Two-Pion Exchange Mechanism in K^+N Scattering*†

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The exchange of two pions resonating in the $T=1$, $J=1$ state between the K^+-meson and nucleon in K^+N scattering is considered in the double dispersion representation to account for the energy dependence of the $J=\frac{1}{2}$ amplitudes. The result here is in qualitative agreement with that of Ferrari *et al.* obtained for K^-N scattering.

THOUGH the experimental information available at present on K^+N scattering may still be considered as meager, there are nevertheless certain conclusions one can draw unambiguously from the existing data.[1] The K^+p cross section is almost isotropic and constant over a wide range of energy (laboratory kinetic energy of K meson $\lesssim 300$ Mev), and the dominant s-wave phase shift is known to be repulsive from the Coulomb interference.[2] The phenomenological analysis of Rodberg and Thaler[3] indicates that the scattering length and the effective range are both positive in this state. For K^+n one can infer that the amplitude in the isotropic spin state, $T=0$, is small compared with that in the $T=1$ state and changes appreciably with energy.

We have investigated the K^+N scattering amplitudes in the double dispersion representation. The analytic properties of the partial wave amplitudes for this process have been reported previously.[4] Lack of crossing symmetry in the present case is an additional complexity which was not encountered in the $\pi\pi$ and πN problems.

In the present approach we shall represent by a phenomenological parameter (essentially the scattering length) the effects of the short-range forces associated with the crossed process $N+\bar{K}\rightarrow N+\bar{K}$ and higher mass intermediate states in the channel $K+\bar{K}\rightarrow N+\bar{N}$, which are expected to be fairly energy insensitive. The longest range force, due to the exchange of two pions between the nucleon and K meson, is then assumed most responsible for the energy dependence of the K-matrix elements ($k \cot\delta$). Recent advances in the study of the $\pi\pi$ interaction[5] and the electromagnetic structure of the nucleon[6] enable us to estimate the strength of this force. Here we adopt the point of view that the virtual process $\pi+\pi\rightarrow K+\bar{K}$ below the physical threshold is strongly enhanced in the $T=1$, $J=1$ state because of the resonance in the initial state, and that the resonance is in fact described by the parameters of Frazer and Fulco[6] deduced from the electromagnetic structure of the nucleon. Our consideration here is very similar to that of Ferrari, Frye, and Pusterla[7] applied to K^-N scattering. We are particularly interested in the agreement or disagreement of the conclusions drawn from the two cases. The possibility of the strong 3-pion exchange mechanism is disregarded in the subsequent discussion.

We shall deal with the amplitude $g_0^{(I)}(W)$ defined by[8]

$$g_0^{(I)}(W)=\frac{W^2}{W+m+m_K}\frac{\exp(i\delta_{0+}^{(I)})\sin\delta_{0+}^{(I)}}{k}. \quad (1)$$

In studying the s-wave phase shift we neglect the left-hand branch cut (in the W plane) associated with the $p_{\frac{1}{2}}$ state[9] completely. The branch cut associated with the 2-pion exchange ($\pi\pi$ cut) starts at $W=(m^2-\mu^2)^{\frac{1}{2}}+(m_K^2-\mu^2)^{\frac{1}{2}}$ and extends to the left.

The discontinuity across the $\pi\pi$ cut is computed from the unitary condition neglecting all other channels. We assume that because of the $\pi\pi$ resonance in the $T=1$, $J=1$ state, the matrix element $\pi+\pi\rightarrow K+\bar{K}$ can be approximated well by that of the $T=1$, $J=1$ state[10]:

$$\langle\bar{K}(q_2)|j_K|2\pi(\kappa_1,\alpha;\kappa_2,\beta)^{\text{in}}\rangle \approx\frac{1}{(8\kappa_{10}\kappa_{20}q_{20})^{\frac{1}{2}}}\frac{1}{2}[\tau_\beta,\tau_\alpha][2\xi F_\pi(t)][3\kappa q P_1(\hat{\kappa}_2\cdot\hat{q}_2)] \quad (2)$$

where $F_\pi(t)$ is the meson form factor of Frazer and Fulco[6] and ξ is taken to be a real, constant parameter. The contribution of the $\pi\pi$ cut computed in this manner

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[1] For comprehensive bibliography, see, for example: D. F. Davis, K. Kwak, and M. F. Kaplon, Phys. Rev. **117**, 846 (1960); O. R. Price, D. H. Stork, and H. K. Ticho, Phys. Rev. **119**, 1702 (1960).
[2] T. F. Kycia, L. T. Kerth, and R. G. Baender, Phys. Rev. **118**, 553 (1960).
[3] L. S. Rodberg and R. M. Thaler, Phys. Rev. Letters **4**, 372 (1960).
[4] S. W. MacDowell, Phys. Rev. **116**, 774 (1959). See also W. R. Frazer and J. R. Fulco, Phys. Rev. **119**, 1420 (1960).
[5] G. F. Chew and S. Mandelstam, University of California Radiation Laboratory Report UCRL-9126, 1960 (unpublished).
[6] W. R. Frazer and J. R. Fulco, Phys. Rev. Letters **2**, 365 (1959); Phys. Rev. **117**, 1609 (1960).
[7] F. Ferrari, G. Frye, and M. Pusterla, Phys. Rev. Letters **4**, 615 (1960).
[8] Our notation here follows that of Frazer and Fulco, reference 4.
[9] See Frazer and Fulco, reference 4.
[10] The quantity $B_1(t)=\xi(t)F_\pi(t)$ satisfies the dispersion relation with branch cuts from $-\infty$ to 0 and $4\mu^2$ to ∞; it has the phase of the $\pi\pi$ phase shift of the $T=1$ and $J=1$ state for $4\mu^2\leq t\leq 16\mu^2$. See B. Lee, Phys. Rev. **120**, 325 (1960); and thesis (unpublished).

can be approximated well in the physical region by a pole,

$$g_0^{(I)\pi\pi}(W)=\beta_{I,1}\frac{\xi R}{W-9.8\mu},\quad R=1.02\mu^3, \tag{3}$$

where $\beta_{1,1}$ is the element of the crossing matrix such that $\beta_{1,1}=\frac{1}{2}$, $\beta_{0,1}=-\frac{3}{2}$.

The resulting approximate dispersion equation:

$$\begin{aligned} g_0^{(I)}(W)&=g_0^{(I)}(W_0)\\ &+\frac{1}{\pi}\int_{W_0}^{\infty}dW'\frac{\mathrm{Im}g_0^{(I)}(W')}{(W'-W)(W'-W_0)}(W-W_0)\\ &+\beta_{I,1}R\xi\left(\frac{1}{W-9.8\mu}-\frac{1}{W_0-9.8\mu}\right),\\ &\qquad\qquad\times W_0=m+m_K, \end{aligned} \tag{4}$$

is solved by the standard N/D method.[11] By requiring that the solution to Eq. (4) agree at the point $W=W_0$ with the effective-range relation given by Rodberg and Thaler for the state $T=1$, we fix the two constants in Eq. (4), and in particular find $\xi=0.5/\mu^2$. In Fig. 1, $k\cot\delta_{0+}^{(1)}$ is plotted for $\xi=0.5$ and $1.0/\mu^2$ and compared with the Rodberg-Thaler result. For the $T=0$ state, we take $a^{(0)}$ (scattering length)$=0$ and $-0.05/\mu$ with $\xi=0.5/\mu^2$. The results are plotted in Fig. 2. Since the exact zero-energy limit of $k^{-1}\tan\delta_{0+}^{(0)}$ is unknown, it is possible to reproduce either a repulsive or an attractive phase shift by slightly adjusting $g_0^{(0)}(W_0)$.

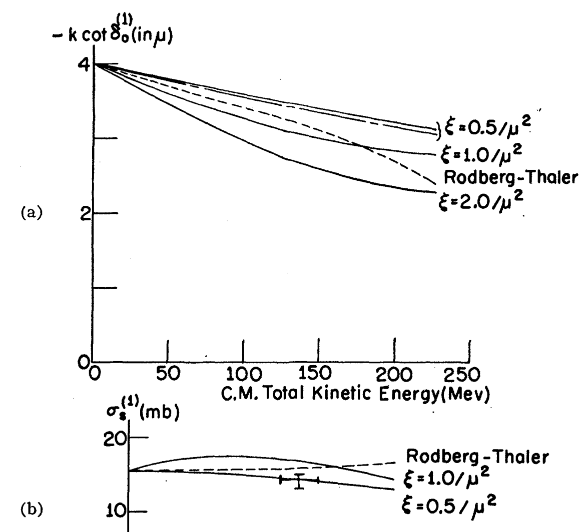

FIG. 1. (a) $-k\cot\delta_0^{(1)}$ vs $(W-W_0)$; the solid lines are obtained by the subtraction method; the dot-dash line is obtained by representing the short-range interactions by a pole at $W=0$; these are compared with the Rodberg-Thaler effective-range relation (dotted line). (b) $\sigma_s^{(1)}$(nuclear) vs kaon kinetic energy in the lab system (E_L). The symbols are the same as in Fig. 1; the cross represents the experimental value of Kyeia *et al.* at $E_L=225\pm25$ Mev.

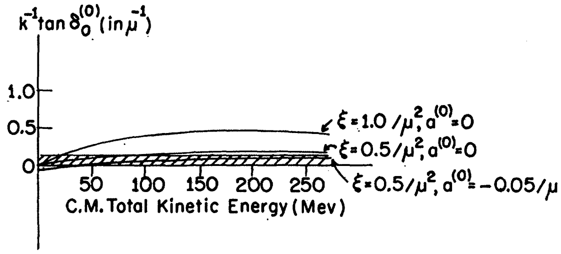

FIG. 2. $k^{-1}\cot\delta_0^{(0)}$ (in μ^{-1}) vs $(W-W_0)$. The shaded area represents the Rodberg-Thaler scattering length with uncertainty.

We may also try to represent the short-range interaction by a pole with a phenomenological residue which is to be determined by fitting the scattering length. Because the shapes of the singularities are symmetrical around the origin in the W plane, we place the pole at the origin, hoping that this represents approximately the short-range effects. If we assume that the dominant effects of the short-range forces come from the singularities on and inside the circle $|W|=(m^2-m_K^2)^{\frac{1}{2}}$, then one expects that this pole represents the effects of these singularities on the left-hand physical branch cut associated with the $p_{\frac{1}{2}}$ state as well as it does on the right-hand physical cut associated with the $s_{\frac{1}{2}}$ state, at least as regards the sign and order of magnitude. For this purpose it is more convenient to use the amplitude $h_0^{(I)}(W)$ of Frazer and Fulco.[4] The approximate dispersion relation takes the form

$$h_0^{(I)}(W)=\frac{\gamma_s^{(I)}}{W}+\frac{\xi\beta_{I,1}\gamma_{\pi\pi}}{W-9.8\mu}+\frac{1}{\pi}\int_{W_0}^{\infty}dW'\frac{\mathrm{Im}h_0^{(I)}(W')}{W'-W}, \tag{5}$$

where $\gamma_{\pi\pi}=0.15\mu^2$.

In order to fit the scattering lengths, $a^{(1)}=0.25/\mu$ and $a^{(0)}=0$, one requires, for $\xi=0.5/\mu^2$,

$$\gamma_s^{(1)}=-98,\quad \gamma_s^{(0)}=3. \tag{6}$$

For given scattering length and ξ, the results of the subtraction method and of the present one practically coincide. As an illustration, the result of the present method with $\gamma_s^{(1)}=-98$, $\xi=0.5/\mu^2$ is plotted and compared with the result of the subtraction method in Fig. 1.

The result $\xi=0.5/\mu^2$ means that, in the framework of our model, the longest range force due to the 2-pion exchange is attractive in the $T=1$ state, and repulsive and 3 times as large in the $T=0$ state. The value $\xi=0.5/\mu^2$ is to be compared with $\xi=1.1/\mu^2$ obtained from K^-N scattering by Ferrari *et al.*[7] In view of the crude models used in both cases, the agreement in sign and order of magnitude is to be noted. In the $T=1$, $s_{\frac{1}{2}}$ state, the short-range force is weakly attractive (this force is too weak to sustain a bound state).

[11] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960).

TABLE I. The contributions of singularities to $f_{1-}(W)$ at the threshold; superscripts, s, $\pi\pi$, and H refer, respectively, the short-range, $\pi\pi$-cut, and the static Chew-Low type hyperon pole contributions.

I	$k^{-2}f_{1-}{}^{s}(W_0)$	$k^{-2}f_{1-}{}^{\pi\pi}(W_0)$	$k^{-2}f_{1-}{}^{H}(W_0)$
0	0	$-0.12/\mu^3$	$-\frac{0.065}{\mu}\frac{1}{2m_K^2}\left(3f_\Sigma^2-f_\Lambda^2\right)$
1	$-0.07/\mu^3$	$+0.04/\mu^3$	$-\frac{0.065}{\mu}\frac{1}{2m_K^2}\left(f_\Sigma^2+f_\Lambda^2\right)$

Assuming that the pole γ_s/W represents approximately the effects of the short-range force in the $p_{\frac{1}{2}}$ state as to the sign and order of magnitude, we can make estimates of the contributions of forces of different ranges to the $k^{-2}f_{1-}{}^{(I)}$. These are tabulated in Table I. We first note the numerical values quoted in Table I are all small. Especially the contributions from the hyperon static poles are negligible even with $f_{\Lambda,\Sigma}{}^2 \simeq 0.1$.[12] Based on Table I, we are in a position to make certain predictions on the $p_{\frac{1}{2}}$ amplitudes which are as yet uncertain experimentally.

(1) In the $T=1$ state, δ_{1-} is likely to be negative (repulsive). In this state there is a large cancellation of the short- and long-range forces, the former slightly dominating. Since the net repulsion comes from the short-range force, whose effect is relatively energy insensitive, $k^{-3}\exp(i\delta_{1-})\sin\delta_{1-}$ will not depend strongly on energy, and therefore stays small.

(2) In the $T=0$ state, δ_{1-} will be negative and small, but larger than that of the $T=1$ state. The repulsion comes almost entirely from the long-range force. Therefore the energy variation of $k^{-3}\exp(i\delta_{1-})\sin\delta_{1-}$ is expected to be appreciable. Our model therefore favors the solution D of the analysis of Chinowsky *et al.*[13]

Recently a new set of parameters for the $\pi\pi$ resonance has been proposed.[14] We believe that the qualitative conclusions of the present paper will remain valid even if these parameters are used. The value of ξ will however be modified considerably. A consistent and simultaneous treatment of K^+N scattering and K^-N reactions with this new set of the $\pi\pi$ resonance parameters is in progress.

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[12] The pseudoscalar coupling of $N-K-Y$, $Y=\Lambda,\Sigma$, is assumed. f_Λ, f_Σ are the unrationalized pseudovector coupling constants.

[13] W. Chinowsky *et al.*, Proceedings of the Tenth Annual International Rochester Conference on High-Energy Nuclear Physics, 1960 (to be published).

[14] J. Bowcock, W. N. Cottingham, and D. Lurie, Phys. Rev. Letters, **5**, 386 (1960).