Energy Levels in Zr^{91} Excited by the (d,p) Reaction*

R. L. PRESTON,† H. J. MARTIN, JR.,‡ AND M. B. SAMPSON
Indiana University, Bloomington, Indiana

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Eleven energy levels in Zr^{91} were excited. The excitation energies of the levels are 0, 1.22, 2.07, 2.19, 2.56, 2.84, 3.05, 3.25, 3.45, 3.65, and 3.87 Mev. Angular distributions were measured for the first four levels and assignments were made using the shell-model and the Butler stripping theory. The assignments are $2d_{5/2}$ for the ground state, $3s_{1/2}$ for the 1.22-Mev state, $2d_{3/2}$ for the 2.07-Mev state, and $1g_{7/2}$ for the 2.19-Mev state. The application of the Butler theory to the data gave reasonable and useful fits. The ground-state Q value was measured and is 5.02 ± 0.03 Mev.

INTRODUCTION

THE work of several experimental and theoretical groups during the past few years indicates that stripping reactions are a useful tool to study the spectroscopy of heavier nuclei. Measurements on Zn by Shull and Ewlyn[1] showed forward peaks that were in agreement with Butler calculations. Work in the Fe region by Schiffer *et al.*[2] demonstrated that the (d,p) reaction will predominantly excite single-particle states. Cohen has extended Schiffer's findings to heavier nuclei[3] and has effectively extracted relative reduced widths to study nuclear structure.[4] A similar study of relative reduced widths in Pb has been made by McEllistrem *et al.*[5] Extentions of the stripping theory to consider distortions in heavier nuclei have been made by a number of authors.[6] The distorted wave calculations by

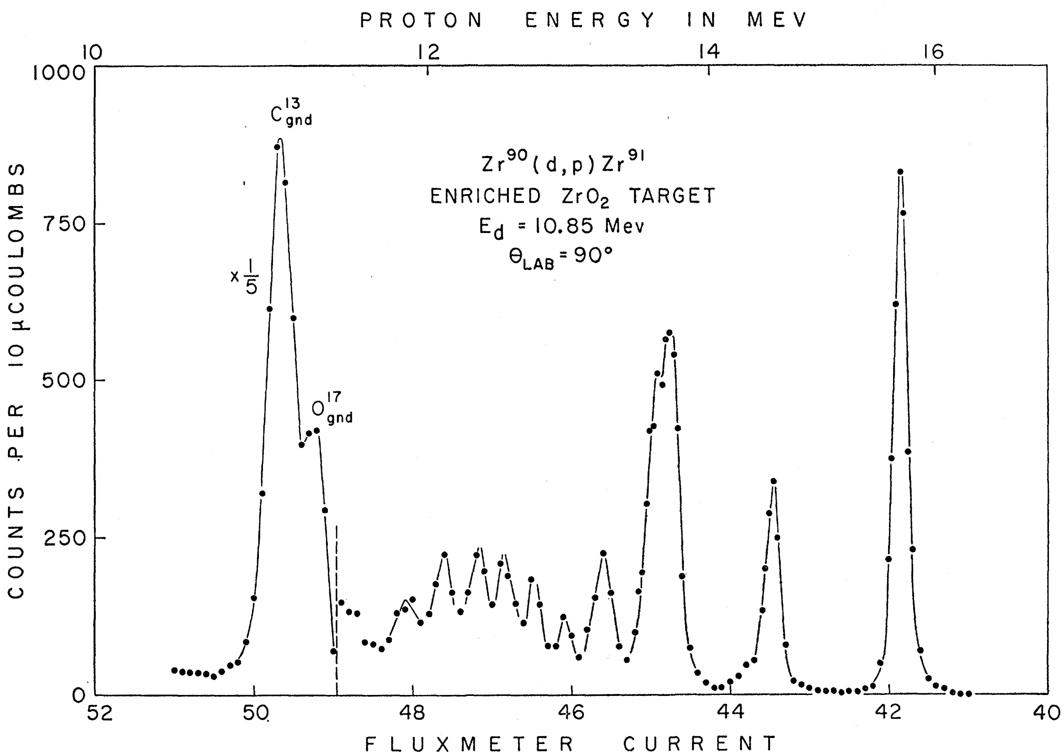

FIG. 1. A proton momentum spectrum for the ZrO_2 target. Angular distributions were measured for the two highest energy groups and for the double group.

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† Present address: 35 Bedford Park Avenue, Richmond Hill, Ontario, Canada.
‡ Now on leave at the Brookhaven National Laboratory, Upton, New York.
[1] F. B. Shull and A. J. Elwyn, Phys. Rev. **112**, 1667 (1958).
[2] J. P. Schiffer, L. L. Lee, and B. Zeidman, Phys. Rev. **115**, 427 (1959).
[3] B. L. Cohen and R. E. Price, Nuclear Phys. **17**, 129 (1960).
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[5] M. T. McEllistrem, H. J. Martin, D. W. Miller, and M. B. Sampson, Phys. Rev. **111**, 1636 (1958).
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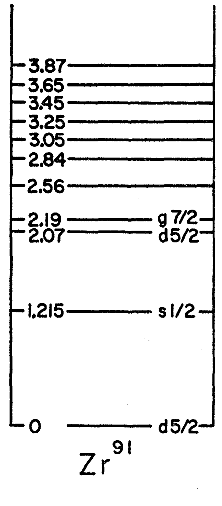

FIG. 2. Levels excited in Zr^{91} by the (d,p) reaction. The spin assignments for the first four states are discussed in the text. Energies are in Mev.

Tobocman[7] are of particular interest because they show that angular distributions, even when strongly disturbed by nuclear and Coulomb effects, may still be used to determine l values.

The work presented in this paper was undertaken with the hope that the properties of the stripping reaction would be of particular value in a study of the zirconium isotopes. Zr^{90} has 40 protons and a closed shell of 50 neutrons. The (d,p) reaction on Zr^{90}, Zr^{91}, and Zr^{92} adds neutrons to this configuration. A study of these reactions gives information about the interaction of the extra neutrons with one another and with the Zr^{90} core. This type of study is initiated by the work presented here. The $Zr^{90}(d,p)Zr^{91}$ reaction is used to find the energy levels of the extra neutron and the angular distributions are used to determine the angular momentum transfer in the reaction.

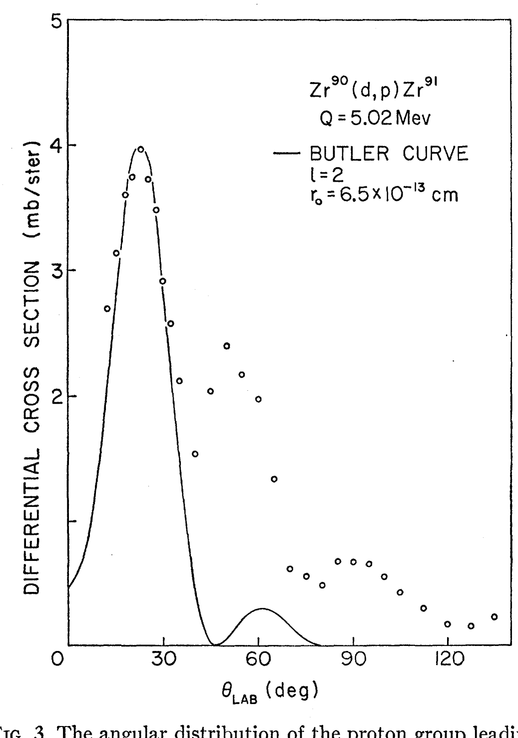

FIG. 3. The angular distribution of the proton group leading to the Zr^{91} ground state. The d-wave Butler fit agrees with the measured spin of $5/2^+$ for the state and with the shell-model expectations.

EXPERIMENTAL PROCEDURES

Natural Zr targets and enriched ZrO_2 targets[8] were used in this experiment. Natural Zr was used to prepare metallic self-supporting evaporated targets with a thickness of about 2 mg/cm^2. The oxide targets were prepared by suspending the powder in ethylene dichloride and adding a small amount of Formvar binder and Aquadag. The suspension was poured on a glass plate, dried, and peeled. These targets also had a total thickness of about 2 mg/cm^2. The Aquadag was added to the targets to strengthen them for the beam bombardment.

The targets were bombarded by the 10.85-Mev deuteron beam from the Indiana University cyclotron. The experimental arrangement and the magnetic spectrometer used in the measurements have been described in earlier papers.[5,9] A proton momentum spectrum is shown in Fig. 1. The oxygen and carbon peaks shown in this spectrum moved through the zirconium peaks as the spectrometer angle is decreased. This effect limited the number of proton groups for which complete angular distributions were obtained.

The natural Zr targets were used to study the properties of the proton group leading to the ground state of Zr^{91}. A normalization of the natural and enriched targets was then made by comparing the ground-state yields.

The error on the absolute cross sections is $\pm 40\%$. The measurements were made with the natural Zr target. The energy loss of 8.78-Mev ThC′ alpha particles passing through the target was measured and used with stopping powers given by Whaling[10] to determine the number of Zr atoms in the target.

Q-value measurements were similar to those described in other work.[5,11] The ground-state Q value was measured with the metallic target. Oxygen and carbon peaks were used to determine the beam energy. Relative Q values were measured with the ZrO_2 target. These measurements determine the difference in the Q values of two proton groups leading to adjacent states in Zr^{91}. This means that a shift in the measured excitation

[7] W. Tobocman, Phys. Rev. **115**, 98 (1959).

[8] ZrO_2, enriched to 98.7% in Zr^{90}, was obtained from the Oak Ridge National Laboratory.

[9] V. K. Rasmussen, D. W. Miller, and M. B. Sampson, Phys. Rev. **100**, 181 (1955); J. R. Rees and M. B. Sampson, Phys. Rev **108**, 1289 (1957).

[10] W. Whaling, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1958), Vol. 34, p. 193.

[11] G. B. Holm, J. R. Burwell, and D. W. Miller, Phys. Rev. **118**, 1247 (1960).

energy of one of the Zr^{91} states will shift the energy of all higher excited states by the same amount. Errors on the absolute Q values are ± 30 kev. Relative Q-value errors are ± 20 kev.

RESULTS

Eleven proton groups were seen in the reaction. All of these groups appear to lead to single states in Zr^{91}. Figure 2 shows these states on a level diagram of Zr^{91}.

The spin of the Zr^{91} ground state has been measured[12] and is $\frac{5}{2}+$. Beta-decay studies[13,14] of the 1.22-Mev state indicate a spin of $\frac{1}{2}+$ or $\frac{3}{2}+$. Early (d,p) reaction studies by Schull excited a 2.10-Mev Zr^{91} state.[15] Day has used the (n,n',γ) reaction to excite levels at 1.19, 1.46, 1.87,

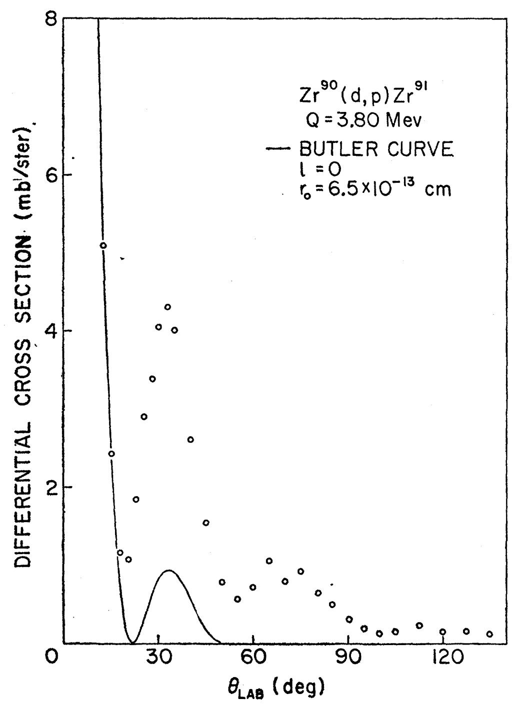

FIG. 4. The angular distribution for the proton group leading to the 1.22-Mev Zr^{91} state.

2.07, and 2.15 Mev.[16] The work reported here replaces the 2.10-Mev state by two states, one at 2.07-Mev excitation and one at 2.19-Mev excitation. Three of the levels seen by Day were also excited in the present work.

The angular distribution for the proton group leading to the ground state of Zr^{91} is shown in Fig. 3. The position of the first peak is fitted by a d-wave Butler curve. This is in agreement with the $2d_{5/2}$ assignment of the shell model for the 51st neutron. Other neutron states

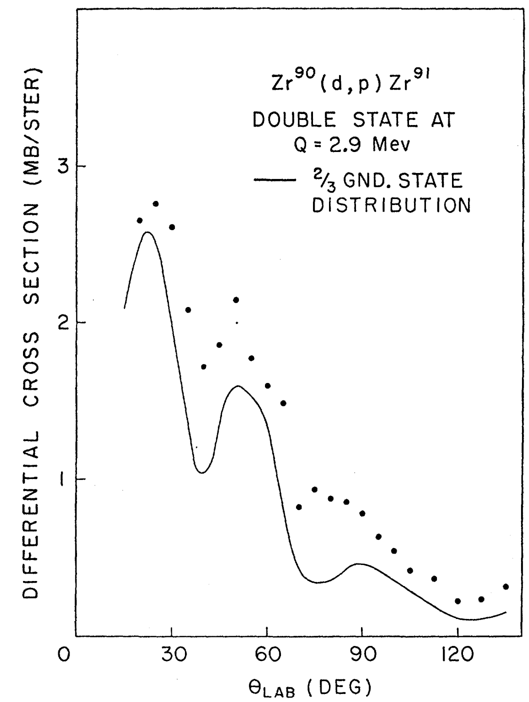

FIG. 5. The angular distribution for the double proton group to the 2.07-Mev state and the 2.19-Mev state. This distribution has many of the features of the ground-state distribution.

predicted by the shell model for the 50–82 shell are $1g_{7/2}$, $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$.

An s-wave Butler curve fits the angular distribution

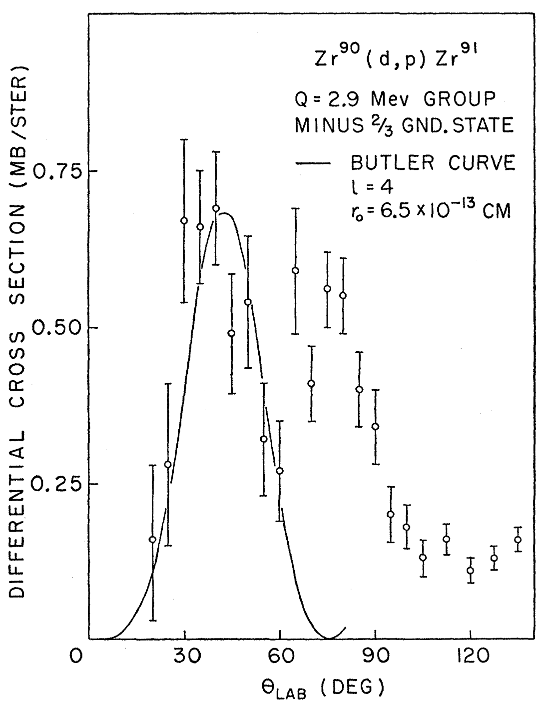

FIG. 6. The angular distribution of the proton group to the 2.19-Mev state, assuming that the 2.07-Mev state is $d_{3/2}$. This assumption is discussed in more detail in the text.

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[12] J. E. Mack, Revs. Modern Phys. **22**, 64 (1950).
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[14] O. E. Johnson and W. G. Smith, Phys. Rev. **118**, 1315 (1960).
[15] F. B. Shull, Phys. Rev. **87**, 216(A) (1952).
[16] R. B. Day (private communication).

of the proton group leading to the 1.22-Mev Zr^{91} state. This is shown in Fig. 4. Cohen finds the same distribution for this state[3] and makes a $3s_{1/2}$ assignment for the state.

Figure 5 shows the total angular distribution for two proton groups, one leading to the 2.07-Mev state and the other to the 2.19-Mev state. The data indicated that the angular distributions for the two groups were different but they could not be measured separately. The curve in Fig. 5 represents a $2d_{3/2}$ shell-model state and is the experimental distribution for the ground-state group multiplied by 0.67. This curve is subtracted from the data and the resultant angular distribution is shown in Fig. 6. This distribution is fitted by a g-wave Butler curve. The expectation of shell-model states leads to a $2d_{3/2}$ assignment for the 2.07-Mev Zr^{91} state and a $1g_{7/2}$ assignment for the 2.19-Mev state.

TABLE I. A comparison of the experimental and calculated reduced widths relative to the ground state.

Q	l	$(\gamma^2/\gamma^2 \text{ gnd})^a$ (Butler+expt)	$(\gamma^2/\gamma^2 \text{ gnd})^b$ (HO)
5.02	2	1.0	1.0
3.80	0	1.7	2.5
2.83	4	0.5	0.5

[a] This column shows the experimental reduced widths relative to the ground state when the Butler theory is assumed.
[b] This column shows calculated reduced widths relative to the ground state using harmonic oscillator wave functions for the neutron.

Complete angular distributions were not measured for the other proton groups. A partial angular distribution for the proton group to the 2.56-Mev state was peaked at large angles. It is consistent with the expected $1h_{11/2}$ state but a definite assignment is not possible.

CONCLUSIONS

Angular distributions were measured for proton groups leading to four energy levels in Zr^{91}. All of the distributions were consistent with Butler theory calculations and shell-model expectations. Distorted-wave calculations by Tobocman[17] also support the assignments given here. Tobocman has calculated the distributions for the ground-state group and for the first excited state group and his assignments agree with the ones given here.

Finally, an extraction of the relative reduced widths of the states can be made by using the Butler theory. A comparison of the experimental widths with calculated widths is shown in Table I.

[17] W. Tobocman (private communication).

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Quantum Mechanical Three-Body Problem. II

LEONARD EYGES
Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts*

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We present a method for treating the following quantum mechanical three-body problem: to find the ground-state eigenvalue and eigenfunction for a system of three identical particles between any pair of which there is an attractive central force. An essential point of the method is to assume the wave function Ψ has a special analytic form, $\Psi=\psi(\mathbf{r}_{12},\boldsymbol{\varrho}_3)+\psi(\mathbf{r}_{13},\boldsymbol{\varrho}_2)+\psi(\mathbf{r}_{23},\boldsymbol{\varrho}_1)$, where $\mathbf{r}_{12}=\mathbf{r}_1-\mathbf{r}_2$, $\boldsymbol{\varrho}_3=\mathbf{r}_3-\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$ and $\mathbf{r}_{13}$, $\boldsymbol{\varrho}_2$ and $\mathbf{r}_{23}$, $\boldsymbol{\varrho}_1$ are defined analogously. The Schrödinger equation for the system can then be written as an integral equation for $\phi(\mathbf{k},\boldsymbol{\kappa})$, the Fourier transform of ψ. We expand this in Legendre polynomials,

$$\phi(\mathbf{k},\boldsymbol{\kappa})=\sum_{l=0}^{\infty}\phi_l(k,\kappa)P_l(\cos\gamma),$$

and this yields a set of coupled integral equations for the $\phi_l(k,\kappa)$. These can be truncated and to a good approximation one can neglect all ϕ_l except ϕ_0, thereby reducing the problem to a single integral equation for a function of two variables.

We propose an iterative scheme for solving this equation for the ground-state eigenfunction, and suggest a simple but accurate nonvariational method for deriving the energy eigenvalue therefrom. We test this proposed solution by working it out in detail for the case of exponential interparticle potentials. The results for the eigenvalue compare favorably with variational calculations by other authors. Finally, we discuss the accuracy of the approximations and the possible sources of error in the wave function.

I. INTRODUCTION

IN this paper we consider the quantum-mechanical problem of finding the ground-state energy eigenvalue and eigenfunction for a system of three identical particles in which identical attractive forces act between each pair. The general features of the method we discuss are applicable to other three-body and many-body problems, but for reasons that are more or less obvious the symmetrical three-body problem we mention is the simplest of these. The results we get in this problem are encouraging enough to make it hopeful that progress can be made along similar lines in more complicated problems. The symmetrical problem we dis-

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