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Gamma-Ray Correlation Function in the Adiabatic Approximation*†

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The gamma-ray correlation function following inelastic excitation of an even-A nucleus by a spinless projectile has been analyzed employing only the adiabatic approximation and theorems relevant to elastic scattering. The direction making equal angles with the incident and scattered directions in the scattering plane, the adiabatic recoil direction, is a convenient axis for quantization. In particular, the intermediate (excited) nucleus is populated with only even-M states, from which follows that the gamma distribution is unchanged by a rotation of π about this axis. For a $0^+-2^+-0^+$ excitation de-excitation, the gamma distribution in the scattering plane reduces to the form $\sin^2[2(\theta_\gamma-\theta_0)]$, where θ_0 is the adiabatic recoil axis. Comparison is made to the similar predictions of plane-wave Born approximation theories (in which the recoil direction for finite energy transfer is the symmetry axis) and to distorted-wave Born approximation calculations (for which, in general, there is no simple expression for the symmetry axis). Analysis of experiments verify the general features of the model, but further data obtained from forward scattering would be desirable to distinguish between the predictions of the adiabatic and Born approximations. Brief comments are made regarding gamma-ray polarization.

1. INTRODUCTION

In the process in which a medium-energy nuclear particle is inelastically scattered and excites a low-lying nuclear state, the angular correlation of the succeeding gamma ray with the inelastically scattered particle frequently has been found to lie close to the predictions of rather simple direct-interaction theories, such as plane-wave Born approximation (with no finite range exchange term in the perturbing potential)[1] or inelastic diffraction scattering models.[2,3] In particular, both models predict that the angular correlation pattern in the scattering plane for a $0^+-2^+-0^+$ excitation de-excitation is proportional to $\sin^2 2\theta_\gamma$, where θ_γ is measured from an appropriate recoil axis.

It is paradoxical that such an angular correlation pattern has been observed even in cases where the angular distribution of the inelastically scattered particles deviate markedly from the results of the simple direct theories. It is the primary purpose of this note to point out that gamma-ray correlation patterns very similar or equivalent to the above predictions follow from the single, less sweeping assumption that the inelastic scattering amplitude may be calculated in the adiabatic approximation for the relevant nuclear coordinates.

Somewhat analogous conclusions recently have been obtained independently by Satchler[4] using the adiabatic approximation plus the further assumptions that the scattered amplitude may be calculated in the distorted-wave Born approximation and that the perturbing potential be local. Correlation functions have been computed by Banerjee and Levinson[5] in the distorted-wave Born approximation (without use of the adiabatic approximation); the predictions of such arduous calculations for the symmetry angle of the correlation functions in the scattering plane are close to those resulting from the use of the adiabatic approximation in their best cases, but for some parameters, significant deviations from the adiabatic approximation (and experiment) are obtained.

The theory is developed in the next section while application to experiment will be made in the third section. Particular attention will be given in the third section to the differences between the predictions which follow from the plane-wave Born approximation and our results based on the adiabatic approximation.

2. THEORY

We consider the scattering of a spinless projectile with coordinate $\mathbf{r}$ from a nucleus whose relevant coordinates are represented by (the set) α and take for the Hamiltonian of this system

$$H=K+V(\mathbf{r},\alpha)+H(\alpha), \tag{1}$$

where K is the kinetic energy of the projectile, $V(\mathbf{r},\alpha)$ is the (complex) interaction potential, and $H(\alpha)$ is the Hamiltonian for the nuclear coordinates. The formal solution for the scattered amplitude[6] from initial state, a, to final state, b, is

$$\langle b|T|a\rangle=\left\langle b\left|V+V\frac{1}{E-K-V-H(\alpha)+i\epsilon}V\right|a\right\rangle; \tag{2}$$

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† The contents of this paper were first presented at the 1959 winter meeting of the American Physical Society [Bull. Am. Phys. Soc. 4, 460 (1959)].

1 G. R. Satchler, Proc. Phys. Soc. (London) **A68**, 1037 (1955).
2 J. S. Blair, Phys. Rev. **115**, 928 (1959).
3 J. S. Blair, D. Sharp, and L. Wilets, Bull. Am. Phys. Soc. **5**, 34 (1960), and to be published.
4 G. R. Satchler, Nuclear Phys. **18**, 110 (1960).
5 M. K. Banerjee and C. A. Levinson, Ann. Phys. **2**, 499 (1957).
6 The scattered amplitude we employ, $\langle b|T|a\rangle$, is more properly termed the "T matrix" and is related to the differential cross section by $d\sigma_{ba}/d\Omega=(2\pi/\hbar v_a)|\langle b|T|a\rangle|^2\rho_b$, where v_a and ρ_b are the incident flux and energy density of final states, respectively. Thus, for the elastic scattering problem with fixed α the more customary scattered amplitude, $f(\mathbf{k}_b,\mathbf{k}_a,\alpha)$ is related to our scattered amplitude by $f(\mathbf{k}_b,\mathbf{k}_a,\alpha)=-(\mu/2\pi\hbar^2)t(\mathbf{k}_b,\mathbf{k}_a,\alpha)$, where μ is the reduced mass of the projectile target system.

here $|a\rangle$ and $|b\rangle$ are the eigenfunctions for the free Hamiltonian, $H_0=K+H(\alpha)$, so that

$$|a\rangle=\exp(i\mathbf{k}_a\cdot\mathbf{r})\Phi_a(\alpha)\equiv|\mathbf{k}_a\rangle|\Phi_a(\alpha)\rangle.$$

The adiabatic approximation neglects nuclear motion during the period of collision [which is equivalent to setting $H(\alpha)=0$ in the denominator of Eq. (2)] and disregards the difference between the initial and final kinetic energies of the particle, i.e., sets $k_a=k_b=k$. We then may write

$$T_{ba}(\text{adiab})=\langle\Phi_b(\alpha)|t(\mathbf{k}_b,\mathbf{k}_a,\alpha)|\Phi_a(\alpha)\rangle, \tag{3}$$

where $t(\mathbf{k}_b,\mathbf{k}_a,\alpha)$ is the exact scattered amplitude[6] for the elastic scattering problem with static α,

$$t(\mathbf{k}_b,\mathbf{k}_a,\alpha)=\left\langle\mathbf{k}_b\left|V+V\frac{1}{E-K-V+i\epsilon}V\right|\mathbf{k}_a\right\rangle. \tag{4}$$

In principle, the coordinates α could refer to any nuclear coordinates; the usual criteria for the validity of the adiabatic approximation[7,8] suggest, however, that the approximation is most relevant when these coordinates have a collective character.

The importance of the adiabatic approximation is that theorems and results pertaining to purely elastic scattering may now be employed in discussions of inelastic scattering. (Indeed, the adiabatic approximation enables one to understand the many qualitative similarities between elastic and inelastic scattering.) Of particular importance is the reversibility (reciprocity) theorem, as stated by Glauber,[9]

$$t(\mathbf{k}_b,\mathbf{k}_a,\alpha)=t(-\mathbf{k}_a,\,-\mathbf{k}_b,\,\alpha); \tag{5}$$

i.e., the elastic scattered amplitude from the incident direction $\mathbf{k}_a$ into the final direction $\mathbf{k}_b$ is the same as the scattered amplitude from $-\mathbf{k}_b$ into $-\mathbf{k}_a$. The reversibility theorem (5) is satisfied by a large class of physical potentials, both real and complex. The property of the potential required for spinless projectiles is

$$\int\psi_a V\psi_b d\tau=\int(V\psi_a)\psi_b d\tau,$$

which is not the usual Hermiticity condition, since complex conjugation is not employed.

It is convenient to expand the scattered amplitude in eigenfunctions of the angular momentum, λ, of the relevant nuclear coordinates,

$$t(\mathbf{k}_b,\mathbf{k}_a,\alpha)=\sum_{\gamma\lambda\mu}C_{\gamma\lambda\mu}(\mathbf{k}_b,\mathbf{k}_a)\psi_{\gamma\lambda\mu}(\alpha), \tag{6}$$

where other quantum numbers are specified by γ. Now, the scattered amplitude, as can be seen from (4), is

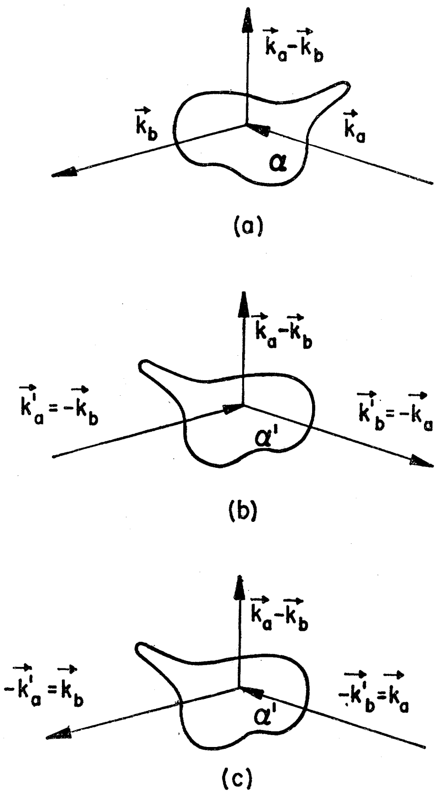

FIG. 1. Schematic representation of the symmetry properties of the scattering process. Sketch (a) represents the original scattering process. The scattered amplitude is unchanged when the entire physical system is rotated by π about the recoil direction, as is shown in sketch (b). By the reversibility theorem, this latter amplitude is equal to that corresponding to sketch (c). Thus, the scattered amplitudes of (a) and (c) are equal, which is the content of Eq. (8).

invariant with respect to either a rotation of the coordinate system or a rigid rotation of $\mathbf{k}_a$, $\mathbf{k}_b$, and the potential. Refer to Fig. 1 for the following. A rotation by π about the $\mathbf{k}_a-\mathbf{k}_b$ direction (z axis) leads to

$$t(\mathbf{k}_b,\mathbf{k}_a,\alpha)=t(-\mathbf{k}_a,\,-\mathbf{k}_b,\,\alpha'), \tag{7}$$

where α' are the rotated nuclear coordinates. But using the reversibility theorem, Eq. (5), in the form

$$t(-\mathbf{k}_a,\,-\mathbf{k}_b,\,\alpha')=t(\mathbf{k}_b,\mathbf{k}_a,\alpha'),$$

we obtain

$$\sum_{\gamma\lambda\mu}C_{\gamma\lambda\mu}(\mathbf{k}_b,\mathbf{k}_a)\psi_{\gamma\lambda\mu}(\alpha)=\sum_{\gamma\lambda\mu}C_{\gamma\lambda\mu}(\mathbf{k}_b,\mathbf{k}_a)\psi_{\gamma\lambda\mu}(\alpha'). \tag{8}$$

Since $\psi_{\gamma\lambda\mu}(\alpha)=(-)^\mu\psi_{\gamma\lambda\mu}(\alpha')$, we conclude that only even-μ are contained in the sum (6).

For the special case, $\Phi_a(\alpha)=|I=0,\,M=0\rangle$ (which is appropriate to excitation of even-even nuclei), the above result requires that the matrix element of the scattered amplitude,

$$T_{IM;00}(\text{adiab})=\langle IM|t(\mathbf{k}_b,\mathbf{k}_a,\alpha)|00\rangle\equiv T_{IM} \tag{9}$$

vanish if M is odd.

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[7] D. M. Chase, Phys. Rev. **104**, 838 (1956).
[8] S. I. Drozdov, Soviet Phys.—JETP **1**, 591, 588 (1955).
[9] R. J. Glauber, in *Lectures in Theoretical Physics* (Interscience Publishers, Inc., New York, 1959), Vol. 1.

These conclusions may be illustrated by consideration of the simple example of an axially symmetric scattering potential which is a function of the quantity $R=R_0[1+\beta_l Y_{l0}(\hat{\theta})]$, where $\hat{\theta}$ is the polar angle referred to the body axis, which is oriented in the direction of the unit vector $\mathbf{d}$. The scattered amplitude for a fixed orientation of the nucleus will be a function of the various parameters of the potential, including R_0 and β_l, and the scalar quantities K^2, κ^2, $\mathbf{K}\cdot\mathbf{d}$, $\boldsymbol{\kappa}\cdot\mathbf{d}$, where $\mathbf{K}=\mathbf{k}_a-\mathbf{k}_b$ and $\boldsymbol{\kappa}=\mathbf{k}_a+\mathbf{k}_b$. {The combination $[(\mathbf{K}\cdot\boldsymbol{\kappa})\cdot\mathbf{d}]^2$ may be expressed as a combination of the preceding.} Since $\boldsymbol{\kappa}$ changes sign (while $\mathbf{K}$ does not) under the interchange, $\mathbf{k}_a\rightarrow\mathbf{k}_b$, $\mathbf{k}_b\rightarrow-\mathbf{k}_a$, the reversibility theorem, Eq. (5), requires that no terms odd in $\boldsymbol{\kappa}\cdot\mathbf{d}$ occur in the scattered amplitude. Comparison to the explicit form of the spherical harmonics shows that this is equivalent to the statement that only even-M states of an even-even nucleus will be excited.

Let us now apply these results to the angular correlation of the gamma ray from the excited nucleus; we restrict ourselves to the excitation of an even-even nucleus and subsequent gamma-ray decay to the ground state. For a given scattering angle, θ, the probability for finding the gamma-ray radiation at angle θ_γ, ϕ_γ with respect to adiabatic recoil axis, $\mathbf{K}$ ($\phi_\gamma=0$ and π correspond to a gamma ray in the scattering plane), is proportional to the correlation function

$$W(\theta,\theta_\gamma,\phi_\gamma)=\sum_{M'M''} A_{M'M''}{}^I a_{M'M''}{}^I. \qquad (10)$$

Here, the nuclear statistical matrix, $A_{M'M''}{}^I$, is defined as

$$A_{M'M''}{}^I=T_{IM'}{}^*T_{IM''}. \qquad (11)$$

The corresponding statistical matrix for unpolarized gamma rays, $a_{M'M''}{}^I$, is well known:

$$a_{M'M''}{}^I=(2I+1)(4\pi)^{\frac{1}{2}}(-)^{M'+1} \times\sum_{\nu=0,\text{even}}^{2I} Y_{\nu,M''-M'}(\theta_\gamma,\phi_\gamma)\frac{1}{(2\nu+1)^{\frac{1}{2}}} (I,I,-M',M''|\nu,M''-M')(I,I,-1,1|\nu,0). \qquad (12)$$

Since states with odd M' are not populated and $Y_{\nu,m}(\theta_\gamma,\phi_\gamma)=(-)^m Y_{\nu,m}(\theta_\gamma,\phi_\gamma+\pi)$, we may verify explicitly the anticipated result that the gamma-ray distribution is unchanged by a rotation of π about the adiabatic recoil direction and in the scattering plane is symmetric about that axis. These results have been obtained also by Satchler[4] for the special case when the adiabatic scattered amplitude is calculated in the distorted-wave Born approximation.

For the interesting case where a 2+ level is excited, we now show in the adiabatic approximation that the correlation function in the scattering plane assumes the simple form

$$W(\theta,\theta_\gamma,\phi_\gamma=0)=b\sin^2(2\theta_\gamma). \qquad (13)$$

As noted earlier, this is also the prediction of direct interaction theories based either on the plane-wave Born approximation[1] or the inelastic diffraction model.[2,3]

To obtain the correlation function given by Eq. (13), crucial use is made of the additional symmetry property

$$T_{IM}=(-)^M T_{I-M}, \qquad (14)$$

which holds quite generally for spinless projectiles exciting a "natural parity" level of an even-even nucleus.[10] This lemma may be established as follows: From the symmetry of the scattering process and the absence of any projectile spin, the asymptotic wave function describing inelastic scattering,

$$\psi_{\text{sc}}\propto\sum_M T_{IM}|I,M\rangle, \qquad (15)$$

is unchanged (except for a possible over-all change in phase) when the coordinate system is reflected through the scattering plane. Such a reflection is equivalent to an inversion through the origin and a rotation of π about the axis perpendicular to the scattering plane, so that the transformed wave function is

$$(-)^I\sum_M T_{IM}\sum_{M'} d_{M'M}{}^I(\pi)|IM'\rangle =\sum_M T_{IM}(-)^M|I-M\rangle, \qquad (16)$$

since $d_{M'M}{}^I(\pi)=\delta_{M',-M}(-)^{I+M}$. Comparison of the coefficients of the nuclear wave functions in Eq. (15) and (16) yields the stated symmetry property. This lemma also may be proved from inspection of partial wave decomposition of T_{IM}.

The above lemma alone permits us to use, for spinless projectiles, a form of the 0–2–0 correlation function given first by Banerjee and Levinson,[5]

$$W(\theta,\theta_\gamma,\phi_\gamma=0)=a+b\sin^2[2(\theta_\gamma-\theta_0)], \qquad (17)$$

where a, b, and θ_0, which are functions of θ, are defined in Eqs. (4.5), (4.6), and (4.7) of reference 2. With the aid of this result we may derive Eq. (13) in exactly the same manner as is indicated in the steps preceding Eq. (4.8) of reference 2. However, we can see directly how the general form Eq. (17) reduces to the special form Eq. (13) when only even-M intermediate states exist in a 0–2–0 excitation-de-excitation. Indeed, the existence of an even-M state assures symmetry about the z axis, since then all components of the photon function have the same symmetry with respect to rotation by π about z. The absence of an isotropic component—or radiation along the z axis—can be seen as follows: The ($L=2$, $M=\pm2$) photons clearly have a node at the poles, since the angular dependence of the vector potential only contains components $Y_{L,M}$ and $Y_{L,M\pm1}$. The (2,0) photon has no intensity along the z axis since it has no component of angular momentum in that direction, and a photon must have ±1 units in its direction of propagation.

It is worthwhile to consider the correlation function

[10] A. Bohr, Nuclear Phys. **10**, 486 (1959).

for circularly polarized gamma rays decaying to the ground state of an even-even nucleus. In this case the gamma-ray statistical matrix is given by Eq. (12) with the change that the ν sum runs over only odd values of ν from 1 to $2I-1$. The necessary condition for the circular polarization correlation function to be identically zero is that the nuclear statistical matrix, Eq. (11), be multiplied by $(-1)^{M''-M'}$ under the exchange $M'\rightarrow -M''$, a result which follows from the antisymmetry property of the gamma-ray statistical matrix with respect to this exchange. This condition is trivially satisfied in the plane-wave Born approximation since only the $M=0$ state (with respect to the recoil direction) is populated. In the adiabatic approximation, this condition on the nuclear statistical matrix is equivalent to the requirement that $T_{IM'}{}^*T_{IM''}=T_{IM'}T_{IM''}{}^*$ where use is made of Eq. (14) and the result that T_{IM} vanishes for odd M. The preceding requirement is not satisfied in general although it is obeyed for those adiabatic approximation models in which the phase of the scattered amplitude is independent of M, in particular for the Fraunhofer[2] or the sharp-cutoff[3] models. For a $0-2-0$ excitation de-excitation, the circularly polarized gamma-ray correlation function in the adiabatic approximation is proportional to

$$(T_{20}{}^*T_{22}-T_{20}T_{22}{}^*)[Y_{3,2}(\theta_\gamma,\phi_\gamma)-Y_{3,-2}(\theta_\gamma,\phi_\gamma)] \propto (T_{20}{}^*T_{22}-T_{20}T_{22}{}^*)(xyz/r^3), \quad (18)$$

where x, y, and z are the coordinates of the gamma-ray counter in a coordinate system where the adiabatic recoil axis is the z axis and the x axis lies in the scattering plane. Satchler[4,11] has given a detailed discussion of circularly polarized gamma-ray correlation functions in the distorted-wave Born approximation.

3. APPLICATION TO EXPERIMENT

There is one rather important distinction to be made between the predictions of this paper and those resulting from use of the plane-wave Born approximation. For a theory based on the adiabatic approximation, the symmetry axis is the direction of momentum transfer that would obtain were the excitation energy equal to zero. Hence, the symmetry axis makes an angle of $(\pi/2-\theta/2)$ to the incident beam where θ is the scattering angle in the center-of-mass system (both the angle of the symmetry axis and the scattering angle are taken to be positive although they lie on opposite sides of the incident beam direction).

On the other hand, for plane-wave Born approximation theories, the symmetry axis is predicted to be the actual recoil direction for finite energy transfer $\mathbf{K}=\mathbf{k}_a-\mathbf{k}_b$. The actual recoil direction will frequently lie close to the adiabatic recoil direction but will differ significantly from it for scattering in forward directions when the excitation energy is non-negligible compared with the initial projectile energy. Indeed, as the scattering angle diminishes to zero, the adiabatic recoil angle approaches $\pi/2$ while the actual recoil angle approaches 0.

A striking feature of most of the observed $0-2-0$ correlation functions when the gamma rays are in the scattering plane has been their correspondence to Eq. (17), whether the projectiles be spinless alpha particles or protons. In Table I we compare the angle of the observed symmetry axis θ_0, to the actual recoil angle, θ_R, and to the adiabatic recoil angle, $\theta_R(\text{ad})=(\pi/2-\theta/2)$ for various experiments involving medium-energy projectiles. (All angles are measured with respect to the incident beam direction.) The same information is given in Table II for some experiments involving protons of rather low energy; while the adiabatic and the plane-wave assumptions are both clearly unjustified

[11] G. R. Satchler, Nuclear Phys. **16**, 674 (1960).

TABLE I. Calculated and observed angles between symmetry axis and direction of incident beam.

	Scattering angle in the laboratory system θ_{lab} (deg)	Actual recoil angle θ_R (deg)	Adiabatic recoil angle $\theta_R(\text{ad})$ (deg)	Observed symmetry angle θ_0 (deg)	
C^{12}, 4.43 Mev	28	65	71	69	43 Mev, Shook[a]
	59	49	52	57	
Mg^{24}, 1.37 Mev	35	68	69	73	43 Mev, Shook[a]
Mg^{24}, 1.37 Mev	30	70	75	73	16.6-Mev p, Yoshiki[b]
	42.5	65	68	66	
	70	52	54	60	
	95	40	41	44	
	120	28	29	26	
C^{12}, 4.43 Mev	15	69	82	73 ±5	39.3-Mev p, Adams, Hintz[c]
	30	67	76	68.2±2	
	45	61	65	62.5±5	
	60	55	58	55.3±2	
	80	45	47	46.4±2	
	110	31	32	34 ±5	

[a] See reference 12.
[b] See reference 16.
[c] See reference 18.

TABLE II. Calculated and observed angles between symmetry axis and direction of incident beam.

	Scattering angle in the laboratory system θ_{lab} (deg)	Actual recoil angle θ_R (deg)	Adiabatic recoil angle $\theta_R(\text{ad})$ (deg)	Observed symmetry angle θ_0 (deg)	
C^{12}, 4.43 Mev	90	24	40	51	6.2-Mev p, Hausman, Dell, and Bowsher[a]
	120	16	26	33	
Ne^{20}, 1.63 Mev	60	51	58.5	62	
	90	39	43	56	
	120	26	28.5	50	
S^{32}, 2.24 Mev	60	48	59	70	
	120	25	37	90	
Mg^{24}, 1.37 Mev	68	49	55	49	6.2-Mev p, Lackner, Dell, and Hausman[b]
	92.5	38.5	42	38.5	
	121	26	28	26	
Mg^{24}, 1.37 Mev	45	59	66.5	58	7.01-Mev p, Seward[c]
	60	53	59	48	
	90	40	44	42	

[a] See reference 21.
[b] See reference 20.
[a] See reference 22.

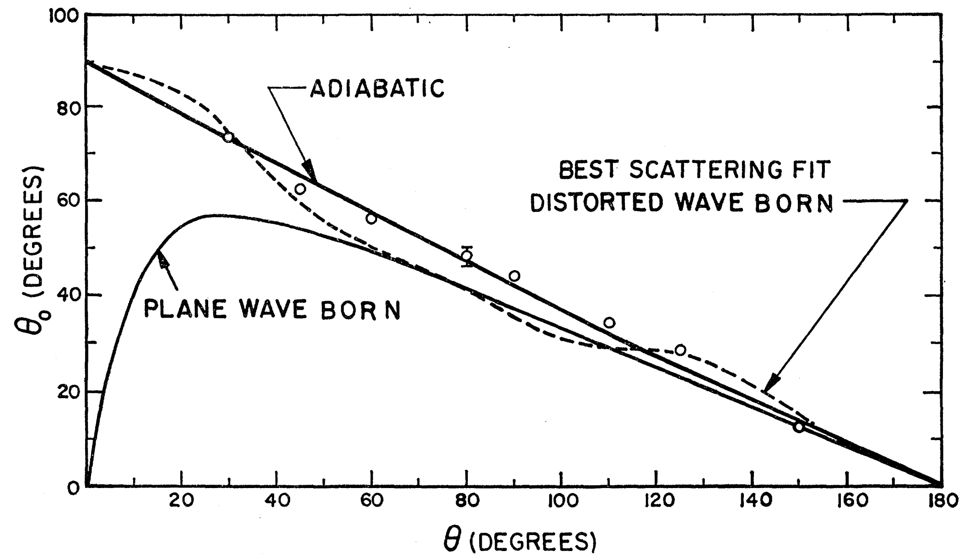

FIG. 2. Plot of symmetry angle, θ_0, versus laboratory scattering angle, θ_{sc}, for 16.6-Mev protons exciting the 4.43-Mev level of C^{12}. The circles represent the experimental points of Sherr and Hornyak.[14] The solid curves correspond to (a) θ_R, the prediction of the plane-wave Born approximation with no finite range nucleon-nucleon exchange interaction, (b) θ_R(ad), the adiabatic recoil angle, and (c) the symmetry angle computed by Banerjee and Levinson[5,15] in the distorted-wave Born approximation using the parameters which give the best fit to the angular distributions of the inelastically scattered protons.

for low-energy protons, it is interesting to observe that some correspondence is found between θ_0 and either θ_R or θ_R(ad). Further comments on the experiments are given below for various targets and bombarding particles.

C^{12} and Mg^{24}—43-Mev Alpha Particles

Shook[12] has observed the angular distribution of gamma rays following excitation of the lowest 2+ levels of these nuclei as well as the alpha-particle angular distributions. The correlation patterns are reasonably consistent with the simple result given in Eq. (13). This should be no surprise since it has been noted in previous papers[2,13] that the elastic and inelastic scattering cross sections of alpha particles at this energy correspond, particularly at moderately small scattering angles, to the predictions of the strong absorption diffraction model for inelastic scattering; one of the assumptions of this model was the adiabatic approximation. Since the uncertainties in the determination of θ_0 are comparable to the differences between θ_R and θ_R(ad), it does not appear possible to say which predicted symmetry angle is favored in this experiment.

C^{12}—16.6-Mev Protons

A detailed study of the dependence of correlation pattern on the scattered angle has been provided by Sherr and Hornyak.[14] Their measured correlation functions were of the form given by Eq. (17) for spinless projectiles. The isotropic contribution, however, was distinctly different from zero in contrast to the simple prediction, Eq. (13); the ratio a/b attained a value as large as 0.5. The results for θ_0 are not included in Table I, but rather are indicated in Fig. 2. Also shown are curves corresponding to (a) θ_R, the prediction of the plane-wave Born approximation when there is no finite range exchange term in the perturbing potential, (b) $\theta_R(\text{ad})=(\pi/2-\theta/2)$, the adiabatic recoil angle, and (c) the symmetry angle computed by Banerjee and Levinson[5,15] in the distorted-wave Born approximation using the parameters which give the best fit to the angular distributions of the inelastically scattered protons. (By construction there will be no exchange term in the perturbing potential when it is assumed that the proton interacts with a collective potential; however, such an exchange term does arise when the perturbing potential is taken to be the sum of two-body interactions between an incident proton and the target nucleons and the total wave function is appropriately antisymmetrized.[5]) It will be observed that the adiabatic result is here superior to the plane-wave Born approximation prediction and to the curve computed in distorted-wave Born approximation. (The adiabatic result also provides a markedly better fit than some other distorted-wave calculations shown in Fig. 16 of reference 15; in these calculations the correlation function is rather sensitive to the choice of parameters in the distorted-wave calculation.)

[12] G. B. Shook, Phys. Rev. **114**, 310 (1959).
[13] J. S. Blair, G. W. Farwell, and D. K. McDaniels, Nuclear Phys. **17**, 641 (1960).
[14] R. Sherr and W. F. Hornyak, Bull. Am. Phys. Soc. **1**, 197 (1956).
[15] C. A. Levinson and M. K. Banerjee, Ann. Phys. **3**, 67 (1958).

Mg^{24}—16.6 Mev Protons

Yoshiki[16] has observed that for the smaller scattering angles, $\theta_{\text{lab}}=30°$, 42.5°, and 70°, the correlation pattern is well approximated by the simple form given in Eq. (13). For larger angles it is necessary to take into account in the correlation function a term proportional to $\sin^2(\theta_\gamma-\theta_0')$; such a term may be present for the case of projectiles with nonzero spin[5,16,17] and is indicative of spin-flip processes. Because of this complication, we believe that the comparison between adiabatic recoil angle and the observed symmetry angle is significant only for the first three angles listed in Table I.

C^{12}—39.3-Mev Protons

The results of Adams and Hintz[18] at this higher energy are similar to those of Sherr and Hornyak in that the correlation functions are consistent with the general formula for spinless projectiles while at the same time they contained a substantial isotropic component. The observed symmetry angles tend to lie between θ_R and $\theta_R(\text{ad})$ and, indeed, are closest to θ_R. Adams and Hintz have shown that the observed symmetry angle is in good agreement with that distorted-wave Born approximation calculation which uses parameters which give the best fit to the inelastic angular distribution of 40-Mev protons.[19]

C^{12}, Ne^{20}, S^{32}, and Mg^{24}—6.2—7.01 Mev Protons

The correlation patterns observed by Hausman, Dell, Bowsher, and Lackner[20,21] and also by Seward[22] conform well to the spinless form, Eq. (17). The isotropic contributions are generally quite large, however. Further, the correlation patterns computed by Seward[22] on the basis of the Hauser-Feshbach[23] compound nuclear model suggest that the compound nuclear model may provide a satisfactory explanation of the correlation patterns where θ_0 is close to 90° or 45°. (Seward makes the interesting observation that the Hauser-Feshbach compound nuclear model does not necessarily imply that the correlation pattern is symmetric about 90°; his computed correlation functions, however, approximate such symmetry.) Thus, comparison between the direct-interaction predictions and experiment is most relevant when the predicted symmetry angles are distinctly different from 45° and 90°.

Not listed in Table II are (i) the data of Hausman, Dell, and Bowsher[21] on the gamma correlations following proton excitation of the 1.78-Mev level of Si^{28}, which exhibited symmetry about 90°, and (ii) the data of Seward[22] on excitation of the 1.37-Mev level of Mg^{24} by 6.66-Mev protons, which for the two smallest proton scattering angles also gave a correlation pattern symmetric about 90°. The observed symmetry angles given by Lackner, Dell, and Hausman[20] for Mg^{24} are not best fit values but rather are the actual recoil angles, which were found to be in good agreement with experiment; this accounts for the exact equivalence between θ_R and θ_0 indicated in Table II.

Before summarizing our conclusions, we should emphasize that most of the experiments have involved proton scattering and that for such cases the theoretical results of the preceding section are relevant only to the extent that the spin of the proton does not affect the scattering. This is not a generally valid assumption; it is a familiar fact that the spin-orbit potential plays a large role in determining elastic scattering amplitudes so that we anticipate that it will similarly affect the inelastic scattering. Further, Yoshiki's correlation patterns for large-angle scattering show explicitly the presence of spin-flip processes. Thus, when we encounter discrepancies between the proton experiments and the simple adiabatic predictions, it is not clear whether we should attribute these to the proton spin or to a breakdown of the adiabatic assumption.

4. SUMMARY

The adiabatic approximation leads to predictions for the gamma correlation function which are similar to those predicted by the Born approximation. The physical content of the two theories differ in that: (1) The adiabatic approximation admits arbitrarily strong interactions; in the Born approximation, the scattered amplitudes are calculated to only first order in the interaction. (ii) The Born approximation admits finite excitation energy, so that initial and final momenta may be different in magnitude; the adiabatic approximation assumes equal initial and final momenta. The distorted-wave Born approximation and the adiabatic approximation become equivalent as both the excitation cross section and excitation energy vanish.

The functional form, $\sin^2 2[\theta_\gamma-\theta_0(\theta)]$, for the gamma distribution in the scattering plane is common to both the adiabatic and plane-wave Born approximations, but the two differ with respect to their prescription for $\theta_0(\theta)$. Although the symmetry angles $[\theta_R(\text{ad};\theta)$ and $\theta_R(\theta)$, respectively$]$ are similar over most of the range of θ, they differ significantly for scattering into near forward directions. The adiabatic symmetry angle, $\theta_R(\text{ad};\theta)$, varies linearly from $\pi/2$ to 0 as θ varies from 0 to π. The recoil angle, $\theta_R(\theta)$, varies rapidly near the forward direction, starting at $\theta_R(0)=0$ and approaching $\theta_R(\text{ad};\theta)$ at angles larger than $\Delta E/E$, where ΔE is the

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energy loss. At exactly forward, the two directions $\theta_R(\text{ad};0)=\frac{1}{2}\pi$ and $\theta_R(0)=0$, are equivalent, as far as concerns the correlation pattern.

In what alpha-particle work there is available, the $\sin^2[2(\theta_\gamma-\theta_0)]$ form appears to be reasonably well satisfied, but it is not possible to distinguish between the adiabatic and plane-wave Born approximation predictions. More proton than alpha-particle work is available. In many cases the gamma distribution can be fitted with the form for spinless projectiles, and a comparison of the models is significant. The work of Sherr and Hornyak (16.6-Mev protons on C^{12}) clearly fits the adiabatic prediction for the symmetry angle better than the plane-wave or distorted-wave Born predictions. However, the symmetry angles observed by Adams and Hintz (39.3-Mev protons on C^{12}) are intermediate between the adiabatic and the plane-wave Born predictions, somewhat favoring the latter, and are in good agreement with distorted-wave Born approximation computations. Experiments by Yoshiki (16.6-Mev protons on Mg^{24}) somewhat favor the adiabatic approximation.

More experiments conducted at forward scattering (angles of the order of $\Delta E/E$) would be highly desirable in distinguishing between the models; no such alpha-particle data are yet available.

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Decay of Hf^{180m}†,*

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The energies and relative intensities of the gamma radiation and the relative intensities of the conversion electrons following the decay of 5.5-hr Hf^{180m} have been measured using the curved-crystal gamma-ray spectrometer (recently calibrated for accurate intensity measurements), the ring-focused beta-ray spectrometer, and the semicircular spectrometer at the California Institute of Technology. The measured transition energies are: 57.54±0.01, 93.33±0.02, 215.25±0.13, 332.5±0.3, and 443.8±0.6 kev. The energy levels deduced from these values are not entirely consistent with the two-parameter rotational formula. Conversion coefficients derived from the measurements were absolutely normalized using a method involving information available from the decay scheme. All of the 57.54-kev transition conversion coefficients are anomalously high if compared with the theoretical $E1$ coefficients. No admixture of $M2+E3$ can account for the anomaly. The K conversion coefficients of the $E2$ transitions are all about 10% low with exception of the 93.33-kev transition. The L coefficients have a varying deviation, the maximum being 12%. The K conversion coefficient of the 501.3-kev transition has the value 0.037±0.012 which is consistent with the theoretical $E3$ value of 0.040.

INTRODUCTION

THE energy levels of the Hf^{180} nucleus have for some time served as a classical example of a rotational excitation spectrum.[1] Above the 0^+ ground state four excited states with spin 2+, 4+, 6+, and 8+ are known. The energies of these levels can be computed using a two-parameter formula of the form

$$E_I=(\hbar^2/2J)I(I+1)+BI^2(I+1)^2, \quad (1)$$

where J is the moment of inertia parallel to the symmetry axis, I the nuclear spin, and B is a constant taking into account the rotation-vibration interaction and other second order effects.[1] The Hf^{180m} γ-ray energies have now been measured with enough precision to provide a useful check of the validity of the two-parameter formula in this overdetermined set of data. It seemed to us worth while to undertake this precision measurement using the crystal diffraction spectrometer. The result to be described shows definite deviation from Eq. (1) for the 6+ and 8+ levels.

In addition, a precise evaluation of γ-ray and conversion-electron intensities seemed feasible and worth while in the Hf^{180} decay because of its simple cascade decay scheme. This evaluation results in precise absolute internal conversion coefficients for all $E2$ transitions of the rotational cascade as well as for the $E1$ and $E3$ transition from the 9− intrinsic state (see Fig. 1).[1a] Anomalies of the conversion coefficients of the 57.5-kev transition have been reported and discussed by Scharff-Goldhaber *et al.*[2] and Gvozdev and Rusinov.[3]

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[1] See, for example, S. A. Moszkowski in *Encyclopedia of Physics* (Springer-Verlag, Berlin, 1957), Vol. 39, p. 485ff; and A. K. Kerman, in *Nuclear Reactions* (North-Holland Publishing Company, Amsterdam), p. 429.

[1a] *Note added in proof.* From a recent experiment by M. Deutsch and R. W. Bauer, Proc. Conf. Nuclear Structure, Kingston, 1960, p. 592, a spin assignment of 8—follows for the 1142-kev state.

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