Domain Configurations About Nonmagnetic Particles in Iron

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The closure domain configuration about a nonmagnetic particle in iron is considered in terms of magnetostatic, interfacial wall, and magnetostrictive energies. Equations for the total energy of the domain configuration and the value of the geometrical parameter θ, have been derived. The effects of particle size and shape are considered.

I. INTRODUCTION

WHEN a nonmagnetic particle is located within a ferromagnetic domain, a certain amount of magnetostatic energy is associated with the free magnetic poles which are formed at the particle-domain interface. A consideration of this energy led Néel[1] to postulate the existence of closure domains which could minimize this energy. By a semiquantitative consideration of the opposing effects of two energy terms, magnetostatic energy and interfacial wall energy, he showed that a spike-like configuration would be expected. Such a domain structure was later found experimentally by Williams[2] in silicon iron. It is the purpose of this paper to consider the closure domain configuration around nonmagnetic particles in iron quantitatively, taking into account the influence of magnetostrictive energy, as well as the magnetostatic and wall energies. This treatment involves a calculation of the important geometrical parameters, as well as the total energy of the domain configuration. Such a detailed consideration is necessary in order to permit the development of a quantitative model of the residual induction and the Barkhausen effect in ferromagnetic materials.[3]

II. ENERGIES INVOLVED

In this determination of the equilibrium domain configurations about a nonmagnetic particle, three types of energy will be considered: magnetostatic energy, interfacial wall energy, and magnetostrictive energy. The total energy of the closure domain will be computed by summing each of these three energies, i.e.,

$$E_t = E_m + E_w + E_{ms}, \tag{1}$$

where E_t = total energy of the closure domain, E_m = magnetostatic energy, E_w = interfacial wall energy, and E_{ms} = magnetostrictive energy.

Magnetostatic Energy

The magnetostatic energy of a unit volume of material is given by[4]

$$\sigma_m = -\tfrac{1}{2}HI, \tag{2}$$

where H = magnetic field strength and I = saturation magnetization. Consider a Bloch wall as illustrated in Fig. 1. The magnetostatic energy per unit area of an incremental thickness of material within the Bloch wall, $d\sigma_m$, is given by

$$d\sigma_m = -\tfrac{1}{2}HIdx. \tag{3}$$

Therefore,

$$\sigma_m = -\frac{1}{2}\int_{x=0}^{x=d} HIdx. \tag{4}$$

But since there is no external field,

$$H = B = 4\pi I$$

so that

$$\sigma_m = -2\pi I^2 d \tag{5}$$

per unit area, where d = thickness of Bloch wall, I = algebraic sum of the normal components of saturation magnetization of each domain.

Interfacial Wall Energy

For a 180° twist Bloch wall in which the separate domains have their saturation magnetization vectors in the ⟨100⟩ directions, Kittel[5] has shown

$$\sigma_{w(180tw)} = 1.8 \text{ ergs/cm}^2. \tag{6}$$

By the same analysis, the energy of a 90° twist wall can also be found:

$$\sigma_{w(90tw)} = 0.9 \text{ erg/cm}^2. \tag{7}$$

A relation has been derived by Stewart[6] which relates the interfacial energy of a 90° twist wall to that of a 90° tilt wall:

$$\sigma_{w(90tl)} = 1.7\sigma_{w(90tw)}. \tag{8}$$

Therefore,

$$\sigma_{w(90tl)} = 1.5 \text{ ergs/cm}^2. \tag{9}$$

Magnetostrictive Energy

The magnetostrictive energy can be computed by determining the elastic work done by the creation of closure domains.

$$\sigma_{ms} = \frac{1}{2}\int Y\epsilon^2 dV, \tag{10}$$

[1] L. Néel, Cahiers phys. **25**, 21 (1944).
[2] H. J. Williams, Phys. Rev. **71**, 646 (1947).
[3] W. D. Nix and R. A. Huggins (to be published).
[4] C. Kittel and J. K. Galt, *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1956), Vol. 3, p. 472.
[5] See reference 4, p. 478.
[6] K. H. Stewart, *Ferromagnetic Domains* (Cambridge University Press, New York, 1954), p. 99.

where Y = Young's modulus and ϵ = magnetostriction constant. The magnetostriction constant for pure iron, taken from Carr,[7] is 32×10^{-5}.

III. PROPOSED MODEL

In order to look at the problem of the closure domain configuration in detail, let us consider a spike-shaped closure domain as predicted by Néel and later experimentally verified by Williams. In this analysis it will be assumed that the nonmagnetic particle is cubic with dimension a and lies parallel and perpendicular to the {100} planes in bcc iron. Figure 2 illustrates the notation to be used. Let us now compute the total energy of the proposed domain configuration. To do this we will calculate the magnetostatic energy, the interfacial wall energy, and the magnetostrictive energy of the closure domain, in that order. The total energy will then be computed from (1).

Magnetostatic Energy

The total magnetostatic energy can be computed from (5). The value of I_{AC} (algebraic sum of the normal components of the magnetization vectors associated with the interface AC in Fig. 2) may be found from trigonometric considerations:

$$I_{AC}=\sqrt{2}I_s \sin\alpha. \tag{11}$$

The length of line AC may also be computed:

$$AC=(b-\tfrac{1}{2}a \sin45°)\cos\alpha+(\tfrac{1}{2}a \cos45°)\sin\alpha. \tag{12}$$

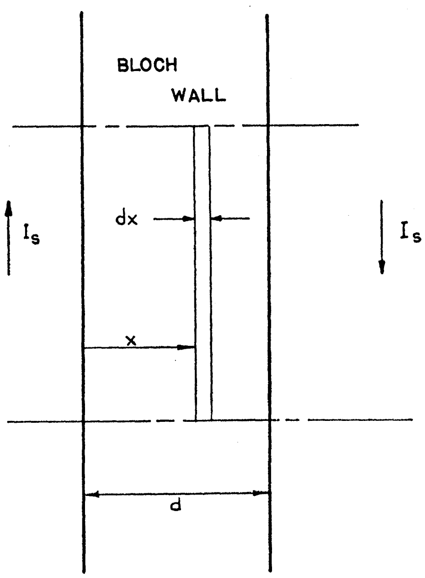

FIG. 1. 180° twist Bloch wall; notation used in magnetostatic energy calculation.

[7] W. J. Carr, Jr., *Magnetic Properties of Metals and Alloys* (American Society for Metals, Cleveland, Ohio, 1959), p. 209.

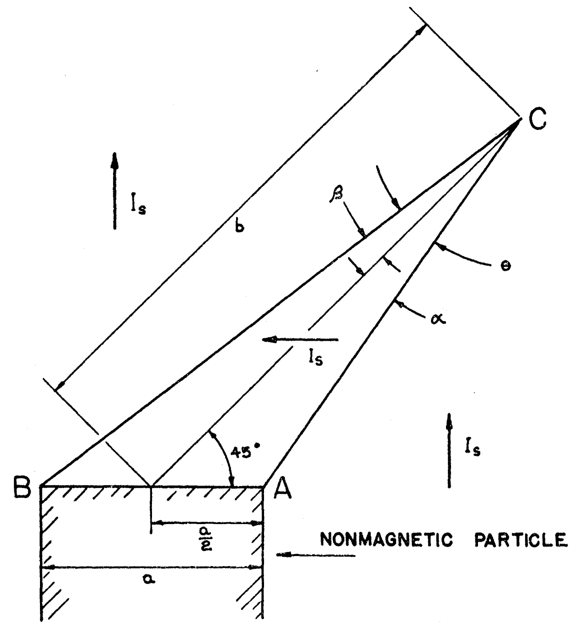

FIG. 2. Notation used with closure domain.

Similarly I_{BC} and length BC may be found:

$$I_{BC}=\sqrt{2}I_s \sin\beta, \tag{13}$$

$$BC=(b+\tfrac{1}{2}a \sin45°)/\cos\beta. \tag{14}$$

Since only interfaces AC and BC have normal magnetic vector components, the total magnetostatic energy will be found by summing the magnetostatic energy of interfaces AC and BC:

$$E_m=E_{m(AC)}+E_{m(BC)}. \tag{15}$$

From (5) and (11), we have

$$E_{m(AC)}=4\pi d[I_s^2 \sin^2\alpha(AC)a]. \tag{16}$$

Similarly

$$E_{m(BC)}=4\pi d[I_s^2 \sin^2\beta(BC)a]. \tag{17}$$

From (15), (16), and (17),

$$E_m=4\pi dI_s^2a[\sin^2\alpha(AC)+\sin^2\beta(BC)]. \tag{18}$$

From (12), (14), and (18), we have

$$E_m=4\pi dI_s^2a\left[\frac{b \sin^2\beta}{\cos\beta}+\frac{(\frac{1}{2}a)0.707 \sin^2\beta}{\cos\beta}+b \sin^2\alpha \cos\alpha\right.$$
$$\left.-(\tfrac{1}{2}a)0.707 \sin^2\alpha \cos\alpha+(\tfrac{1}{2}a)0.707 \sin^3\alpha\right]. \tag{19}$$

Interfacial Wall Energy

The total interfacial wall energy will now be derived by computing the energy of each of the four interfaces defining the closure domain:

$$E_w=\sigma_{w(90tl)}(AC+BC)a+\sigma_{w(90tw)}(ab \cos45°). \tag{20}$$

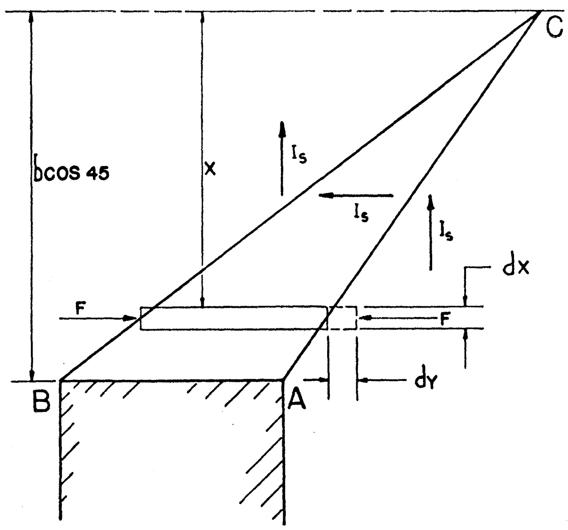

FIG. 3. Model for magnetostrictive energy calculation.

From (12), (14), and (20), we have

$$E_w=\sigma_{w(90tl)}\left[ab\cos\alpha-(\tfrac{1}{2}a^2)0.707\cos\alpha+(\tfrac{1}{2}a^2)0.707\sin\alpha+\frac{ab}{\cos\beta}+\frac{(\frac{1}{2}a^2)0.707}{\cos\beta}\right]+\sigma_{w(90tw)}[0.707ab]. \quad (21)$$

To simplify the relations (19) and (21), one must find a relation between b, a, and θ. First let us assume that

$$\alpha\cong\beta\cong\theta/2.$$

It follows that

$$b=0.707(a/2)\left[\frac{\cos(\theta/2)}{\sin(\theta/2)}+1\right]. \quad (22)$$

Using (22) the relations (19) and (21) may now be rewritten, eliminating b.

$$E_m=4\pi dI_s^2 0.707(a^2/2)\sin^2(\theta/2)\times\left[\frac{2}{\cos(\theta/2)}+\frac{1}{\sin(\theta/2)}+\frac{\cos^2(\theta/2)}{\sin(\theta/2)}+\sin(\theta/2)\right], \quad (23)$$

$$E_w=\sigma_{w(90tl)}0.707(a^2/2)\times\left[\frac{\cos^2(\theta/2)}{\sin(\theta/2)}+\sin(\theta/2)+\frac{1}{\sin(\theta/2)}+\frac{2}{\cos(\theta/2)}\right]+\sigma_{w(90tw)}0.707(a^2/2)\left[0.707\frac{\cos(\theta/2)}{\sin(\theta/2)}+0.707\right]. \quad (24)$$

Magnetostrictive Energy

Figure 3 defines some of the terms used in the computation of the magnetostrictive energy. When a closure domain is formed within the boundaries of the triangle ABC, the lattice undergoing this magnetic reversal experiences an elastic strain. The elastic work expended by an incremental volume of material is given by

$$dE=\tfrac{1}{2}Fdy. \quad (25)$$

Letting ϵ = magnetostriction constant (strain associated with magnetization) and $Y=F/\epsilon adx$, then

$$E_{ms}=\frac{1}{2}\frac{a^2\epsilon^2Y}{b\cos45^\circ}\int_0^{b\cos45^\circ}xdx,$$

$$E_{ms}=\tfrac{1}{4}a^2\epsilon^2Y(b\cos45^\circ). \quad (26)$$

From (22) and (26) the total magnetostrictive energy can be computed as

$$E_{ms}=0.0625a^3\epsilon^2Y\left[\frac{\cos(\theta/2)}{\sin(\theta/2)}+1\right]. \quad (27)$$

Total Energy

The total energy of the closure domain may now be found from (1), (23), (24), and (27). First let $\theta/2$ be ϕ.

$$E_t=[4\pi dI_s^2 0.707(a^2/2)\sin^2\phi+\sigma_{w(90tl)}0.707(a^2/2)]\times\left[\frac{\cos^2\phi}{\sin\phi}+\sin\phi+\frac{1}{\sin\phi}+\frac{2}{\cos\phi}\right]+\sigma_{w(90tw)}0.707(a^2/2)\left[0.707\frac{\cos\phi}{\sin\phi}+0.707\right]+0.0625a^3\epsilon^2Y\left[\frac{\cos\phi}{\sin\phi}+1\right]. \quad (28)$$

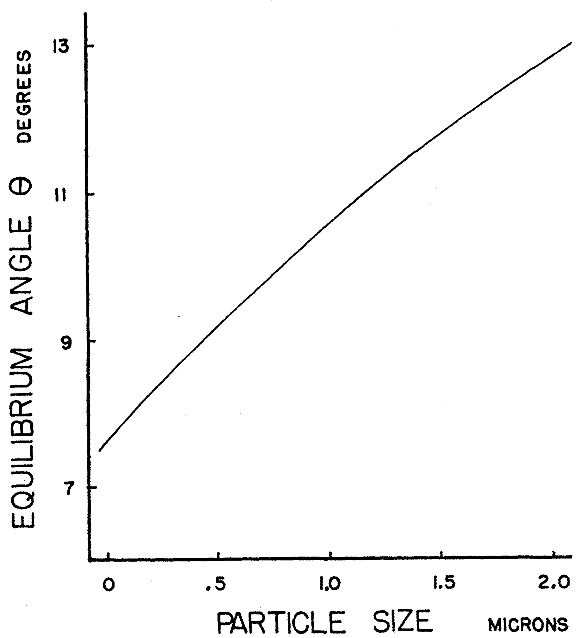

FIG. 4. Equilibrium angle of the closure domain vs the particle dimension.

Now let

$$4\pi d I_s^2 0.707(a^2/2)=M,$$
$$0.707(a^2/2)\sigma_{w(90tl)}=N,$$
$$\sigma_{w(90tw)}(0.707a)^2(\tfrac{1}{2})+0.0625a^3\epsilon^2 Y=P.$$

Then

$$E_t=(M\sin^2\phi+N)\left[\frac{\cos^2\phi}{\sin\phi}+\sin\phi+\frac{1}{\sin\phi}+\frac{2}{\cos\phi}\right]+P\left[\frac{\cos\phi}{\sin\phi}+1\right]. \quad (29)$$

IV. DETERMINATION OF EQUILIBRIUM SHAPE OF CLOSURE DOMAINS

In the previous section the total energy of the closure domain has been derived as a function of the angle ϕ. In order to determine the equilibrium value of ϕ which would be observed in pure iron, we will minimize the total energy. By differentiating (29) with respect to ϕ, neglecting small terms, and setting equal to zero, we find

$$(M\sin^2\phi-N)[\cos^3\phi+\cos\phi]+4M\sin^3\phi=P. \quad (30)$$

Now solving for the constants M, N, and P, letting

$$I_s=1700 \text{ gauss},$$
$$d=10^{-5} \text{ cm},$$
$$\sigma_{w(90tl)}=1.5 \text{ ergs/cm}^2,$$
$$\sigma_{w(90tw)}=0.9 \text{ erg/cm}^2,$$
$$\epsilon=32\times10^{-6},$$
$$a=0.5\times10^{-4} \text{ cm},$$
$$Y=2\times10^{12} \text{ dynes/cm}^2,$$

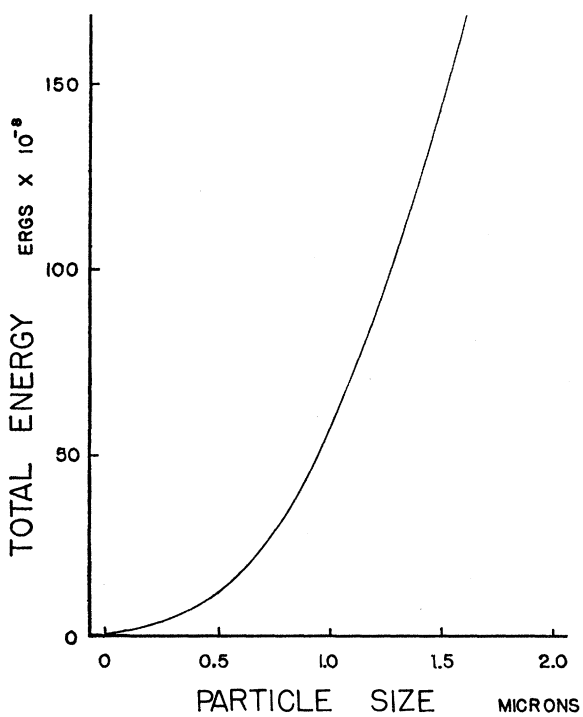

FIG. 5. Total energy of the closure domain vs the particle dimension.

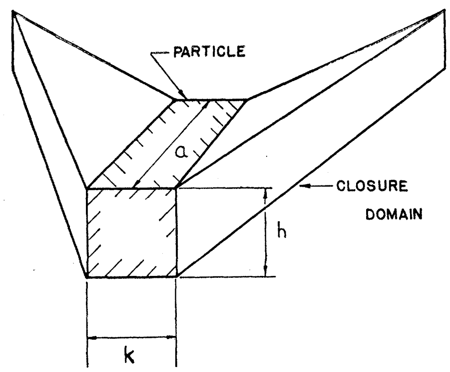

FIG. 6. Notation for noncubic particles.

we find $M=3.2\times10^{-7}$, $N=1.32\times10^{-9}$, $P=2.16\times10^{-9}$. Now solving Eq. (30) for ϕ, we find that $\phi=4.61°$, $\theta=9.22°$. It should be pointed out that in order to compute the angle θ, it was necessary to assume a value of the particle size. This is due to the dependence of the magnetostrictive energy on the value of a^3, rather than on a^2, as is the case for the other energy terms. The dependence of θ on the value of a is illustrated in Fig. 4. Figure 4 indicates that as the particle size approaches zero the equilibrium angle approaches a value of about 7.6 degrees.

In order to determine the sensitivity of these results to the values of the constants which were chosen, the rate of change of the angle θ with respect to each of these constants has been determined:

$$\partial\theta/\partial\sigma_{w(90tl)}=0.991 \text{ degree/erg cm}^2,$$
$$\partial\theta/\partial\sigma_{w(90tw)}=0.319 \text{ degree/erg cm}^2,$$
$$\partial\theta/\partial I_s=-0.319\times10^{-2} \text{ degree/gauss},$$
$$\partial\theta/\partial d=-2.45 \text{ degrees/micron}.$$

From these results it can be seen that the value of the equilibrium angle which has been calculated is not strongly dependent on the selection of these constants.

Having calculated the equilibrium angle for the closure domain as a function of particle size, from (29) we can determine the total energy of the domain configuration. This is shown in Fig. 5 as a function of the particle dimension a. It should be noted that particles would be expected to have two such closure domains.

In the discussion thus far we have assumed cubic particles. Let us now look at the effect of particle shape, considering particles which have rectilinear cross sections. Figure 6 illustrates a particle having dimensions a, h, and k. Since the value of the dimension k has no effect upon the closure domain configuration, we shall only be concerned with the dimensions a and h. By allowing the product ah to be constant, the singular effect of the shape parameter a/h on both θ and E_t may

be evaluated. For such particles, Eqs. (29) and (30) become

$$E_t=[4\pi dhI_s^2 0.707(a/2)\sin^2\phi+\sigma_{w(90tl)}h(0.707)(a/2)]\times\left[\frac{\cos^2\phi}{\sin\phi}+\sin\phi+\frac{1}{\sin\phi}+\frac{2}{\cos\phi}\right]+\sigma_{w(90tw)}0.707(a/2)\left[0.707a\frac{\cos\phi}{\sin\phi}+0.707a\right]+0.0625a^2\epsilon^2Yh\left[\frac{\cos\phi}{\sin\phi}+1\right], \quad (31)$$

and

$$[4\pi dhI_s^2 0.707(a/2)\sin^2\phi-\sigma_{w(90tl)}h(0.707)(a/2)]\times[\cos^3\phi+\cos\phi]+16\pi dhI_s^2 0.707(a/2)\sin^3\phi=[\sigma_{w(90tw)}(0.707a)^2(\tfrac{1}{2})+0.0625a^2\epsilon^2Yh]. \quad (32)$$

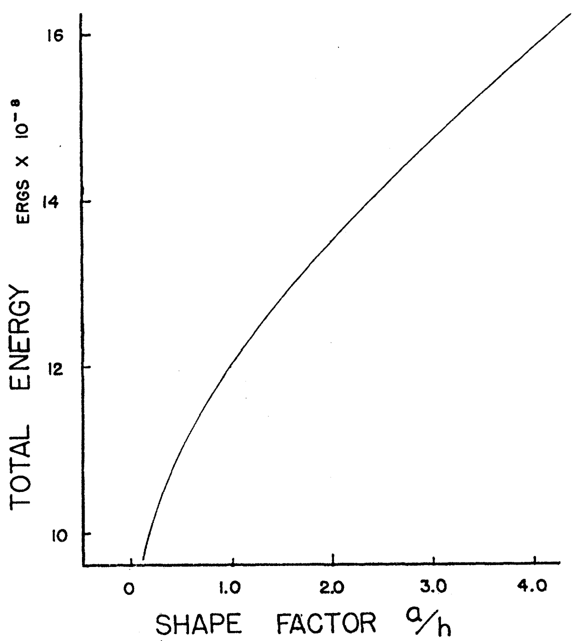

FIG. 7. Total energy of the closure domain vs the shape factor a/h for a value of $ah=0.25\times10^{-8}$ cm^2.

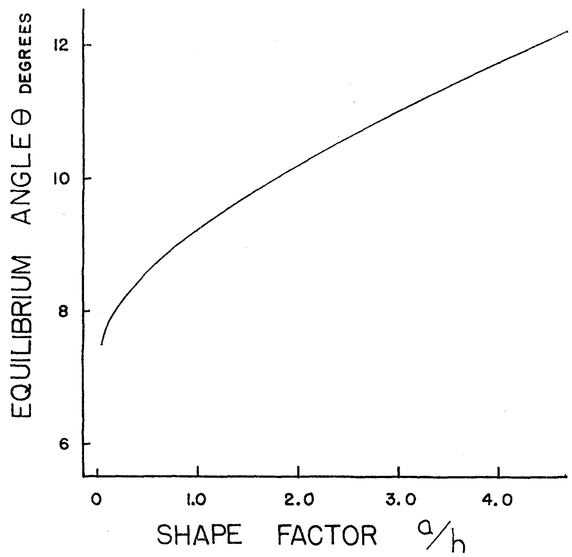

FIG. 8. Equilibrium angle of the closure domain vs the shape factor a/h for a value of $ah=0.25\times10^{-8}$ cm^2.

From a consideration of Fig. 7, in which the total energy of a closure domain about a particle having a value of $ah=0.25\times10^{-8}$ cm^2 is plotted against the shape parameter a/h, it can be seen that the closure domain will be oriented so as to minimize the ratio a/h.

In Fig. 8 the effect of the value of the parameter a/h on θ is shown for the same value of ah. One finds that the value of θ is less for noncubic particles.

V. BULK MAGNETIC BEHAVIOR

During the magnetization process in ferromagnetic materials, domain boundaries move over large distances. The over-all magnetic behavior of the bulk material has been shown to be affected by the interaction of these moving domain boundaries and nonmagnetic inclusions. A quantitative look at this phenomenon requires that detailed consideration be given to the interaction of the closure domain configuration with a moving domain boundary. A later paper will treat this process, the Barkhausen effect, and the magnitude of the residual induction.