Boson Currents in the Theory of Weak Interactions*

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A possible new method of introducing boson currents into the weak interaction is suggested. The Bose fields are assumed to obey the first-order Kemmer equation. In this way, the boson and fermion currents enter the interaction in a symmetric manner. Three coupling constants are introduced according to the degree of isotopic spin and strangeness symmetries of the currents. The interaction so obtained gives rise in a natural way to the $|\Delta T|=\frac{1}{2}$ selection rule and its violation, and also to fast pionic modes and slow leptonic modes of hyperon decay. First-order calculations are performed for $K_{l3}{}^{+}$, τ, and τ' decays. Calculations of the decay rates yield sufficient information to determine the approximate magnitudes of the effective coupling constants. The first-order calculations are at variance with the experimentally uniform distribution of τ events in the Dalitz plot.

I. INTRODUCTION

THE inclusion of pion currents in the vector part of the strangeness-conserving weak interaction current has been suggested by Feynman and Gell-Mann[1] in order to obtain a nonrenormalized vector coupling constant. Sugawara[2] has presented a modified viewpoint, wherein certain boson terms are also included in the strangeness-nonconserving current. However, in Sugawara's scheme, other possible boson terms must be explicitly excluded in an *ad hoc* manner.

In this work we present a third viewpoint, wherein all boson currents consistent with a few basic hypotheses are included in the weak interaction. Like Sugawara, we do not assume divergencelessness of currents. Furthermore, we find it necessary to abandon strict universality in the coupling strength, and introduce, instead, a "fine structure" splitting of the weak-interaction coupling constant. This splitting can be regarded as a consequence of the different symmetries involved in the weak interaction processes, and is similar to the nonequality of the K-baryon and π-baryon coupling constants in the usual theory of the strong interactions. We shall show that boson currents introduced in this manner give rise to the $|\Delta T|=\frac{1}{2}$ selection rule with a violation of about 5–10% in amplitude, and also to fast pionic modes and slow leptonic modes of hyperon decay.

We begin by discussing in Sec. II the general form of the boson currents entering the weak interaction. In Secs. III and IV we obtain the specific structures of the boson currents and the weak-interaction Hamiltonians inducing leptonic and nonleptonic decays, respectively. In Secs. V and VI we give the results of some first-order calculations on $K_{e3}{}^{+}$, $K_{\mu3}{}^{+}$, and $K_{\pi3}{}^{+}$ decays. In Sec. VII we summarize our results and determine, as far as possible, the magnitudes of the coupling constants involved in our weak interaction Hamiltonian. Finally, in the Appendix we discuss briefly, for reference throughout the paper, the Kemmer equation for spinless particles and its second quantization.

We use the following notations and conventions: Unless otherwise noted, Greek indices run from 0 to 3, Latin indices i and j run from 1 to 3, Latin indices a and b run from 0 to 4. Repeated indices are summed over. We use the metric tensor $g^{\mu\nu}$, where $g^{00}=-g^{11}=-g^{22}=-g^{33}=1$, and all other components are zero. $A^{\dagger}$, A^{*}, and $\tilde{A}$ are the Hermitian conjugate, complex conjugate, and transpose, respectively, of A. Our Dirac γ matrices obey $\gamma^{0\dagger}=\gamma^{0}$, $\gamma^{i\dagger}=-\gamma^{i}$, $\gamma^{5}\equiv\gamma^{0}\gamma'\gamma^{2}\gamma^{3}=-\gamma^{5\dagger}$, $\gamma^{5*}=-\gamma^{5}$.

II. STRUCTURE OF THE BOSON CURRENTS

We shall assume that the weak interaction is generated by a local, four field interaction Hamiltonian of the general form

$$H_I=\sum_{i,j=1}^{n} J_{i\mu}{}^{\dagger}J_j{}^{\mu},$$

where the set of currents $J_{i\mu}$ consists of fermion currents and boson currents. We shall further assume that the fermion currents are of the usual V-A form:

$$J^{\mu}\propto\bar{\psi}\gamma^{\mu}(1-i\gamma^{5})\psi,\text{ for fermions.}\qquad(1)$$

Our first task, then, is to determine the structure of the boson currents.

In the usual theory of the strong interactions and in the electromagnetic interactions, the bosons play a different role than the fermions; they are the quanta which are exchanged by the fermions to give rise to an effective force. This is expressed by the fact that the corresponding interaction Hamiltonians treat the Bose fields and Fermi fields quite differently. However, in the weak interaction, there is no reason why bosons and fermions should not enjoy the same status. Therefore, it would seem desirable that if bosons are going to enter the weak interaction at all, they should enter it in a manner as nearly symmetric to that of the fermions as possible.

To this end, we shall assume that our free Bose fields

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[1] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[2] M. Sugawara, Phys. Rev. **113**, 1361 (1959).

obey the first-order Kemmer equation[3] for spinless particles:

$$(\partial_\mu \beta^\mu - im)\psi(x) = 0, \tag{2}$$

where $\psi(x)$ has five components, and the β^μ are 5×5 numerical matrices satisfying the commutation relations (A2) of the Appendix. Equation (2) has the same form as the Dirac equation.

Under a homogeneous Lorentz transformation carrying x^μ into $x^{\mu\prime} = a^\mu{}_\nu x^\nu$, $\psi(x)$ transforms into

$$\psi'(x') = S\psi(x), \tag{3}$$

where S is a 5×5 matrix satisfying

$$a^\nu{}_\mu \beta^\mu = S^{-1}\beta^\nu S. \tag{4}$$

If we define η^0 by

$$\eta^0 \beta^{\mu\dagger} \eta^{0\,-1} = \beta^\mu, \quad \eta^0 = \eta^{0\dagger}, \tag{5}$$

and $\bar{\psi}$ by

$$\bar{\psi} \equiv \psi^\dagger \eta^{0\,-1}, \tag{6}$$

we find that

$$\eta^0 S^\dagger \eta^{0\,-1} = S^{-1}, \tag{7}$$

and hence the quantity

$$\bar{\psi}\beta^\mu\psi, \tag{8}$$

transforms as a vector under Lorentz transformations.[4]

The structure (8), however, is not the most general form of a vector which can be formed from two Kemmer fields. Clearly, for any matrix A satisfying

$$[A,S] = 0 \quad \text{for all } S, \tag{9}$$

the structure $\bar{\psi}\beta^\mu A\psi$ is also a vector. Since for the parity transformation, $S = \eta^0$, we also have that for any A satisfying

$$[A,S] = 0, \tag{10a}$$

for all S corresponding to proper Lorentz transformations, and

$$\{A,\eta^0\} = 0, \tag{10b}$$

the structure $\bar{\psi}\beta^\mu A\psi$ is a pseudovector. There exists no matrix A, however, satisfying conditions (10), since it is not possible to form a pseudovector from two scalar fields and their derivatives. On the other hand, it is clear from (A4) that S must have the general form

$$S = \begin{pmatrix} \mathrm{x} & \mathrm{x} & \mathrm{x} & \mathrm{x} & 0 \\ \mathrm{x} & \mathrm{x} & \mathrm{x} & \mathrm{x} & 0 \\ \mathrm{x} & \mathrm{x} & \mathrm{x} & \mathrm{x} & 0 \\ \mathrm{x} & \mathrm{x} & \mathrm{x} & \mathrm{x} & 0 \\ 0 & 0 & 0 & 0 & \pm1 \end{pmatrix},$$

[3] N. Kemmer, Proc. Roy. Soc. (London) **A173**, 91 (1939).

[4] We assume that the π and K mesons with which we are dealing have relative parity $+1$. If the two fields in (8) had relative parity -1, then (8) would transform as a pseudovector.

and, hence, any matrix of the form

$$A = \begin{pmatrix} a & & & & \\ & a & & & \\ . & & a & & \\ & & & a & \\ & & & & b \end{pmatrix}, \tag{11}$$

where a and b are c numbers, satisfies condition (9). This gives us considerable freedom in choosing the form of our boson currents.

In order to eliminate this arbitrariness in the structure of the boson currents, we shall impose on all strangeness-conserving, baryon and meson currents involved in nonleptonic decays a restriction, first proposed by Weinberg,[5] on the transformation properties of the currents under the operation G, which consists of charge conjugation followed by a rotation of 180° about the 2 axis in isotopic spin space. The requirement will be that these strangeness-conserving, baryon and meson currents will all be of the "first class"[5]; i.e., the vector part of the current transforms into itself under G, and the axial vector part transforms into minus itself. Of course, this restriction implies that the concept of isotopic spin is relevant in nonleptonic decays, a point of view which we shall adopt because of the strong experimental evidence in support of the selection rule $|\Delta T| = \frac{1}{2}$.

Consider, now, the current containing two K-meson fields, which we denote by (K,K). Denoting a particle and its field by the same symbol, we write in isotopic spin space

$$\psi_K = \begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix},$$

and the two simplest forms for the (K,K) current are

$$J_{\mathrm{I}}{}^\mu = \lambda \bar{\psi}_K \beta^\mu A \psi_K,$$
$$J_{\mathrm{II}i}{}^\mu = \lambda \bar{\psi}_K \beta^\mu A \tau_i \psi_K,$$

where the τ_i are the Pauli spin matrices, and λ is a real c number. Under G, ψ_K transforms into $i\tau_2\psi_K{}^C$, and hence

$$J_{\mathrm{I}}{}^\mu \underset{G}{\rightarrow} -\lambda \bar{\psi}_K A \beta^\mu \psi_K,$$

$$J_{\mathrm{II}i}{}^\mu \underset{G}{\rightarrow} \lambda \bar{\psi}_K A \beta^\mu \tau_i \psi_K.$$

Clearly, if $J_{\mathrm{I}}{}^\mu$ is to be of the first class, we must have

$$\{A,\beta^\mu\} = 0, \tag{12}$$

and if $J_{\mathrm{II}}{}^\mu$ is to be of the first class, we must have

$$[A,\beta^\mu] = 0. \tag{13}$$

Now the only matrix of the form (11) satisfying (13) is $A = I$, and the only matrix of the form (11) satisfying (12) is $A = C$, where C, in the representation used here, is the charge conjugation matrix given by Eqs. (A5)

[5] S. Weinberg, Phys. Rev. **112**, 1375 (1958).

and (A6). For the choice $A=I$, it is easy to see that the (π^0,π^0) current vanishes (except for a c number) via the field Eq. (2). We shall see, however, in Sec. IV, that the existence of the (π^0,π^0) current gives the correct branching ratios for the pionic modes of Λ decay and for the two modes of $K_{\pi 3}{}^+$ decay. We therefore choose $A=C$, and our boson currents have the structure

$$J^\mu \propto \bar{\psi}\beta^\mu C\psi \tag{14}$$

for bosons.[6]

It is interesting to note that the connection (A4) between the Kemmer field and the scalar field gives for the (1,2) current

$$\begin{aligned} J^\mu &\propto \bar{\psi}_{(1)}\beta^\mu C\psi_{(2)} \\ &= i[(m_2/m_1)^{\frac{1}{2}}(\partial^\mu\phi_1{}^\dagger)\phi_2+(m_1/m_2)^{\frac{1}{2}}\phi_1{}^\dagger(\partial^\mu\phi_2)], \end{aligned} \tag{15}$$

where m_1 and m_2 are the masses of particle 1 and particle 2, respectively. This differs from the form

$$J^\mu \propto [(\partial^\mu\phi_1{}^\dagger)\phi_2-\phi_1{}^\dagger(\partial^\mu\phi_2)], \tag{16}$$

suggested by Feynman and Gell-Mann[1] and used by Sugawara.[2]

III. LEPTONIC DECAYS

We now come to the problem of explicitly constructing the currents entering the weak interaction from the fields describing the known weakly interacting particles. This must be done in accordance with a few basic principles, some of which have already been discussed. All currents satisfying these principles will then be coupled together to form the weak interaction.

For leptonic decays, the nonexistence of such reactions as

$$\mu^\pm \to e^\pm+e^\pm+e^\pm,$$
$$\mu^-+p \to e^-+p,$$
$$K^+ \to \mu^\pm+e^\pm+\pi^+,$$

leads to the restriction $\Delta Q=+1$ for the currents. Since no decays involving a change in strangeness of more than one unit have been observed, we shall also require the currents to obey $\Delta S=0, +1$. Denoting by (1,2) the structure $\bar{\psi}_{(1)}\gamma^\mu(1-i\gamma^5)\psi_{(2)}$ if $\psi_{(1)}$ and $\psi_{(2)}$ are Dirac fields, or the structure $\bar{\psi}_{(1)}\beta^\mu C\psi_{(2)}$ if $\psi_{(1)}$ and $\psi_{(2)}$ are Kemmer fields, we are led to the following strangeness-conserving currents:

$$\begin{aligned} &J_1{}^\mu=(G_2)^{\frac{1}{2}}(\nu,e^-); && J_2{}^\mu=(G_2)^{\frac{1}{2}}(\nu,\mu^-);\\ &J_3{}^\mu=(G_2)^{\frac{1}{2}}(p,n); && J_4{}^\mu=(G_2)^{\frac{1}{2}}(\pi^0,\pi^-);\\ &J_5{}^\mu=(G_2)^{\frac{1}{2}}(\Sigma^+,\Sigma^0); && J_6{}^\mu=(G_2)^{\frac{1}{2}}(\Sigma^+,\Lambda);\\ &J_7{}^\mu=(G_2)^{\frac{1}{2}}(\Sigma^0,\Sigma^-); && J_8{}^\mu=(G_2)^{\frac{1}{2}}(\Lambda,\Sigma^-);\\ &J_9{}^\mu=(G_2)^{\frac{1}{2}}(\Xi^0,\Xi^-); && \end{aligned}$$

[6] Note that since $\bar{\psi}\beta^\mu C\psi$ transforms under CP in exactly the same manner as $\bar{\psi}\gamma^\mu(1-i\gamma^5)\psi$, the CP invariance of the interaction Hamiltonian is not destroyed by the introduction of currents of the form (14).

and strangeness-nonconserving currents:

$$\begin{aligned} &J_{10}{}^\mu=(G_3)^{\frac{1}{2}}(n,\Sigma^-); && J_{11}{}^\mu=(G_3)^{\frac{1}{2}}(p,\Sigma^0);\\ &J_{12}{}^\mu=(G_3)^{\frac{1}{2}}(p,\Lambda); && J_{13}{}^\mu=(G_3)^{\frac{1}{2}}(\Sigma^0,\Xi^-);\\ &J_{14}{}^\mu=(G_3)^{\frac{1}{2}}(\Lambda,\Xi^-); && J_{15}{}^\mu=(G_3)^{\frac{1}{2}}(\Sigma^+,\Xi^0);\\ &J_{16}{}^\mu=(G_3)^{\frac{1}{2}}(\pi^0,K^-); && J_{17}{}^\mu=(G_3)^{\frac{1}{2}}(K^0,\pi^-); \end{aligned}$$

and the corresponding interaction Hamiltonian for leptonic decays:

$$\begin{aligned} H_{I(L)}=&\sum_{i=3}^{17}\sum_{j=1}^{2}(J_{i\mu}{}^\dagger J_j{}^\mu+\text{H.c.})\\ &+\sum_{i=1}^{2}J_{i\mu}{}^\dagger J_i{}^\mu+(J_{1\mu}{}^\dagger J_2{}^\mu+\text{H.c.}), \end{aligned} \tag{17}$$

where the factors $(G_2)^{\frac{1}{2}}$ and $(G_3)^{\frac{1}{2}}$ are coupling constants. We have explicitly written the coupling constant for the strangeness-nonconserving currents as different from the coupling constant for the strangeness-conserving currents, for we do not wish to assume *a priori* that the two kinds of currents are coupled with the same strength. Indeed, in keeping with the principle that interactions become weaker as their symmetries decrease, it would not seem unreasonable for the strangeness-nonconserving currents to be coupled more weakly. It should be mentioned that such a nonuniversality in coupling strength is already believed to exist in the strong interactions.

Couplings between $J_1{}^\mu$, $J_2{}^\mu$, and $J_3{}^\mu$, of course, give rise to the usual V-A theory for neutron decay, muon decay, and muon capture. These require

$$G_2=(1.00\pm.01)\times10^{-49}\ \text{erg-cm}^3. \tag{18}$$

Coupling $J_{10}{}^\mu$ and $J_{12}{}^\mu$ to the lepton currents gives rise to the decays

$$\Lambda \to p+\binom{e^-}{\mu^-}+\bar{\nu},$$

$$\Sigma^- \to n+\binom{e^-}{\mu^-}+\bar{\nu},$$

with a coupling strength of $(G_2G_3)^{\frac{1}{2}}$. Calculations of these first-order processes are well known to require[7]

$$G_2G_3\approx\tfrac{1}{12}G_2{}^2, \tag{19}$$

in order to fit the experimental decay rates. As in the usual theory, the processes $\Sigma^+\to n+l^++\nu$ are forbidden by the $\Delta S=\Delta Q$ requirement on the strangeness-nonconserving currents.

The interesting point is that by coupling the boson currents to the lepton currents, we obtain such first-order processes as Fig. 1(A) for $K^+\to\pi^0+l^++\nu$, with a coupling strength of $(G_2G_3)^{\frac{1}{2}}$, and Fig. 1(B) for $\pi^+\to\pi^0+e^++\nu$, with a coupling strength of G_2.

[7] R. H. Dalitz, Revs. Modern Phys. 31, 823 (1959).

Sugawara[8] has shown, using the conventional boson current (16), that Fig. 1(A) with a coupling strength of G_2 yields a decay rate for K_3^+ about 100 times greater than the experimental rate. However, as we have already seen (and as was noted by Sugawara), the leptonic modes of hyperon decay indicate $G_3<G_2$. In Sec. V we shall give the results of some calculations based on Figs. 1(A) and 1(B).

IV. NONLEPTONIC DECAYS

The important distinction, here, between leptonic and nonleptonic decays is the fact that experimental evidence seems to favor the selection rule $|\Delta T|=\frac{1}{2}$ for the latter. The most significant experimental data supporting the $|\Delta T|=\frac{1}{2}$ rule are the following ratios of decay rates[9]:

$$\Gamma(\Lambda\rightarrow p+\pi^-)/\Gamma(\Lambda\rightarrow n+\pi^0)\approx 2, \tag{20}$$

$$\Gamma(K_1^0\rightarrow \pi^++\pi^-)/\Gamma(K^+\rightarrow \pi^++\pi^0)\approx 500, \tag{21}$$

$$\Gamma(K^+\rightarrow \pi^++\pi^++\pi^-)/\Gamma(K^+\rightarrow \pi^++\pi^0+\pi^0) \equiv\Gamma(\tau)/\Gamma(\tau')\approx 3.3. \tag{22}$$

On the other hand, (21) also indicates that there must be a violation of the $|\Delta T|=\frac{1}{2}$ rule of the order of 5–10% in amplitude, for a strict $|\Delta T|=\frac{1}{2}$ rule would absolutely forbid $K^+\rightarrow\pi^++\pi^0$.

Now, if any selection rule such as $|\Delta T|=\frac{1}{2}$ is to exist, it must be a result of the structure of the interaction Hamiltonian. Therefore, we do not expect the currents involved in nonleptonic decays to be the same as the currents inducing the leptonic decays. They must, instead, have a definite structure in isotopic spin space. These currents will then be coupled together to form the interaction Hamiltonian for nonleptonic decays, which we shall denote by $H_{I(NL)}$. The total weak interaction Hamiltonian, of course, will then be the sum of $H_{I(L)}$ and $H_{I(NL)}$.

In addition to imposing the structures (1) and (14) on the currents involved in $H_{I(NL)}$, we shall postulate four requirements to be obeyed by these currents and then show how these requirements lead directly to the $|\Delta T|=\frac{1}{2}$ selection rule and its violation.

(1) We require that the strangeness-conserving currents be of the first class under the G transformation, as discussed in Sec. II.

(2) We shall consider only the simplest structures in isotopic spin space. In particular, we shall only consider currents having the transformation properties of scalars, spinors, or vectors in isotopic spin space.[10]

[8] M. Sugawara, Phys. Rev. **112**, 2128 (1958).

[9] For a discussion of experimental evidence for the $|\Delta T|=\frac{1}{2}$ rule, see, for example, M. Gell-Mann and A. H. Rosenfeld, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, California, 1957), Vol. 7, p. 407.

[10] More complicated structures could be introduced with, perhaps, correspondingly weaker couplings, but there seems to be little evidence for or against this. Requirement 2 is introduced, however, only for purposes of simplification.

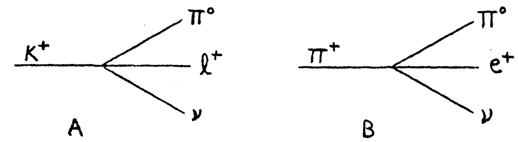

FIG. 1. Lowest order diagrams for K_{l3}^+ and π_{e3}^+ decays.

(3) In keeping with the principle that the greater the symmetry, the stronger the coupling strength, we shall allow, in addition to the weaker coupling already introduced for strangeness-nonconserving currents, a stronger coupling for those currents having the particularly simple structure of a scalar in isotopic spin space. It will be found that this structure is only possible for strangeness-conserving currents, and hence we need introduce only one more coupling constant, G_1, with $G_1>G_2>G_3$.

(4) Finally, we impose $\Delta S=0, 1$ (to avoid $\Delta S=2$ transitions) and the reasonable requirement $\Delta Q=0, 1$.

We take for the baryons and mesons the following structures in isotopic spin space:

$$\psi_\Lambda=\Lambda \quad \psi_N=\begin{pmatrix} p \\ n \end{pmatrix} \quad \psi_\Xi=\begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix} \quad \psi_K=\begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix},$$
$$\psi_\pi=\begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi^0 \end{bmatrix} \quad \psi_\Sigma=\begin{bmatrix} \Sigma_1 \\ \Sigma_2 \\ \Sigma^0 \end{bmatrix}, \tag{23}$$

where

$$\begin{aligned} \pi_1&=(1/\sqrt{2})(\pi^++\pi^-),\\ \pi_2&=(i/\sqrt{2})(\pi^+-\pi^-),\\ \Sigma_1&=(1/\sqrt{2})(\Sigma^++\Sigma^-),\\ \Sigma_2&=(i/\sqrt{2})(\Sigma^+-\Sigma^-). \end{aligned} \tag{24}$$

In constructing the currents from these fields, we make use of the spin matrices

$$\tau_1=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2=\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
$$\tau_+=\tfrac{1}{2}(\tau_1+i\tau_2)=\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

for spinors, and

$$T_1=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}, \quad T_2=\begin{bmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{bmatrix},$$
$$T_3=\begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad T_+=T_1+iT_2=\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & -i \\ 1 & i & 0 \end{bmatrix}$$

for vectors.

It is an easy matter to see that the fields (23) have the following transformation properties under G:

$$\begin{aligned} &\psi_\Lambda\rightarrow\psi_\Lambda^C, \quad \psi_N\rightarrow i\tau_2\psi_N^C, \quad \psi_\Xi\rightarrow i\tau_2\psi_\Xi^C,\\ &\psi_K\rightarrow i\tau_2\psi_K^C, \quad \psi_\pi\rightarrow-\psi_\pi, \quad \psi_\Sigma\rightarrow-\psi_\Sigma^C. \end{aligned} \tag{25}$$

Denoting the structure $\gamma^\mu(1-i\gamma^5)$ by A^μ, we then find that the only currents allowed under requirements 1 through 4 are the strangeness-conserving currents

$$\begin{array}{ll} \lambda(G_2)^{\frac{1}{2}}\bar{\psi}_N A^\mu \tau_+ \psi_N, & \lambda(G_2)^{\frac{1}{2}}\bar{\psi}_N A^\mu \tau_3 \psi_N, \\ \lambda(G_2)^{\frac{1}{2}}\bar{\psi}_\Xi A^\mu \tau_+ \psi_\Xi, & \lambda(G_2)^{\frac{1}{2}}\bar{\psi}_\Xi A^\mu \tau_3 \psi_\Xi, \\ \lambda(G_1)^{\frac{1}{2}}\bar{\psi}_K \beta^\mu C \psi_K, & \lambda(G_1)^{\frac{1}{2}}\bar{\psi}_\pi \beta^\mu C \psi_\pi, \\ \lambda(G_2)^{\frac{1}{2}}\bar{\psi}_\Sigma A^\mu T_+ \psi_\Sigma, & \lambda(G_2)^{\frac{1}{2}}\bar{\psi}_\Sigma A^\mu T_3 \psi_\Sigma, \end{array} \tag{26}$$

and the strangeness-nonconserving currents

$$\begin{array}{c} \lambda(G_3)^{\frac{1}{2}}\bar{\psi}_N A^\mu \psi_\Lambda, \quad \lambda(G_3)^{\frac{1}{2}}\bar{\psi}_N A^\mu \tau_i \psi_{\Sigma_i}, \quad \lambda(G_3)^{\frac{1}{2}}\bar{\psi}_\Lambda A^\mu \psi_\Xi, \\ \lambda(G_3)^{\frac{1}{2}}\bar{\psi}_{\Sigma_i} A^\mu \tau_i \psi_\Xi, \quad \lambda(G_3)^{\frac{1}{2}}\bar{\psi}_{\pi_i} \beta^\mu C \tau_i \psi_K, \end{array} \tag{27}$$

where λ is a real, positive c number, whose only relevance is connected with the relative strength of leptonic and nonleptonic decays. λ may be regarded as an adjustable parameter of the theory. However, since the symmetrical $\pi^+\pi^+\pi^-$ state is $\frac{4}{5}$ in a $T=1$ state, it seems logical to choose λ such that the selection rule $|\Delta T|=\frac{1}{2}$ introduces a factor $(\frac{4}{5})^{\frac{1}{2}}$ in that part of $H_{I(NL)}$ inducing τ decay. We thus choose $\lambda=10^{-\frac{1}{4}}$.

Writing out the expressions (26) and (27) in full, we obtain the following set of currents entering $H_{I(NL)}$:

$$J_{18}{}^\mu=\lambda(G_2)^{\frac{1}{2}}(p,n), \qquad J_{26}{}^\mu=\lambda(G_2)^{\frac{1}{2}}[(p,p)-(n,n)],$$
$$J_{19}{}^\mu=\lambda(G_2)^{\frac{1}{2}}(\Xi^0,\Xi^-), \quad J_{27}{}^\mu=\lambda(G_2)^{\frac{1}{2}}[(\Xi^0,\Xi^0)-(\Xi^-,\Xi^-)],$$
$$J_{28}{}^\mu=\lambda(G_1)^{\frac{1}{2}}[(\bar{K}^0,\bar{K}^0)+(K^-,K^-)],$$
$$J_{29}{}^\mu=\lambda(G_1)^{\frac{1}{2}}[2(\pi^-,\pi^-)+(\pi^0,\pi^0)],$$
$$J_{20}{}^\mu=\lambda(G_2)^{\frac{1}{2}}\sqrt{2}[(\Sigma^0,\Sigma^-)-(\Sigma^+,\Sigma^0)],$$
$$J_{30}{}^\mu=\lambda(G_2)^{\frac{1}{2}}[(\Sigma^+,\Sigma^+)-(\Sigma^-,\Sigma^-)],$$
$$J_{21}{}^\mu=\lambda(G_3)^{\frac{1}{2}}(p,\Lambda), \quad J_{31}{}^\mu=\lambda(G_3)^{\frac{1}{2}}(n,\Lambda),$$
$$J_{22}{}^\mu=\lambda(G_3)^{\frac{1}{2}}[\sqrt{2}(n,\Sigma^-)+(p,\Sigma^0)],$$
$$J_{32}{}^\mu=\lambda(G_3)^{\frac{1}{2}}[\sqrt{2}(p,\Sigma^+)-(n,\Sigma^0)],$$
$$J_{33}{}^\mu=\lambda(G_3)^{\frac{1}{2}}(\Lambda,\Xi^0), \quad J_{23}{}^\mu=\lambda(G_3)^{\frac{1}{2}}(\Lambda,\Xi^-),$$
$$J_{34}{}^\mu=\lambda(G_3)^{\frac{1}{2}}[\sqrt{2}(\Sigma^-,\Xi^-)+(\Sigma^0,\Xi^0)],$$
$$J_{24}{}^\mu=\lambda(G_3)^{\frac{1}{2}}[\sqrt{2}(\Sigma^+,\Xi^0)-(\Sigma^0,\Xi^-)],$$
$$J_{35}{}^\mu=\lambda(G_3)^{\frac{1}{2}}[\sqrt{2}(\pi^-,K^-)+(\pi^0,\bar{K}^0)],$$
$$J_{25}{}^\mu=\lambda(G_3)^{\frac{1}{2}}[\sqrt{2}(\pi^+,\bar{K}^0)-(\pi^0,K^-)].$$

$H_{I(NL)}$ is then given by

$$H_{I(NL)}=\sum_{i,j=18}^{25} J_{i\mu}{}^\dagger J_j{}^\mu+\sum_{i,j=26}^{35} J_{i\mu}{}^\dagger J_j{}^\mu.$$

Now, coupling either of the boson currents in (26) to any of the currents in (27) yields a term in $H_{I(NL)}$ which transforms as a spinor in isotopic spin space, and hence gives rise to the $|\Delta T|=\frac{1}{2}$ selection rule. Coupling any of the other currents in (26) to any of the currents in (27) produces a violation of the $|\Delta T|=\frac{1}{2}$ rule of amplitude $\approx(G_2/G_1)^{\frac{1}{2}}$ compared to the $|\Delta T|=\frac{1}{2}$ amplitude. It should be pointed out that although a $|\Delta T|=\frac{1}{2}$ interaction can be postulated directly using only fermion currents, with or without intermediate bosons, the advantage here is that the introduction of the boson currents yields the $|\Delta T|=\frac{1}{2}$ rule and its violation in a more natural way.

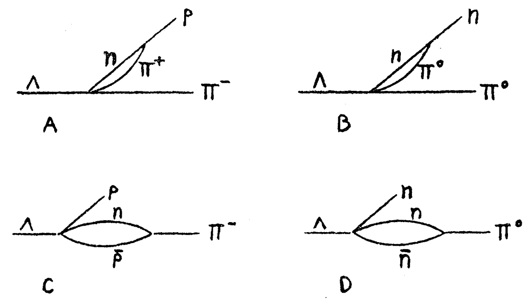

FIG. 2. Lowest order diagrams for pionic Λ decays.

Note that the only effect of couplings between the currents of (26) would be to give a small ($\sim10^{-12}$) correction to isotopic spin and parity conservation in strong-interaction processes.

In Fig. 2 are shown some lowest order diagrams contributing to pionic Λ decay. A and B give rise to a $T=\frac{1}{2}$ final state, with a coupling strength of $(G_1G_3)^{\frac{1}{2}}$, and C and D give rise to a mixed T state, with a coupling strength of $(G_2G_3)^{\frac{1}{2}}$. Since $G_1>G_2$, we expect A and B to be the dominant diagrams. These, of course, should yield the experimental branching ratio (20) characteristic of a $T=\frac{1}{2}$ final state. Note, however, that the existence of the (π^0,π^0) current is essential for this result. It is for this reason, and a similar situation, below, for τ decays, that we have chosen the matrix C in (14) for our boson currents.

Pionic Σ and Ξ decays proceed via diagrams similar to those of Fig. 2. At present, the experimental information on pionic Σ decays is consistent with, but does not support, the $|\Delta T|=\frac{1}{2}$ rule. Of course, not enough information is known at present about Ξ decays to determine whether or not they are consistent with $|\Delta T|=\frac{1}{2}$.

$K_1{}^0\rightarrow2\pi$ can proceed via the diagrams of Figs. 3 and 4. The diagrams of Fig. 3 give rise to the $|\Delta T|=\frac{1}{2}$ rule with a coupling strength of $(G_1G_3)^{\frac{1}{2}}$, and those of Fig. 4 violate the $|\Delta T|=\frac{1}{2}$ rule with a coupling strength of $(G_2G_3)^{\frac{1}{2}}$. The experimental value of the branching ratio

$$\frac{\Gamma(K_1{}^0\rightarrow\pi^0+\pi^0)}{\Gamma(K_1{}^0\rightarrow\pi^0+\pi^0)+\Gamma(K_1{}^0\rightarrow\pi^++\pi^-)}\lesssim\frac{1}{3},$$

still not too certain, but recent evidence[11] seems to indicate that it is consistent with the value $\frac{1}{3}$ expected from a pure $|\Delta T|=\frac{1}{2}$ transition.

For the decay $K^+\rightarrow\pi^++\pi^0$, only the $T=2$ final state is allowed by Bose-Einstein statistics for a spinless K particle. Thus a $|\Delta T|=\frac{1}{2}$ transition is forbidden, and the decay must proceed through such diagrams as those of Fig. 5, with a coupling strength of $(G_2G_3)^{\frac{1}{2}}$. The

[11] F. S. Crawford *et al.*, Phys. Rev. Letters **2**, 266 (1959).

experimental ratio (21) now follows if we choose

$$G_1 \lesssim 500 G_2, \tag{28}$$

where we have written $\lesssim$ because of the possibility that the complexity of the diagrams of Fig. 5 may tend to further suppress this mode.

In Fig. 6 are shown some diagrams giving rise to τ and τ' decays. The particularly simple diagrams A and B give rise to a $T=1$ final state, with a coupling strength of $(G_1G_3)^{\frac{1}{2}}$, whereas C and D violate the $|\Delta T|=\frac{1}{2}$ rule with a coupling strength of $(G_2G_3)^{\frac{1}{2}}$. Similar diagrams lead to the $K_2^0 \to 3\pi$ decay modes. Calculations based on A and B are presented in Sec. VI, where it will be shown that these diagrams lead approximately to the experimental branching ratio (22). Again, this result depends on the existence of the (π^0,π^0) current.

V. K_{l3}^+ DECAYS

Diagram a of Fig. 1, representing the lowest order contribution to K_{l3}^+ decay, results from a coupling of J_{16}^μ with J_1^μ or J_2^μ. The square of the matrix element, summed over lepton spins, for this diagram is

$$|M|^2 = \frac{G_2G_3}{2(2\pi)^4 p^0 p'^0 k^0 k'^0 M_{\pi^0} M_K} \times(-p_\mu p'^\mu K_\nu K^\nu + 2K_\mu p^\mu K_\nu p'^\nu), \tag{29}$$

where $K_\mu \equiv M_K k_\mu' - M_{\pi^0} k_\mu$, M_{π^0} and M_K are the masses of the π^0 and K^+ mesons, and p_μ, p_μ', k_μ, and k_μ' are the 4 momenta of the electron (or muon), neutrino, K^+ meson, and π^0 meson, respectively.

Taking (29) in the center-of-mass of the K particle and integrating over the final momenta, we can obtain the (first-order) energy spectra and first-order expressions for the decay rates $\Gamma(K_{l3}^+)$. The electron energy spectrum coincides with that of Dalitz,[12] and the muon energy spectrum coincides roughly with Dalitz's $S=0$ curve.[12,13] Hence, these curves will not be reproduced here. The first-order decay rates are

$$\begin{aligned}\Gamma(K_{e3}^+) &= (7.0\times 10^{105}) G_2G_3 \text{ sec}^{-1},\\ \Gamma(K_{\mu3}^+) &= (3.4\times 10^{105}) G_2G_3 \text{ sec}^{-1}.\end{aligned} \tag{30}$$

Comparing these with the experimental rates,[14]

$$\begin{aligned}\Gamma(K_{e3}^+) &= (5.1\pm 0.8)\times 8.16\times 10^5 \text{ sec}^{-1},\\ \Gamma(K_{\mu3}^+) &= (3.9\pm 0.5)\times 8.16\times 10^5 \text{ sec}^{-1},\end{aligned} \tag{31}$$

yields

$$\begin{aligned}&\text{from } K_{e3}^+: \quad G_2G_3 \approx 6.2\times 10^{-100} \text{ erg}^2\text{-cm}^6,\\ &\text{from } K_{\mu3}^+: \quad G_2G_3 \approx 9.8\times 10^{-100} \text{ erg}^2\text{-cm}^6.\end{aligned} \tag{32}$$

The neglect of all radiative corrections, in addition to

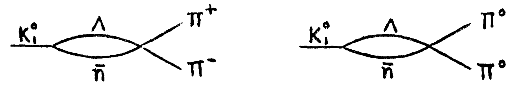

FIG. 3. Lowest order diagrams for $K_1^0 \to 2\pi$ decays, with $|\Delta T|=\frac{1}{2}$. For convenience, only $S=-1$ intermediate states are shown.

the experimental errors in (31), make these relations only approximate. However, we shall see that a number of such relations all lead to the same approximate values for G_1 and G_3.

The theoretical branching ratio $\Gamma(K_{e3}^+)/\Gamma(K_{\mu3}^+) = 2.1$ obtained from (30) is somewhat higher than the experimental ratio of about 1.3 obtained from (31), the discrepancy, perhaps, being a consequence of the first-order approximation.

The square of the matrix element for diagram B of Fig. 1 can be obtained from (29) by replacing M_K by the mass of the charged pion. Because of the specific structure (14) of our boson currents, we obtain the anomalously small result

$$\Gamma(\pi^+ \to \pi^0 + e^+ + \nu) \approx 2\times 10^{-4} \text{ sec}^{-1},$$

compared with the result $\Gamma = 0.42$ sec^{-1} obtained by Jauch and Yamaguchi[15] using boson currents of the type (16). Thus, in our theory, diagram 1(B) is probably not the dominant diagram inducing this decay mode.

VI. τ AND τ' DECAYS

The lowest order contribution to τ decay is represented by diagram A of Fig. 6, and results from a coupling of J_{29}^μ and J_{35}^μ. The square of the matrix element for this diagram is

$$|M|^2 = \frac{G_1G_3}{40(2\pi)^4 p^0 p'^0 k^0 k'^0 M_\pi M_K} \times[(M_\pi k^\mu - M_K p'^\mu)(k_\mu' + p_\mu) + (M_\pi k^\mu - M_K p^\mu)(k_\mu' + p_\mu')]^2, \tag{33}$$

where M_π and M_K are the masses of the charged π and K mesons, and k_μ, k_μ', p_μ, and p_μ' are the 4 momenta of the K^+ meson, π^- meson, and the two π^+ mesons, respectively.

If we take (33) in the center-of-mass system of the K particle, and transform to the Dalitz variables[16]

$$\begin{aligned}x &= (\sqrt{3}/Q)(T_1 - T_2),\\ y &= (3/Q)T_3 - 1,\end{aligned} \tag{34}$$

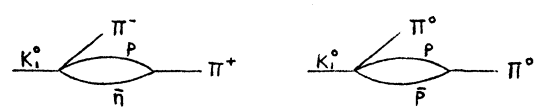

FIG. 4. Lowest order diagrams for $K_1^0 \to 2\pi$ decays, violating $|\Delta T|=\frac{1}{2}$.

[12] R. H. Dalitz, Revs. Modern Phys. **31**, 823 (1959), Figs. 6 and 7.

[13] We note in passing that a scalar coupling of the fields does not, in general, yield these energy spectra.

[14] M. Bruin *et al.*, Nuovo cimento **9**, 422 (1958).

[15] J. M. Jauch and Y. Yamaguchi, Helv. Phys. Acta **32**, 251 (1959).

[16] R. H. Dalitz, Phil. Mag. **44**, 1068 (1953).

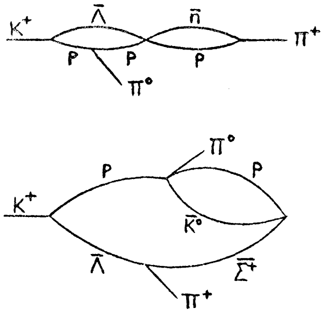

FIG. 5. Lowest order diagrams for $K^+\rightarrow 2\pi$ decay.

where T_1, T_2, and T_3 are the (nonrelativistic) kinetic energies of the two π^+ mesons and the π^- meson, respectively, and $Q=M_K-3M_\pi$, we can obtain an expression for the τ decay rate as an integral over the circular Dalitz plot.[16] This expression is

$$\begin{aligned}\Gamma(\tau)&=\frac{G_1G_3Q^4(M_K+M_\pi)^2}{1080\sqrt{3}(2\pi)^3M_\pi}\int_{-1}^{1}dy\int_{-(1-y^2)^{\frac{1}{2}}}^{+(1-y^2)^{\frac{1}{2}}}dx(y-2)^2\\&=\frac{33\pi G_1G_3Q^4(M_K+M_\pi)^2}{4320\sqrt{3}(2\pi)^3M_\pi}.\end{aligned}\tag{35}$$

Comparing (35) with the experimental decay rate,[17]

$$\Gamma(\tau)=(5.66\pm0.30)\times8.16\times10^5\ \mathrm{sec}^{-1},$$

gives

$$G_1G_3\approx9.0\times10^{-98}\ \mathrm{erg}^2\text{-}\mathrm{cm}^6.\tag{36}$$

The distribution $(y-2)^2$ of events in the Dalitz plot predicted by Eq. (35) is in serious disagreement with the experimentally uniform distribution. In general, a uniform distribution, which indicates an over-all s state for the three pions, is to be expected only from a nonderivative coupling and from diagrams in which all three pions are emitted from the same vertex. The simplest such interaction would be a four-boson interaction with scalar coupling. The usual four-fermion weak interaction would not, in general, be expected to yield a uniform distribution.

To what extent the strong radiative corrections and final-state interactions might modify the Dalitz plot distribution is difficult to determine. It is not impossible, however, that the final-state interactions are such as to force the pions into an over-all s state.

In calculating $\Gamma(\tau')$ from diagram B of Fig. 6, we shall retain first-order effects of the π^+-π^0 mass difference, such as the larger Q value, but neglect higher order effects, such as the distortion of the circular shape of the Dalitz plot. The diagram results from a coupling of $J_{29}{}^\mu$ and $J_{35}{}^\mu$, and the square of the matrix element is

$$\begin{aligned}|M|^2=&\frac{G_1G_3}{40(2\pi)^4p^0p'^0k^0k'^0M_\pi M_K}\\&\times[(M_\pi k^\mu-M_Kp'^\mu)(p_\mu+k_\mu')]^2,\end{aligned}\tag{37}$$

where k^μ, p'^μ, p^μ, and k'^μ are the 4 momenta of the K^+, π^+, and two π^0 mesons, respectively. Again taking the center-of-mass system, and transforming to the Dalitz variables (34), where this time T_1, T_2, and T_3 are the kinetic energies of the two π^0 and the π^+ meson, respectively, and $Q=M_K-M_\pi-2M_{\pi^0}$, we obtain for the τ' decay rate

$$\begin{aligned}\Gamma(\tau')&=\frac{G_1G_3Q^4(M_K+M_\pi)^2}{1080\sqrt{3}(2\pi)^3M_\pi}\int_{-1}^{1}dy\int_{-(1-y^2)^{\frac{1}{2}}}^{+(1-y^2)^{\frac{1}{2}}}dx(y+1)^2\\&=\frac{9\pi G_1G_3Q^4(M_K+M_\pi)^2}{4320\sqrt{3}(2\pi)^3M_\pi}.\end{aligned}\tag{38}$$

Comparison with (35) immediately gives

$$\Gamma(\tau)/\Gamma(\tau')=2.3.\tag{39}$$

This is somewhat smaller than the experimental ratio (22). If we take into account the fact that in τ' decay the maximum kinetic energy of either π^0 is about $0.64Q$ rather than $\frac{2}{3}Q$,[18] thereby replacing Q by $(0.64/0.667)Q$ in Eq. (38), we obtain in place of (39)

$$\Gamma(\tau)/\Gamma(\tau')=2.7.\tag{40}$$

This is in fair agreement with (22).

From (40), (36), and (22), we find that our calculation of $\Gamma(\tau')$ indicates

$$G_1G_3\approx7.4\times10^{-98}\ \mathrm{erg}^2\text{-}\mathrm{cm}^6.\tag{41}$$

VII. SUMMARY AND CONCLUSIONS

Relations (18), (19), (28), (32), (36), and (41) can now be used to determine the approximate magnitudes of G_1 and G_3. Admittedly, all of these results are valid only to first order, but the important point is that they all indicate the same approximate values for G_1 and G_3.

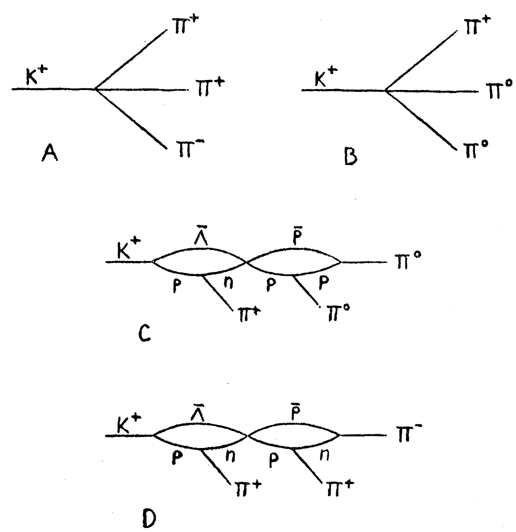

FIG. 6. Diagrams for $K^+\rightarrow 3\pi$ decays.

[17] M. Gell-Mann and A. H. Rosenfeld, see reference 10.

[18] The maximum kinetic energy of the π^+ is slightly smaller still.

A good fit to the data, with G_2 given by (18), is obtained by choosing

$$G_1 \approx 1\times 10^{-47} \text{ erg-cm}^3,$$
$$G_3 \approx 8\times 10^{-51} \text{ erg-cm}^3.$$

This gives a violation of the $|\Delta T|=\frac{1}{2}$ rule of about 5–10% in amplitude. The fact that $G_1G_3 > G_2^2$ may help account for the fact that K_1^0 decay and the pionic modes of hyperon decay are the most rapid of all weak decays.

We have seen that the introduction of boson currents and the subsequent splitting of the coupling strength provides an explanation for a number of experimental facts which have troubled the universal Fermi interaction. In particular, we have seen how the $|\Delta T|=\frac{1}{2}$ rule and its violation are a natural consequence of the theory, and how the interaction leads to fast pionic modes and slow leptonic modes of hyperon decay. We have also found, however, that the vector coupling of the fields leads to a nonuniform distribution of events in the Dalitz plot of τ decays, in disagreement with experiment. It is not impossible that this disagreement is a result of our first-order approximation. Further investigation of this question, especially with regard to final-state interactions, is necessary.

APPENDIX. THE KEMMER EQUATION FOR SPINLESS PARTICLES

The Kemmer Eq. (2) for spin zero particles is a reduction of the Klein-Gordon equation,

$$(\partial_\mu \partial^\mu + m^2)\phi(x) = 0, \tag{A1}$$

to first-order form. The β matrices satisfy the Kemmer-Duffin commutation relations

$$\beta^\sigma\beta^\nu\beta^\mu + \beta^\mu\beta^\nu\beta^\sigma = \beta^\sigma g^{\nu\mu} + \beta^\mu g^{\nu\sigma}. \tag{A2}$$

For the spin-zero case, we choose a 5×5 representation of the β matrices. A particular representation is

$$\beta^0 = \begin{bmatrix} 0&0&0&0&-1\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ -1&0&0&0&0 \end{bmatrix}, \quad \beta^1 = \begin{bmatrix} 0&0&0&0&0\\ 0&0&0&0&1\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&-1&0&0&0 \end{bmatrix},$$
$$\beta^2 = \begin{bmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&1\\ 0&0&0&0&0\\ 0&0&-1&0&0 \end{bmatrix}, \quad \beta^3 = \begin{bmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&1\\ 0&0&0&-1&0 \end{bmatrix}. \tag{A3}$$

In this representation, we see that (2) is equivalent to (A1) if we put

$$\psi(x) = -\frac{i}{m^{\frac{1}{2}}} \begin{bmatrix} -\partial_0\phi(x)\\ \partial_1\phi(x)\\ \partial_2\phi(x)\\ \partial_3\phi(x)\\ im\phi(x) \end{bmatrix}. \tag{A4}$$

The charge-conjugate field $\psi^C(x)$ is given by

$$\psi^C(x) \equiv C^*\bar{\psi}^\dagger(x),$$

where the matrix C satisfies

$$C\beta^\mu = -\beta^{\mu *}C, \quad C^*C = 1. \tag{A5}$$

In the representation (A3), C is given by

$$C = C^* = \begin{bmatrix} -1&&&&\\ &-1&&&\\ &&-1&&\\ &&&-1&\\ &&&&1 \end{bmatrix}. \tag{A6}$$

The relations

$$S^* = CSC^*,$$
$$\eta^0 = C^*\eta^{0*}\tilde{C},$$

where S and η^0 are given by Eqs. (4) and (5), are easily seen to hold.

The Lagrangian density, from which the field Eq. (2) results, is

$$L = -i\bar{\psi}(\partial_\mu\beta^\mu - im)\psi.$$

If we define the operator Γ_x by

$$\Gamma_x(\partial_\mu\beta^\mu - im) = (\partial_\mu\beta^\mu - im)\Gamma_x = \partial_\mu\partial^\mu + m^2,$$

and the function $R(x-x')$ by

$$R(x-x') \equiv \Gamma_x\Delta(x-x'),$$

where $\Delta(x-x')$ is given by[19]

$$\Delta(x-x') = \frac{i}{(2\pi)^3}\int e^{-ip\cdot(x-x')}\delta(-p_\mu p^\mu + m^2)\frac{p_0}{|p_0|}d^4p,$$

we obtain, in the usual manner, the commutation relations

$$[\psi_a(x), \bar{\psi}_b(x')] = -R_{ab}(x-x'),$$
$$[\psi_a(x), \psi_b(x')] = [\bar{\psi}_a(x), \bar{\psi}_b(x')] = 0,$$

for $x-x'$ arbitrary.

The Fourier expansion of $\psi(x)$ has the form

$$\psi(x) = (2\pi)^{-\frac{3}{2}}\int d^3p[a(\mathbf{p})u(\mathbf{p})e^{-ip\cdot x} + b^\dagger(\mathbf{p})v(\mathbf{p})e^{ip\cdot x}],$$

where the integral is to be evaluated at $p^0 = (\mathbf{p}^2 + m^2)^{\frac{1}{2}} \equiv \epsilon(\mathbf{p})$. $u(\mathbf{p})$ and $v(-\mathbf{p})$ are positive and negative energy solutions, respectively, of $(p_\mu\beta^\mu + m)f(\mathbf{p}) = 0$, and are given, in the representation (A3), by

$$u(\mathbf{p}) = [2m\epsilon(\mathbf{p})]^{-\frac{1}{2}} \begin{bmatrix} \epsilon(\mathbf{p})\\ -p_1\\ -p_2\\ -p_3\\ m \end{bmatrix}, \quad v(\mathbf{p}) = [2m\epsilon(\mathbf{p})]^{-\frac{1}{2}} \begin{bmatrix} \epsilon(\mathbf{p})\\ -p_1\\ -p_2\\ -p_3\\ -m \end{bmatrix}.$$

[19] For a discussion of the function $\Delta(x)$, see J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1955), Appendix A1.

The operators $a(\mathbf{p})$ and $b(\mathbf{p})$ satisfy

$$[a(\mathbf{p}),a^\dagger(\mathbf{p}')]=[b(\mathbf{p}),b^\dagger(\mathbf{p}')]=\delta(\mathbf{p}-\mathbf{p}'),$$

all other commutators vanishing.

The energy-momentum and charge operators are

$$P^\mu=\int[a^\dagger(\mathbf{p})a(\mathbf{p})+b^\dagger(\mathbf{p})b(\mathbf{p})]p^\mu d^3p,$$

$$Q=e\int[b^\dagger(\mathbf{p})b(\mathbf{p})-a^\dagger(\mathbf{p})a(\mathbf{p})]d^3p,$$

where e is the charge of the positron. Thus $a(\mathbf{p})$ and $b(\mathbf{p})$ annihilate negatively charged and positively charged mesons, respectively.

For the neutral π-meson field, we impose the restriction $\psi=\psi^C$ and take for our Lagrangian density

$$L=-\tfrac{1}{2}i\bar{\psi}(\partial_\mu\beta^\mu-im)\psi. \tag{A7}$$

The commutation relations are[20]

$$[\psi_a(x),\psi_b(x')]=-[R(x-x')C^*\bar{\eta}^0]_{ab},$$

and the Fourier expansion becomes

$$\psi(x)=(2\pi)^{-\frac{3}{2}}\int d^3p[c(\mathbf{p})u(\mathbf{p})e^{-ip\cdot x}-c^\dagger(\mathbf{p})v(\mathbf{p})e^{ip\cdot x}],$$

where $[c(\mathbf{p}),c^\dagger(\mathbf{p}')]=\delta(\mathbf{p}-\mathbf{p}')$ and

$$[c(\mathbf{p}),c(\mathbf{p}')]=[c^\dagger(\mathbf{p}),c^\dagger(\mathbf{p}')]=0.$$

The energy-momentum operator for neutral pions is

$$P^\mu=\int c^\dagger(\mathbf{p})c(\mathbf{p})p^\mu d^3p.$$

The Kemmer Eq. for spin one-half particles has been discussed elsewhere.[21]

[20] Because of the form of the generator obtained from (A7), care must be taken with regard to a factor of $\frac{1}{2}$ when deriving the canonical commutation relations. See, for example, J. Schwinger, Phil. Mag. 44, 1171 (1953).

[21] A. O. Barut, M. Samiullah, Nuovo cimento 17, 876 (1960).

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Spin-Momentum Correlations in Positron-Electron Scattering

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The imaginary part of the fourth-order Bhabha (e^+-e^-) scattering matrix element interferes with the (real) second-order matrix element, to produce a sixth-order dependence of the cross section on the spin of one of the particles (after summing over the spins of the other three particles). The process of extracting the imaginary part of the fourth-order matrix element is presented in some detail in one of the graphs (vacuum polarization).

I

SPIN-MOMENTUM correlations in the scattering of electrons by atomic nuclei (Mott scattering) are well known. Until recently, however, effects of this kind have not been studied in the scattering of leptons by leptons. Calculations have been performed by $\mu-e$ and e^--e^- scattering, and the results found to be very small.[1] Because of the qualitative difference between this case and that of Bhabha (e^+-e^-) scattering, due to the existence of annihilation graphs, it was thought that the effect might be more important in the latter process. In the present paper, results for Bhabha scattering are presented. These are also found to be small.

In reference 1, the imaginary part of the fourth-order scattering matrix element was calculated by using unitarity. We have therefore chosen to give a short exposition of an alternative method. The imaginary part is here extracted from the complete fourth-order matrix element. (See Fig. 1.) After performing two trivial contour integrations the imaginary part is easily isolated. The remaining (angular) integrations are performed *after* the trace calculations.

As was pointed out in reference 1, diagrams of the type of Fig. 2 do not contribute to the spin-momentum correlations. Arguments for the neglect of rescattering by the nucleus (valid for scattering by hydrogen) were also given there.

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[1] A. Barut and C. Fronsdal, Phys. Rev. 120, 1871 (1960).

II

If the positron beam (particle 2, momentum p_2) is partly polarized, with degree of polarization ξ, then the cross section may be written

$$\sigma=\sigma_0(1+\xi P), \tag{1}$$