor would decrease this estimate and introduce some angular dependence.) The ratio of the interfering background to the incoherent background is given by

$$\frac{2C_Z^{\frac{1}{2}}(2A^{\frac{2}{3}}\sigma_{\mathrm{H}}^{\frac{1}{2}})}{2A^{\frac{2}{3}}\sigma_{\mathrm{H}}}\frac{\theta^2}{\Delta^2+\theta^2} \leq 2\left(\frac{C_Z}{\sigma_{\mathrm{H}}}\right)^{\frac{1}{2}} \approx 2. \tag{29}$$

Thus, the contribution due to interference is a factor of about two times the ordinary incoherent source of background. Therefore, the interference plays no important role in the photoproduction process and can be ignored in the subsequent discussion, except for its effect in enhancing the background.

The coherent production alone also contributes to the background, but we quickly note that this term in the cross section is much smaller than the interference term. The coherent cross section is larger than the incoherent by the number of nucleons contributing, but smaller by the angular factor θ^2. Thus the two terms are of the same order of magnitude for $\theta=77^{-\frac{1}{2}}=0.11$ rad. But since, as we shall see below, we are interested in angles smaller than this by at least a factor of three, it is clear that the coherent process by itself is an order of magnitude weaker than the incoherent photoproduction on the individual nucleons, which can be expected to be the main source of background in the experiment.[8]

Proceeding now to discuss the behavior of the differential cross section in the high-energy range, we note that the differential cross section has a pronounced maximum at $\theta=\Delta$, of value

$$(d\sigma/d\Omega)_{\max} = C_Z(k/\mu)^4 = 2.5 \text{ mb/sr}. \tag{30}$$

The numerical value has been obtained by again choosing the energy equal to 1 Gev, for which case $\theta=9.1$ mrad or about one half of a degree. The energy-dependent factor amounts to about 3000 for this case, so that now the differential cross section for the desired process is greater than the background by an order of magnitude. Because of the somewhat indirect nature of the detection of the π^0's by their decay γ rays, it is out of the question to investigate experimentally the very sharp detail in the angular distribution described by Eq. (28). Therefore we need to calculate the average differential cross section for a measurement of flux inside the finite solid angle subtended, say, by a core of half angle equal to θ_c. This can easily be found to be given by the expression

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{av}} = f\left(\frac{\theta_c}{\Delta}\right)\left(\frac{d\sigma}{d\Omega}\right)_{\max}, \tag{31}$$

where the function is

$$f(x) = \frac{4}{x^2}\ln(1+x^2) - \frac{4}{1+x^2}. \tag{32}$$

This function has a broad maximum at $x=1.1$, where it takes on the value 0.81. The average cross section drops to 77% of the maximum at $\theta_c=2.2\Delta$ and then more rapidly to 62% at $\theta_c=3\Delta$. For the purpose of further discussion we shall choose $\theta_c=3\Delta$, since it is clear that not much will be gained in counting rate by going to larger solid angles, which instead would simply bring in more background counts.

Before proceeding, we should now examine the effect of the form factor on the differential cross section which we have studied. Determination of the form factor is not completely unambiguous since a strict application of Eq. (16) would correspond to including π^0's which are produced inside the nucleus. As stated above, these will be absorbed as they propagate outwards toward the surface. To avoid including a contribution from production in the interior, we take the electric field to vanish there, as if all the charge were concentrated at the nuclear surface. A further refinement would be to exclude the integration over the cylindrical region of space of cross-sectional area equal to that of the nucleus and extending from the front side of the nucleus towards the incident photon beam. The pions produced in this region can be expected to propagate forward and to be intercepted by the nucleus. We have not evaluated this second correction, but it seems likely that it is smaller than the first.

The form factor for our "effective" nuclear distribution in which all of the charge Ze is spread uni-

[8] The coherent production has recently been studied at 300 Mev by G. Davidson, Ph.D. thesis, Massachusetts Institute of Technology, August, 1959 (unpublished). *Note added in proof.* C. Chiuderi and G. Morpurgo have recently carried out a detailed investigation (Nuovo cimento, to be published), based on the impulse approximation, of the expected nuclear photoproduction background. Their results are substantially in agreement with the rough estimates obtained here.

formly over the surface of a sphere of radius R is

$$F(K)=(\sin KR)/KR. \quad (33)$$

Making a Taylor series expansion, we find for the average value of the form factor squared, the expression

$$[F(K)^2]_{\mathrm{av}}\approx 1-\tfrac{1}{3}R^2(K^2)_{\mathrm{av}}, \quad (34)$$

where the percentage error in the magnitude of the reduction below unity, due to the neglect of the higher terms, is roughly two-fifths of the reduction itself. The mean value of the transfer momentum squared can be calculated in a straightforward manner from the differential cross section given by Eq. (28) and is found to be

$$(K^2)_{\mathrm{av}}=K_c^2\frac{1-(1+\theta_c^2/\Delta^2)^{-1}\ln(1+\theta_c^2/\Delta^2)}{\ln(1+\theta_c^2/\Delta^2)-(1+\Delta^2/\theta_c^2)^{-1}}$$
$$=1.38(\mu/k)^2\mu^2. \quad (35)$$

K_c is the momentum transfer corresponding to the cutoff angle θ_c, which we have taken equal to 3Δ. For an energy of 1 Gev, we find a reduction in the mean differential cross section of 16%. Inclusion of the higher terms in Eq. (34) would change this to 15%, while the second correction discussed in the preceding paragraph might amount to a further reduction of from five to ten percent. This gives a total reduction in the mean differential cross section of 20–25%. Thus it is clear that at this energy the finite size of the nucleus does not introduce any serious uncertainty into the experimental determination of the pion lifetime. At somewhat lower energies, however, the situation will not be so favorable, as the mean squared form factor will decrease rapidly with energy, with a correspondingly large increase in the uncertainty of the correction. For example, at 600 Mev Eq. (34) above already gives a reduction of about 50%, so that an uncertainty of the least a factor of two in the inferred value of the π^0 lifetime would be unavoidable at this lower energy.

To return to the discussion of background, we should like to make clear that the experiment can be employed to determine an accurate value of the π^0 mean life even if the latter turns out to be considerably longer than the value 5×10^{-17} assumed here. Then at 1 Gev the signal is no longer so much stronger than the background. But, because of the fourth-power dependence in Eq. (30), the situation is repaired by going to slightly higher energy. For example, a factor-of-two increase in energy will compensate for a factor of sixteen in τ. It is clear that some angular dependence is inevitable in the background. Therefore, it is desirable that the latter always be smaller than the signal by an order of magnitude, so that no serious uncertainty is introduced by the subtraction of the background.

We now turn to the question of the absolute counting rate to be expected in the experiment. We have seen that $\theta_c=3\Delta$ is a reasonable choice and limits the background counting rate from the nuclear photomeson effect to the order of 14% of the effect being sought. This choice of angle gives already 16% of the total cross section so that not much would be gained by going to larger cutoff angles. Thus we should expect with this cutoff angle at an energy of 1 Gev a production cross section of about 2.9 μb, with the lifetime assumption made above in Eq. (25). This can be compared with a pair production cross section[9] of 41 barns, corresponding to a radiation length in lead of 0.75 cm. Thus, if the target of one radiation length in thickness is chosen, the chance is roughly 7×10^{-8} per photon of 1 Gev that a desired event will occur. We must multiply this by the efficiency for detecting the decay γ rays of a π^0. Here it is essential to use two counters in coincidence, since the direction of the π^0 must be determined within the angle θ_c. Thus it is clear that each counter should be placed symmetrically at an angle of μ/k about the incident photon direction, and should subtend an angle of the order of θ_c. In order to define the π^0 direction within the decay plane, as well as normal to it, it is necessary to measure the γ energies in each counter and to require them to be equal, to an accuracy of μ/k (14% at 1 Bev). Under these conditions it is easily seen that the role of the second counter is to discriminate against "wide-angle" background π^0's, while admitting the "desirable" small-angle π^0's resulting from the Primakoff process. (We note in passing that registering coincidences in which the γ energies are not equal will give a simultaneous determination of background.) The coincidence efficiency can consequently be estimated from the ratio of the solid angle of a single counter, $\pi\theta_c^2$, to the solid angle over which the decay γ's are distributed, $\pi(\mu/k)^2$. This gives an efficiency of $(\theta_c k/\mu)^2=(3\mu/2k)^2=0.04$. (This efficiency can, of course, be increased by using several such pairs of counters, placed in a ring about the direction of the incidence photons.) Thus the expected counting rate is $3\times10^{-9}\times$the incident photon flux. To take a specific example, if we assume that there are 10^7 incident photons per second of the required energy and of the required collimation of about 30 mrad (2°), we would expect one event every 30 seconds, or about one hundred per hour. On the other hand, it may be necessary to use a target very much thinner than one radiation length, in order to avoid background arising from the electron showers produced in the target. If this is the case, the counting rate would, of course, be reduced accordingly.

IV. DISCUSSION AND SUMMARY

The Hamiltonian operator of Eq. (1) is an approximation to the true interaction of the π^0 field with the electromagnetic field. Therefore we have incurred some error in using it to calculate the matrix element for the Primakoff process. This error can be estimated by writing the matrix element in Lorentz covariant form and by employing the analytic consequences of causality to

[9] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, New York, 1954), 3rd ed., p. 260.

obtain an integral representation for the matrix element. The integration extends over an effective "photon-mass" spectrum. The spectrum begins with the smallest mass 2μ, which is the minimum frequency for which a fluctuating electromagnetic field can experience absorption by producing a real transition in an intermediate state (pion pair). Thus, for small momentum transfers from the Coulomb field (imaginary virtual photon mass), the integrand can be expanded in a power series. This leads to a correction factor in the cross section for the Primakoff process of the form

$$\begin{aligned} f &= 1+a\tfrac{1}{4}(K^2)_{\mathrm{av}}/\mu^2 \\ &= 1+0.4a(\mu/k)^2 \\ &= 1+0.007a, \end{aligned} \tag{36}$$

where a is some numerical coefficient of the order of unity, and we have substituted Eq. (35) and set the energy equal to 1 Gev. Thus it is clear that an error arising from Eq. (1) in the determination of the π^0 lifetime from the Primakoff process of more than a few percent is quite unlikely. A way of measuring experimentally the coefficient a has been recently proposed by Petermann[10] and by Berman and Geffen.[10] Namely, the matrix element, which governs the Primakoff effect, enters also in the creation of Dalitz pairs in the π^0 decay, with values of the mass of the virtual photon ranging from 0 to the π^0 mass μ. Therefore, the measurement of the dependence of the decay rate on the total energy of the emitted electron-positron pair to an accuracy of a few percent should yield a good determination of the coefficient a in the power series expansion of the matrix element.

In principle, the coupling of the π^0 to the electromagnetic field gives rise to other phenomena besides the 2γ decay, the decay into Dalitz pairs, and the Primakoff process. But the probability of these other processes seems to be much too small to make them of anything but academic interest. For example, the π^0 could serve as an intermediate state in the free-photon scattering of light by light and also in Delbruck scattering, but rough estimates place these contributions far beyond the limits of observability. A further process,[11] similar to the production of Dalitz pairs, is the internal conversion of one of the decay photons of the π^0 by an atomic electron. If m and n are the electron mass and density, respectively, one finds for the branching ratio for internal conversion

$$R=4\pi ne^2/m^2\mu. \tag{37}$$

But even for the very high electron density at the center of a lead atom this amounts to only about 6×10^{-5}. The actual branching ratio depends on how long the π^0 remains in such a region of high density, and would therefore give a determination of the lifetime, assuming that one knew the velocity of the π^0's produced at the nucleus. But with a lifetime of 5×10^{-17} sec and a velocity of 10^{10} cm/sec, one should expect a branching ratio of the order of only 10^{-8}. Thus it is clear that internal conversion also does not provide any practical alternative to the Primakoff process for the π^0-lifetime determination.

From an operational point of view, the Primakoff process can be considered as a type of photon splitting, since the incident photon emerges as two lower energy photons. Thus, the observation of it must in principle contend with any other concurrent modes of photon splitting, such as the purely electromagnetic process via an intermediate electron-positron pair. So far as the authors are aware, the cross section for this latter has not yet been calculated from quantum electrodynamics,[12] but it seems likely to be small and to be confined to much smaller angles for the divergent photons. In any case, because of the characteristic relationship between the angle and energy of the decay photons in the Primakoff process, it should be possible to separate the two different types of photon splitting.

To summarize the principal results of this paper, we have seen first of all that the total cross section for the Primakoff effect is too small by two orders of magnitude to permit the separation of it from the background of nuclear photoproduction. But the drastic peaking of the differential cross section in the nearly forward direction changes the situation to an order of magnitude in favor of the Primakoff process. To avoid serious uncertainties associated with the interaction and absorption of the π^0's by the nucleus, a minimum photon energy of 1 Gev is required. At this energy the momentum transfer is sufficiently small that the π^0 production takes place predominantly outside the nucleus. To take advantage of the peaking of the differential cross section, it is necessary to measure the energy of both decay photons. With all of these prerequisites met, it should be possible to determine the π^0 mean life to, say, ten percent accuracy. It should be emphasized that the coherent nuclear photoproduction does not introduce any great uncertainty, since it does not contribute excessively to the background in the experiment.

ACKNOWLEDGMENTS

We wish to express our appreciation to numerous colleagues at CERN for many stimulating discussions, and in particular to Professor G. Bernardini for his continued interest in the problem. We are also indebted to Professor H. Primakoff, Professor G. Morpurgo, and Professor A. Tollestrup for correspondence on various points.

[10] A. Petermann (private communication); S. Berman and D. A. Geffen (to be published). *Note added in proof.* The measurement has been completed by N. P. Samios (Phys. Rev. **121**, 275 1961), who finds $a=1.0\pm0.6$.

[11] G. Bernardini (private communication).

[12] For an estimate of the total cross section see, however, M. Bolsterli, Phys. Rev. **94**, 367 (1954).

APPENDIX I. NUCLEAR EXCITATION

In the discussion preceding Eq. (21) we have mentioned that we neglect any nuclear excitation and consider only that portion of the Primakoff process which leaves the nucleus in the ground state. It is clear that highly excited states of a single-particle nature will not be important because of their intrinsically small matrix elements. But it might be supposed that certain special excited states of a collective nature, such as, for example, the giant dipole resonance, would have sufficiently strong coupling to the electromagnetic field to make a significant contribution to the cross section for the Primakoff process. We shall establish that this is, however, not the case and that our neglect of such states is also well justified.

In place of Eq. (20) where $F(K)$ is evaluated as an expectation value in the ground state, we need to write a similar equation for the differential cross section for the Primakoff process when the nucleus is left in, say, its nth excited state. Then in place of $F(K)$ we have the inelastic form factor

$$\begin{aligned} F_n(K) &= Z^{-1}(\textstyle\sum_p e^{i\mathbf{K}\cdot\mathbf{x}_p})_{n0} \\ &\approx iZ^{-1}K(\textstyle\sum_p x_p)_{n0}, \end{aligned} \tag{38}$$

where the sum is over all protons and the matrix element is taken between the ground and excited state. x_p is the component of $\mathbf{x}_p$ in the direction of $\mathbf{K}$. We have made a dipole approximation to the power series expansion of the exponentials, which will be good enough for the small values of K of interest. Now we need to establish the size of $|F_n(K)|^2$. This can be accomplished by employing the dipole sum rule

$$\textstyle\sum_n |(\sum_p x_p)_{n0}|^2 = 1.4NZ/2AME_{\mathrm{av}}, \tag{39}$$

where the factor 1.4 arises from the nuclear exchange forces[13] and E_{av} is an average excitation energy which can be estimated for lead at about 15 Mev. N and A are the neutron and nucleon number, respectively. Substituting from Eq. (38), we obtain

$$\begin{aligned} \sum_{n\neq 0} |F_n(K)|^2 &\approx \frac{1.4NK^2}{2MZAE_{\mathrm{av}}} \\ &\approx 1.5\times 10^{-28}\ \mathrm{cm}^2\ K^2. \end{aligned} \tag{40}$$

This can be compared with the K^2 correction term of Eq. (34) for the elastic form factor:

$$\tfrac{1}{3}R^2K^2 \approx 1.5\times 10^{-25}\ \mathrm{cm}^2\ K^2.$$

Thus, the contribution of the inelastic processes is about one thousand times smaller than the correction to the elastic process, and therefore is quite negligible.

Actually, the minimum momentum transfer depends sensitively on the excitation energy. Therefore, the above work should not be considered as an accurate determination,[14] but rather as an overestimate. For example, already at an excitation energy of 9 Mev the value of Δ is doubled, which reduces the peak differential cross section by a factor of four.

---

[13] J. Levinger and H. A. Bethe, Phys. Rev. **78**, 115 (1950). M. Gell-Mann, M. L. Goldberger, and W. E. Thirring, Phys. Rev. **95**, 1612 (1954).

[14] Similarly, it would seem that the closure approximation used by Primakoff (reference 4) is not justified.