PHYSICAL REVIEW VOLUME 121, NUMBER 6 MARCH 15, 1961

Analysis of Gamma-Gamma Polarization-Directional Correlations Involving Multipole Mixtures*

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(Received August 19, 1960)

A method is described for the graphical analysis of the gamma-gamma polarization-directional correlation when one or both gamma rays involve a mixture of dipole and quadrupole radiation. The case in which the polarization of the mixed gamma ray is observed is treated in detail.

I. INTRODUCTION

A MEASUREMENT of the γ-γ directional correlation function $W(\theta)$ gives information regarding the spins of the nuclear levels and the multipolarities of the gamma rays involved. Often, however, this information alone does not uniquely determine these quantities. A measurement of the γ-γ polarization-directional correlation function $W(\theta,\varphi)$ provides an additional piece of information with which the assignments can be made unique. It also will give the relative parities of the levels involved in the cascade.

The experimental situation, as it was originally proposed by Metzger and Deutsch,[1] is shown in Fig. 1. The angle between the detectors θ is ordinarily fixed at $\pi/2$. The scintillation detector A is not sensitive to the polarization of γ_1. The polarization of the radiation γ_2 is measured by the polarization-sensitive detector composed of the three scintillation counters B, C, and D. If the polarization is parallel, Compton scattering from counter B into counter D will be favored over scattering into counter C. The polarization-directional correlation experiment therefore consists in a comparison of the rate for parallel polarization $W(\varphi=\pi/2)$ which involves a triple coincidence between A, B, and D, with the rate for perpendicular polarization, $W(\varphi=0)$, which involves a triple coincidence between A, B, and C.

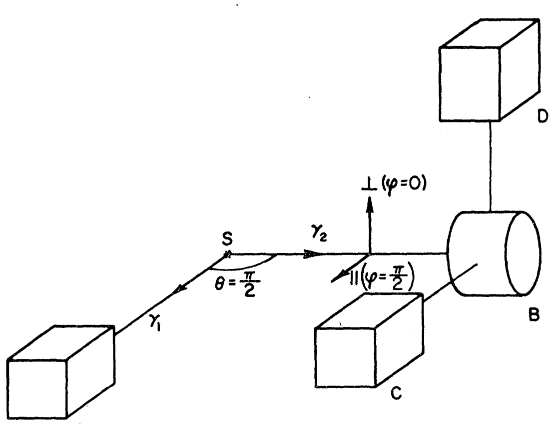

FIG. 1. Schematic diagram of the polarization-directional correlation experiments.

The theoretical results of Biedenharn and Rose[2] for the γ-γ polarization-directional correlation have been rewritten using the F coefficients of Ferentz and Rosenzweig.[3] The relationship between these results and single-transition mixture curves for the γ-γ directional correlation[4,5] has been demonstrated in order to facilitate the analysis of experimental data.

II. POLARIZATION-DIRECTIONAL CORRELATION WITH POLARIZATION OF THE MIXED RADIATION OBSERVED

Denote j_1, j, and j_2 as the initial, intermediate, and final momenta of the gamma cascade. Let the transition between j_1 and j be a mixture of 2^{L_1} and 2^{L_1+1} poles. Denote the multipolarity of the second step of the cascade by L_2 (pure radiation). The polarization-directional correlation function will be denoted by $W(\theta,\varphi)$ for the present case in which the polarization of the mixed gamma ray is observed. $W(\theta,\varphi)$ is of the form[2]:

$$W(\theta,\varphi)=W_1(\theta,\varphi)+\delta^2W_2(\theta,\varphi)+2\delta W_3(\theta,\varphi), \quad (1)$$

where δ^2 is the ratio of the intensity of the 2^{L_1+1} pole to that of the 2^{L_1} pole. The functions $W_1(\theta,\varphi)$ and $W_2(\theta,\varphi)$ correspond to the pure cascades $j_1(L_1)j(L_2)j_2$ and $j_1(L_1+1)j(L_2)j_2$, respectively. The function $W_3(\theta,\varphi)$ is the interference term between L_1 and L_1+1. These can be written as follows:

$$\begin{aligned}
W_1(\theta,\varphi)&=\sum_{K\text{ even}} F_K(L_1L_1j_1j)\\
&\qquad\times F_K(L_2L_2j_2j)\mathcal{P}_K(L_1L_1;\theta\varphi),\\
W_2(\theta,\varphi)&=\sum_{K\text{ even}} F_K(L_1+1L_1+1j_1j) \qquad (2)\\
&\qquad\times F_K(L_2L_2j_2j)\mathcal{P}_K(L_1+1L_1+1;\theta\varphi),\\
W_3(\theta,\varphi)&=\sum_{K\text{ even},\neq 0} F_K(L_1L_1+1j_1j)\\
&\qquad\times F_K(L_2L_2j_2j)\mathcal{P}_K(L_1L_1+1;\theta\varphi),
\end{aligned}$$

* Supported by the U. S. Atomic Energy Commission.
[1] F. Metzger and M. Deutsch, Phys. Rev. **78**, 551 (1950).
[2] L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953).
[3] M. Ferentz and N. Rosenzweig, Argonne National Laboratory Report ANL-5324, 1955 (unpublished).
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[5] R. G. Arns and M. L. Wiedenbeck, Phys. Rev. **111**, 1631 (1958).

where

$$\mathcal{P}_K(LL';\theta\varphi)=P_K(\cos\theta)+(-1)^{\sigma(L')}\left[\frac{(K-2)!}{(K+2)!}\right]^{\frac{1}{2}} \times\frac{C(LL'K;11)}{C(LL'K;1-1)}\cos 2\varphi P_K^2(\cos\theta). \quad (3)$$

The coefficients $F_K(LL'jj')$ are the F coefficients defined by Ferentz and Rosenzweig.[3] The term $(-1)^{\sigma(L')}$ is equal to -1 if L' is $M1$ or $E2$ or to $+1$ if L' is $E1$ or $M2$. The summation over K in (2) runs from zero for $W_1(\theta,\varphi)$ and $W_2(\theta,\varphi)$ and from 2 for $W_3(\theta,\varphi)$ to the least of $2j$, $2(L_1+1)$ or $2L_2$. The $C(LL'K;11)$ and $C(LL'K;1-1)$ are Wigner coefficients. φ is the angle between the polarization vector and the normal to the plane of the two γ rays, and $P_K^2(\cos\theta)$ is the associated Legendre polynomial of order 2.

It is now desirable to relate the above result to the ordinary γ-γ directional correlation which is assumed to be known experimentally. The γ-γ directional correlation function, $W(\theta)$, is given by

$$W(\theta)=W_1(\theta)+\delta^2W_2(\theta)+2\delta W_3(\theta), \quad (4)$$

where the functions $W_i(\theta)$ have a similar meaning to the $W_i(\theta,\varphi)$:

$$W_1(\theta)=\sum_{K \text{ even}} F_K(L_1L_1j_1j)F_K(L_2L_2j_2j)P_K(\cos\theta),$$
$$W_2(\theta)=\sum_{K \text{ even}} F_K(L_1+1L_1+1j_1j)\times F_K(L_2L_2j_2j)P_K(\cos\theta), \quad (5)$$
$$W_3(\theta)=\sum_{K \text{ even},\neq 0} F_K(L_1L_1+1j_1j)F_K(L_2L_2j_2j)P_K(\cos\theta),$$

For the special cases of $\varphi=0$ and $\pi/2$, $W(\theta,\varphi)$ reduces to the form:

$$W(\theta,\ \varphi=0)=W(\theta)-f(\theta),$$
$$W(\theta,\ \varphi=\pi/2)=W(\theta)+f(\theta). \quad (6)$$

Now define a $J(\theta)$:

$$J(\theta)=\frac{W(\theta,\ \varphi=\pi/2)-W(\theta,\ \varphi=0)}{W(\theta,\ \varphi=\pi/2)+W(\theta,\ \varphi=0)}=\frac{f(\theta)}{W(\theta)}. \quad (7)$$

For the special case of a dipole-quadrupole mixture:

$$J(\theta)=-\tfrac{1}{2}(-1)^{\sigma(L_1)}\{P_2^2(\cos\theta)F_2(L_2L_2j_2j)[F_2(11j_1j)+\delta^2F_2(22j_1j)-\tfrac{2}{3}\delta F_2(12j_1j)] -P_4^2(\cos\theta)(\tfrac{1}{6}\delta^2)F_4(L_2L_2j_2j)F_4(22j_1j)\} \times\{1+\delta^2+P_2(\cos\theta)F_2(L_2L_2j_2j)[F_2(11j_1j)+\delta^2F_2(22j_1j)-2\delta F_2(12j_1j)] +P_4(\cos\theta)\delta^2F_4(22j_1j)F_4(L_2L_2j_2j)\}^{-1}. \quad (8)$$

Now define Q_ν as the quadrupole content of the νth (mixed) transition:

$$Q_\nu=\delta_\nu^2/(1+\delta_\nu^2), \quad (9)$$

and

$$a_K^{(\nu)}=F_K(L_\nu L_\nu j_\nu j),$$
$$b_K^{(\nu)}=2F_K(L_\nu L_\nu+1j_\nu j), \quad (10)$$
$$c_K^{(\nu)}=F_K(L_\nu+1L_\nu+1j_\nu j).$$

Equation (8) becomes: (here $Q_1\equiv Q$)

$$J(\theta)=-\tfrac{1}{2}(-1)^{\sigma(L_1)}\{P_2^2(\cos\theta)a_2^{(2)}[(1-Q)a_2^{(1)} -\tfrac{1}{3}[Q(1-Q)]^{\frac{1}{2}}b_2^{(1)}+Qc_2^{(1)}]-\tfrac{1}{6}P_4^2(\cos\theta)Qc_4^{(1)}a_4^{(2)}\} \times\{1+P_2(\cos\theta)a_2^{(2)}[(1-Q)a_2^{(1)}+[Q(1-Q)]^{\frac{1}{2}}b_2^{(1)} +Qc_2^{(1)}]+P_4(\cos\theta)Qc_4^{(1)}a_4^{(2)}\}^{-1}. \quad (11)$$

$J(\theta)$ may now be related to the single-transition mixture curves for the γ-γ directional correlation.[4] These curves are plots of $A_K^{(\nu)}$ vs Q_ν:

$$A_2^{(\nu)}=(1-Q_\nu)a_2^{(\nu)}+[Q_\nu(1-Q_\nu)]^{\frac{1}{2}}b_2^{(\nu)}+Q_\nu c_2^{(\nu)}, \quad (12)$$
$$A_4^{(\nu)}=Q_\nu c_4^{(\nu)}. \quad (13)$$

A straight line, intersecting the ellipse at $Q=0$ and $Q=1$, must be added to the diagram for the present analysis. The equation of this line is

$$\mathcal{A}_2^{(\nu)}=Q_\nu(c_2^{(\nu)}-a_2^{(\nu)})+a_2^{(\nu)}. \quad (14)$$

Using this definition and specializing to the case of $\theta=\pi/2$, Eq. (11) becomes ($v=1$ in present case)

$$J(\pi/2)=(-1)^{\sigma(L_1)}\frac{A_2-4\mathcal{A}_2^{(1)}A_2^{(2)}-(5/4)A_4}{2-A_2+(3/4)A_4}. \quad (15)$$

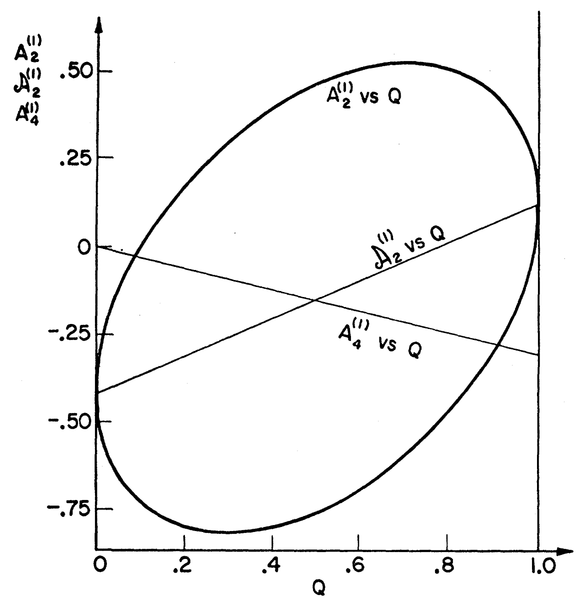

FIG. 2. Illustration of the curves required for graphical analysis of a γ-γ polarization-directional correlation in the case where the polarization of the mixed transition is observed. This method requires that the experimental γ-γ directional correlation coefficients be known.

Using the appropriately chosen curves, $\mathcal{A}_2^{(1)}$ can now be deduced from the experimentally determined directional correlation coefficients, A_2 and A_4 (see Sec. III of reference 5). This is illustrated in Fig. 2. The value of $J(\pi/2)$ to be expected in a polarization-directional correlation experiment then follows at once from Eq. (15). The term $(-1)^{\sigma(L_1)}$ is equal to $+1$ if the dipole is electric, or to -1 if the dipole is magnetic.

III. DOUBLY-MIXED CASCADES

If both steps of the cascade are a mixture of dipole and quadrupole radiation, the result of the polarization-directional correlation experiment is again given by Eq. (15). However, the partial coefficients $A_2^{(2)}$ and $\mathcal{A}_2^{(1)}$ must now be determined from a double-mixture analysis of the γ-γ directional correlation data. This method is treated in detail in Sec. IV of reference 5 and will not be repeated here.

IV. PURE RADIATIONS OR SINGLE-MIXED CASCADES WITH POLARIZATION OF THE PURE RADIATION OBSERVED

If both steps of the cascade are pure radiation or if the polarization of the pure radiation (in a singly-mixed cascade) is observed, the result of the polarization-directional correlation is given as follows:

(a) If one or both gamma rays is a pure dipole,

$$J(\pi/2)=(-1)^{\sigma(L)}\frac{-3A_2}{2-A_2}. \tag{16}$$

(b) If both gamma rays are pure quadrupole, or if the mixed gamma ray has an appreciable quadrupole content,

$$J(\pi/2)=(-1)^{\sigma(L)}\frac{-3A_2-(5/4)A_4}{2-A_2+(3/4)A_4}. \tag{17}$$

Here again $(-1)^{\sigma(L)}$ is equal to $+1$ if the radiation (the polarization of which is observed) is of type $E1$ or $M2$ and to -1 if it is of type $M1$ or $E2$.

PHYSICAL REVIEW VOLUME 121, NUMBER 6 MARCH 15, 1961

Neutron Capture Gamma-Ray Spectra of the Nickel Isotopes*

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(Received September 14, 1960)

The spectra of gamma radiation following pile-neutron capture in separated isotopes of nickel have been measured with a three-crystal scintillation pair spectrometer as well as with a single-crystal scintillation counter. Energies and intensities of the gamma rays from each isotope are reported. Each of the lines observed with natural nickel has been assigned to $Ni^{58}(n,\gamma)Ni^{59}$, $Ni^{60}(n,\gamma)Ni^{61}$, or $Ni^{62}(n,\gamma)Ni^{63}$. In some cases, gamma rays of nearly identical energies are found in two, or in all three, isotopes. Additional gamma rays not resolved in the natural nickel spectrum appear in $Ni^{58}(n,\gamma)Ni^{59}$ at 7.65 Mev, and in $Ni^{60}(n,\gamma)Ni^{61}$ at 7.65 Mev, and in $Ni^{60}(n,\gamma)Ni^{61}$ at 4.70 and 5.55 Mev. The spectrum of $Ni^{62}(n,\gamma)Ni^{63}$ is dominated by a strong line at 6.80 Mev, presumably the ground-state transition.

INTRODUCTION

THE gamma-ray spectrum following thermal-neutron capture in natural nickel has been investigated by Ad'yasevich *et al.*[1] and Kinsey and Bartholomew[2] using external conversion spectrometers, and by Braid[3] using a Compton scintillation spectrometer. In the present work, natural nickel in the form of nickel oxide powder and Ni^{58}, Ni^{60}, and Ni^{62} in the form of isotopically enriched metallic bars and lumps were irradiated in a neutron beam at the Ford Nuclear Reactor at the University of Michigan, and the gamma-ray spectra were detected by means of a single scintillation crystal and a three-crystal scintillation spectrometer.[4] The single crystal was used in the 0–2.0 Mev energy range and the three-crystal spectrometer in the 1.5–10.0 Mev range.

EXPERIMENTAL CONSIDERATIONS

Geometry

The nickel samples, whose masses and isotopic constitutions are indicated in Table I, were placed in a beam of 2×10^6 neutrons/sec approximately 1 cm^2 in area. The Ni^{60} sample was not large enough to intercept the entire beam, requiring a correction in the intensity measurements. Each sample was thick enough to capture or scatter nearly all of the neutrons impinging upon it.

* Work supported by a contract with the U. S. Atomic Energy Commission.
† National Science Foundation Predoctoral Fellow.
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