of E_r, F_r can now be resolved by using *any* of the last four equations (I.11) to (I.14). In addition, valuable checks of the correctness of the obtained solution can be deduced from the last 4 quantities.

Our primary solution relies on the first ten quantities of Table I which on the whole include the easier types of experiments. As discussed in Sec. 4, experiment 14 tends to be somewhat easier than 11 to 13.

We have not investigated the problem of solving the set of equations (I.1) to (I.14) when the experiments are rather inaccurate, as in practice they tend to be. In this case, the use of all 14 experiments is probably desirable.

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High-Energy Nucleon-Nucleon Collisions*

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The single virtual boson exchange interaction model is applied to high-energy inelastic nucleon-nucleon collisions over an energy range of several orders of magnitude. The phase space is discussed simply in terms of three "natural" phase-space variables, and a simple, exact formula is given for the "upper" boundary of these variables. The probability for a particular final-state configuration is then discussed in terms of the available phase space, the magnitude of the phase-space factor, and the magnitude of the average total "cross-section" factors that occur in this model. Qualitative features of experimental data for incident nucleon laboratory energies of 10, 10^2, and 10^3 Bev can be understood on the basis of this model.

EXPERIMENTAL evidence that many high-energy inelastic nucleon-nucleon collisions occur with large impact parameters[1–3] suggests the importance of single π-meson exchange graphs.[4,5] A recently given field-theoretical description of general binary collisions dominated by single-boson exchange graphs[6] is applied in this note to high-energy nucleon-nucleon collisions.[7,8] This model leads naturally to the two "independent" groups of final-state particles that are observed. It is shown that if the plausible assumption is made that the "scattering" cross sections of the exchanged "almost real" pion with the incident nucleons are close to the real cross sections at high energies, then this model leads to qualitative understanding of certain features observed in inelastic nucleon-nucleon scattering all the way from incident nucleon laboratory energy, $E_{iL}\sim 10$ Bev, up to and including ultrarelativistic energies.

The pertinent graph is shown in Fig. 1. Nucleons N and N', with four-momenta p_i and p_i' exchange a π meson with four-momentum Δ_i, leading to two groups of particles, C and C', with total four-momenta P and P'. The nucleon rest mass is M, and the metric is chosen so that $p_i^2=p_i'^2=-M^2$. The "rest masses" W and W' of C and C' are defined by $P^2=-W^2$ and $P'^2=-W'^2$. The rest (barycentric) system of the group of particles C is denoted by (W) and that of the group C' by (W'). For the case considered (at least one pion in each group C, C'), the minimum value of W, and of W', is $m_\pi+M$. The over-all barycentric system is

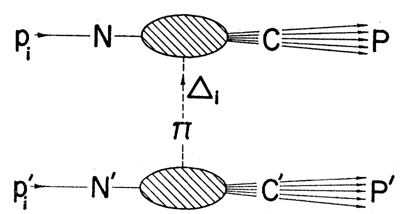

FIG. 1. A general inelastic collision in which nucleons N and N', with four-momenta p_i and p_i', interact by the exchange of a single π-meson π, with four-momentum Δ_i, leading to two groups of final state particles, C, with total four-momentum P, and C', with total four-momentum P'. Each group contains at least one π meson.

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[7] A closely related treatment of nucleon-nucleon collisions is given by Dremin and Chernavskii (see footnote 4). D. S. Chernavskii, I. M. Dremin, I. M. Gramenitski, and V. M. Maksimenko, Lebedev Institute of the Academy of Sciences, U.S.S.R., Report A-27, 1960 (unpublished), treat the $N-N$ interaction at 9 Bev; D. S. Chernavskii and I. M. Dremin, Lebedev Institute of the Academy of Science, U.S.S.R., Report A-28, 1960 (unpublished), treat the $N-N$ interaction at 10^2 Bev.

[8] A related, but phenomenological treatment of the same problem is given by N. Yajima, S. Takagi, and K. Kobayakawa, Progr. Theoret. Phys. (Kyoto) **24**, 59 (1960). This paper has extensive references to related papers. A similar model is suggested by E. M. Friedländer, Phys. Rev. Letters **5**, 212 (1960).

denoted by (U), and U is the total energy in this system, defined by $(p_i+p_i')^2=-U^2$. In the system (U), p_i has components $(\mathbf{p}_{iU},E_{iU})$, and we adopt the convention that N is incident from the left and the target nucleon N' is incident from the right. Also, P has components $(\mathbf{P}_U,W_U)$, and the components of p_i' and P' are similarly denoted. For small values of Δ^2, where Δ^2 denotes Δ_i^2, the differential cross section may be written as[6,9]

$$\frac{d\sigma_{N+N'\to C+C'}(\Delta^2,U)}{d(\Delta^2)}=\frac{2}{(2\pi)^3 p_{iU}^2U^2}\frac{1}{(\Delta^2+m_\pi^2)^2}\times\int dWdW'\; p_W W^2\sigma_{\pi+N\to C}(\Delta^2;W)\times p'_{W'}W'^2\sigma_{\bar{\pi}+N'\to C'}(\Delta^2;W'),\quad (1)$$

where $p_{iU}=|\mathbf{p}_{iU}|$; p_W is defined by the equation

$$W=(p_W^2+M^2)^{\frac{1}{2}}+(p_W^2+m_\pi^2)^{\frac{1}{2}};\qquad (2)$$

$\sigma_{\pi+N\to C}(\Delta^2;W)$ is the total "cross section" for the reaction $\pi+N\to C$ at energy W in the system (W); the π is the exchanged "almost real" pion; and $p'_{W'}$ and $\sigma_{\bar{\pi}+N'\to C'}(\Delta^2;W')$ are similarly defined. It should be noted that the integrations over the phase space variables of the groups of particles C and C' are thus included in these "cross sections."

The variables Δ^2, W, and W' are independent; however, the limits of their ranges are related because of over-all energy-momentum conservation, as given by

$$\Delta^2U^2\left(1-\frac{W^2+W'^2+2M^2+\Delta^2}{U^2}\right)=(W^2-M^2)(W'^2-M^2)+(W'^2-W^2)^2\frac{M^2}{U^2}.\qquad (3)$$

Equation (3) gives the minimum value of Δ^2 for given W and W', and alternatively for given Δ^2 and W (or W') it gives the maximum value of W' (or W). Δ^2 is at its minimum value for $W=W'=m_\pi+M$ and increases with W and W'. Its minimum value decreases towards zero as U increases. For sufficiently large U^2 and small Δ^2, Eq. (3) is well approximated by

$$\Delta^2U^2=(W^2-M^2)(W'^2-M^2).\qquad (4)$$

In discussing the phase space, it is useful to think of a fixed value of Δ^2 and to consider W and W' as variables. The allowed region of phase space, in the $W-W'$ plane, is shown in Fig. 2 for three different incident energies and for $\Delta^2=m_\pi^2$. It is bounded by the solid curve, given by Eq. (3), and by the lines W, $W'=m_\pi+M$. The approximate boundary, given by Eq. (4), is shown by the dotted curve. Except at the end points, the approximation is good for $\Delta^2=m_\pi^2$ and $U\gtrsim 14M$. The phase space is symmetrical about the line $W=W'$, shown as a dashed line, and for simplicity in what follows, we consider that half for which $W\leqslant W'$.

For sufficiently large U, restriction of Δ^2 to small values leads to important kinematical results. (1) In the system (U) the "center-of-mass" motion of the group C, given by $\mathbf{P}_U$, is almost parallel to $\mathbf{p}_{iU}$ and towards the right, i.e., $\mathbf{P}_U\cdot\mathbf{p}_{iU}=P_Up_{iU}\cos\theta\approx P_Up_{iU}$, and of course $\mathbf{P}'_U$ is almost parallel to $\mathbf{p}'_{iU}$ and towards the left. (2) The particles of group C are contained within a forward cone and those of group C' within a backward cone. The opening angle of the forward cone in (U) depends on the energy and angular distribution of the particles of C in (W) and on the Lorentz transformation connecting (W) and (U). The γ of this transformation is given by $\gamma=W_U/W$, where

$$W_U=(U^2+W^2-W'^2)/(2U).$$

In general, the larger γ is, the narrower is the cone. Similar considerations hold for the backward cone, with $\gamma'=W'_U/W'$ and

$$W'_U=(U^2+W'^2-W^2)/(2U).$$

W'_U is related to W_U by the equation $W_U+W'_U=U$. For small Δ^2, $W_U\approx W'_U\approx U/2$. Because W' can be large when W is small (see $W\leqslant W'$ half of phase space in Fig. 2), γ and γ' can differ considerably.

The probability of a final-state configuration with given W and W' depends upon the phase-space factor $p_WW^2p'_{W'}W'^2$, and on the pion-nucleon "cross sections" which appear in Eq. (1). The phase-space factor is maximum and roughly constant for values of W, W' on the boundary, except for values of W close to $m_\pi+M$, at which point p_W vanishes. This result is a consequence of the "winged" nature of the allowed region of phase space in the $W-W'$ plane, i.e., of the roughly hyperbolic boundary. One obtains the surprising result, with the help of Eq. (4) and the approximation $p'_{W'}\approx W'/2$, that the phase-space factor has a small but absolute maximum on the boundary at W close to the energy of the $\frac{3}{2}-\frac{3}{2}$ pion-nucleon resonance, $W_{\mathrm{res}}\approx 1.3M$.

If we now assume that the "cross sections" for the collisions of the virtual pion with each of the incident nucleons, which appear in Eq. (1), are close to those for real pion collisions with nucleons at the same barycentric energies W and W'; then, with the previous kinematical analysis, Eq. (1) can be used to discuss certain qualitative features of high-energy nucleon-nucleon collisions. For this purpose, we sum over all possible final states C and C' allowed for given W, W', and charge of the virtual pion, so that the cross sections for the particular processes $\pi+N\to C$ and $\bar{\pi}+N'\to C'$ are replaced by $\sigma_{\pi N}{}^{\mathrm{tot}}(W)$ and $\sigma_{\bar{\pi}N'}{}^{\mathrm{tot}}(W')$. In addition, we sum over the three charge states of the virtual pion, and consider, for the present approximate purpose,

[9] The notation of Eq. (1) has been slightly altered from that of references 5 and 6 by the use of a semicolon to separate Δ^2 from the other variables in the arguments of off-the-mass-shell "cross sections," and thus to help distinguish them from real differential cross sections such as the left-hand member of Eq. (1).

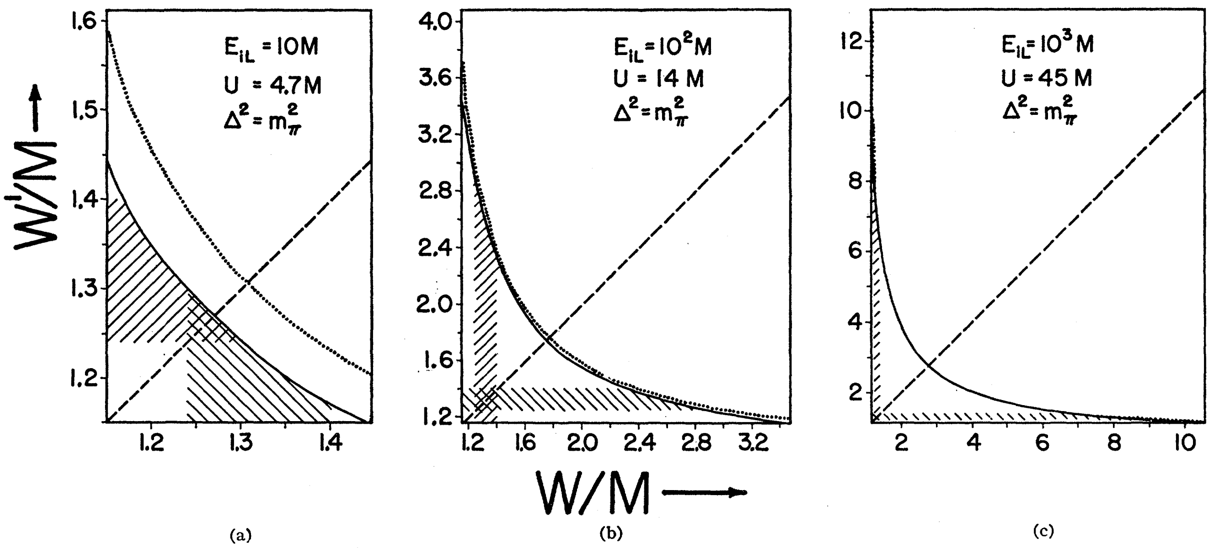

FIG. 2. Phase space for the graph of Fig. 1 for three incident laboratory energies, E_{iL}, and for $\Delta^2=m_\pi^2$. W and W' are the "internal energies" of the groups C and C'. The allowed region in the $W-W'$ plane is bounded by the solid curve, given by Eq. (3), and is symmetrical about the dashed line, $W=W'$. The dotted curve is the approximate boundary given by Eq. (4). The hatched vertical strip is the region in which W is close to W_{res}, the energy of the $\frac{3}{2}-\frac{3}{2}$ pion-nucleon resonance, and the horizontal strip that in which W' is near W_{res}.

an "average" pion-nucleon cross section with a large absolute maximum at the $\frac{3}{2}-\frac{3}{2}$ resonance,[10] the well-known three smaller maxima at higher energies, and an asymptotic value of about 30 mb reached at $W\approx 2M$. The region in which W is close to the $\frac{3}{2}-\frac{3}{2}$ pion-nucleon resonance is shown in Fig. 2 as the hatched vertical strip, and that in which W is near W_{res} as the hatched horizontal strip. These maxima of the "average" total cross section will be important only if those values of W and W' that satisfy $(m_\pi+M)\leqslant W,\, W'\leqslant 2M$ constitute a large part of the phase space.

In nucleon-nucleon collisions at incident laboratory energy, $E_{iL}\sim 9$ Bev, there is evidence that in large impact parameter collisions the final state often consists of two $\frac{3}{2}-\frac{3}{2}$ "isobars."[2,4] In Fig. 2(a) the phase space is shown for $E_{iL}=10M(U^2\approx 2.2\times 10M^2)$ and $\Delta^2=m_\pi^2$. In most of the $W\leqslant W'$ half of the phase space, W' is close to W_{res}. Although the region with both W and W' near W_{res} (shown as the doubly hatched area) is a relatively small part of the phase space (for $\Delta^2=m_\pi^2$), it is heavily weighted both by the phase-space factor and by the "cross-section" factors. Of course, the entire allowed region in Fig. 2(a) corresponds to almost completely elastic $\pi-N$ scattering, and thus each of C and C' consists of a nucleon and a single pion.

As E_{iL} increases, and the region with both W and W' near the $\frac{3}{2}-\frac{3}{2}$ resonance becomes less important [see Fig. 2(b)], then, because of the "winged" nature of the phase space, the region with only one of W, W' near the $\frac{3}{2}-\frac{3}{2}$ resonance increases in importance. Thus, the combined effect of the available phase space, the phase-space factor, and the pion-nucleon cross-section factors is to produce very asymmetric events. A large number of events of this kind are observed in nucleon-nucleon collisions with $E_{iL}\sim 10^2$ Bev.[3]

For higher values of E_{iL}, W and W' are large throughout most of the phase space [see Fig. 2(c)], thus leading to the typical "two-center" production of secondaries seen in ultrarelativistic events ($E_{iL}\gtrsim 10^3$ Bev).[1]

There seems to be a natural explanation within the framework of this model for the "small inelasticities" observed in many of these ultrarelativistic events,[1] i.e., the fact that, in (U), each of the two nucleons proceeds almost undeflected from its original trajectory and loses only a small fraction of its energy in producing secondary particles. Small inelasticity in (U) is equivalent to small inelasticity in (W) for the nucleon N in its collision with the virtual pion, provided W is large, and similarly for the nucleon N'. There is indication of this small inelasticity in pion-nucleon scattering data at barycentric energy $W\approx 3M$ (pion laboratory momentum$=5$ Bev/c)[11] and in recent data at higher energies.[12]

In order to estimate the cross section for ultrarelativistic $N-N$ collisions for small values of Δ^2, we assume that the important part of the phase space occurs for values of W and W' for which the average pion-nucleon cross section is constant, i.e., for W, W'

[10] We ignore the Δ^2 dependence of the off-the-mass-shell cross section near the $\frac{3}{2}-\frac{3}{2}$ pion-nucleon resonance indicated by the static nucleon approximation, F. Salzman and G. Salzman (see footnote 5).

[11] W. D. Walker, Phys. Rev. **108**, 872 (1957); G. Maenchen, W. Fowler, W. Powell, and R. Wright, Phys. Rev. **108**, 850 (1957).

[12] *Proceedings of the Tenth Annual Conference on High-Energy Nuclear Physics at Rochester, 1960* (Interscience Publishers, New York, to be published).

$\gtrsim\sqrt{2}M$. By carrying out the W and W' integrations in Eq. (1), with the help of Eqs. (2) and (4), one obtains

$$\frac{d\sigma}{d(\Delta^2)}\approx\frac{3}{16\pi^3}[(\sigma_{\pi N})_{\rm av}]^2\frac{\Delta^4}{(\Delta^2+m_\pi^2)^2}\ln\left(\frac{\Delta U}{M^2}\right), \quad (4)$$

where we have taken $p_{iU}\approx U/2$, and assumed that $4\ln[\Delta U/(M^2)]\gg 1$. The logarithmic dependence on U of the cross section has also been obtained by Berestetski and Pomeranchuk, and by Gribov.[13] For $E_{iL}=10^4$ Bev ($U\approx 140$ Bev), at which energy these approximations should be valid, and with the assumption that $(\sigma_{\pi N})_{\rm av}\approx 30$ mb, one obtains for the total contribution to the $N-N$ cross section from Δ^2 below $(2m_\pi)^2$ a value of about 2 mb. Of course, this is only a first estimate, but it already indicates that a cross section of several millibarns can be expected from very small values of Δ^2.

[13] Reported by A. P. Rudik, reference 12.

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We wish to thank Professor A. Bohr, Professor U. Fano, and Professor V. Weisskopf for stimulating discussions of parts of this work during the 1960 Summer Institute of Theoretical Physics at Boulder, where the general ideas of the single-boson-exchange interaction were presented by one of us (G.S.). We have also benefited from discussions with Professor N. Dobrotin, Professor J. Gierula, Professor M. Miesowicz, and Professor V. Veksler at the Tenth Annual Conference on High-Energy Nuclear Physics at Rochester.

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Cross Section and Decay Asymmetries of Neutral V Particles Produced in Copper by π^- Mesons*

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The charged mode of decay of 189 Λ and 77 K^0 particles produced by 1.12-Bev/c π^- mesons in the copper wall of the Berkeley 10-inch liquid hydrogen bubble chamber have been analyzed and their asymmetries calculated. The asymmetries have been examined for several interesting subgroups of the sample and for several directions of quantization. A fore-aft asymmetry in Λ decay has previously been reported in cosmic-ray interactions with complex nuclei and in some machine events of a similar nature. This asymmetry has recently been reviewed and summarized by Salmeron and Zichichi. They find a 3-standard-deviation effect, $\langle\alpha P_{\text{fore-aft}}\rangle_{\rm av}=-0.56\pm0.15$. In contrast, the asymmetries found here for both Λ and K^0 are consistent with the assumption of the conservation of parity in the production process π^-+p (in nucleus) $\rightarrow\Lambda+K^0$.

The cross section for the associated production of ΛK^0 and Σ^0K^0 has been determined as 10.5 ± 0.9 mb per copper nucleus. This value is somewhat larger than that expected on the assumption that the nucleons interact in first approximation as free nucleons possessing Fermi momentum according to an $A^{\frac{2}{3}}$ scaling of the hydrogen cross sections. This scaling predicts a value for this cross section of 7.4 ± 0.8 mb/nucleus.

Subsequent interactions of the hyperons with another nucleon in the same nucleus in which they were produced are frequent. A lower limit for the probability of scattering of the Λ can be set at 0.23 ± 0.07. This is in accordance with the observed $\Lambda+p$ cross sections.

INTRODUCTION

If parity is not conserved in strong interactions, then the final-state particles may have a component of polarization in the plane of production. An asymmetry in this plane may result from the parity-nonconserving decay of unstable particles.[1] In particular, with respect to the line of flight of the particle, a fore-aft asymmetry is observed. Such an asymmetry has been observed in the decay of Λ's produced in *complex* nuclei by cosmic rays,[2] and in corresponding machine experiments.[3] The situation has been reviewed by Salmeron and Zichichi.[4] They have applied criteria which supposedly eliminate all biases and give an over-all result for $\langle\alpha P_{\text{fore-aft}}\rangle_{\rm av}$ of -0.56 ± 0.15, a 3-standard-deviation effect. No such asymmetries in the plane of production have been found at Berkeley in $\pi^-+p\rightarrow\Lambda+K^0$ hydrogen events.[1] Many V particles which were produced in the copper wall of the bubble chamber are also observed. An unbiased sample of these has been analyzed in a search for possible asymmetries.

* Work done under the auspices of the U. S. Atomic Energy Commission.

[1] F. S. Crawford, M. Cresti, M. L. Good, F. T. Solmitz, and M. L. Stevenson, Phys. Rev. Letters **1**, 209 (1958); F. Eisler, R. Plano, A. Prodell, N. Samios, M. Schwartz, J. Steinberger, P. Bassi, V. Borelli, G. Puppi, G. Tanaka, P. Woloschek, V. Zoboli, M. Conversi, P. Franzini, I. Mannelli, R. Santangelo, V. Silvestrini, D. A. Glaser, C. Graves, and M. L. Perl, Phys. Rev. **108**, 1353 (1957).

[2] See, for example, W. A. Cooper, H. Filthuth, L. Montanet, J. A. Newth, G. Petrucci, R. A. Salmeron, and A. Zichichi, Nuovo cimento **8**, 471 (1958); R. Armenteros, Proceedings of the Bagnères Congress on Cosmic Radiation, 1953 (unpublished).

[3] H. Blumenfeld, W. Chinowsky, and L. M. Lederman, Nuovo cimento **8**, 296 (1958), and references given by them.

[4] R. Salmeron and A. Zichichi, Nuovo cimento **11**, 461 (1959).