Method to Obtain the Energy Dependence of Reaction Cross Sections When a Compound Nucleus is Formed*

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It is found that measuring the energy distributions of emitted particles at two different incident energies and two reaction cross sections is sufficient to obtain the energy dependence of a reaction cross section when it is known that a compound nucleus is formed. The cross sections obtained are a function of the excitation energy of the residual nucleus. They are the usual reaction cross sections at zero excitation energy when the assumption is made that the reaction cross section does not vary with the excitation energy of the target nucleus. By comparing the reaction cross sections thus obtained with measured nonelastic cross sections, the compound elastic cross section may also be obtained. An example using this method is given in which two measured energy distributions and two measured cross sections were used to obtain the energy dependence of the reaction cross section of nickel in the 2.5- to 7-Mev energy range.

INTRODUCTION

TOTAL reaction cross sections are difficult to obtain. To obtain an individual reaction cross section for a particular incident particle at a particular energy, the reaction cross section of all the emitted particles must be determined. Many such reaction cross sections are needed to obtain information of the characteristics of nuclear structure. There is some evidence, at the present time, that the primary mode of reaction for incident neutrons and charged particles is via the formation of a compound nucleus, at relatively low incident energies and over a wide range of target nuclei. A method is here described in which many reaction cross sections as a function of energy are obtained for a particle when it is known that a compound nucleus is formed.

THEORY AND THE CALCULATIONAL PROCEDURE

A universal relation describing the reciprocity of a nuclear reaction is[1]

$$\sigma_{(b,a)}/\lambda\!\!\!^{-}_b{}^2=\sigma_{(a,b)}/\lambda\!\!\!^{-}_a{}^2, \tag{1}$$

where $\sigma(b,a)$ is the cross section for the reaction of a particle b on a nucleus Y in a particular configuration C_Y resulting in the emission of a particle a, and a nucleus X in a configuration C_X; $\sigma(a,b)$ is the cross section for the reaction of a particle a with a nucleus X in a configuration C_X, resulting in the emission of a particle b, and a nucleus Y in a configuration C_Y; $\lambda\!\!\!^{-}_b$ is the de Broglie wavelength of particle b; and $\lambda\!\!\!^{-}_a$ is the de Broglie wavelength of particle a.

The formation of a compound nucleus and its decay is one of the few reactions where this reciprocity theorem can be used. The reactions in which we will be interested will have the nucleus Y at an excitation E, corresponding to C_Y, and the nucleus X in its ground state, corresponding to C_X.

The decay rate of a compound nucleus C with the emission of a particle b is given by the following expression[2]:

$$I(\alpha,\epsilon)=K(\alpha)\sigma(E,\epsilon)P(E), \tag{2}$$

where $I(\alpha,\epsilon)$ is the differential scattering cross section per Mev per steradian divided by ϵ (the kinetic energy of the relative motion of the emitted particle b and the residual nucleus) for an incident energy α (the kinetic energy of the relative motion of the incident particle a and the target nucleus) all evaluated in the c.m. system; $K(\alpha)$ is a function of the incident energy α, and is independent of E or ϵ; $\sigma(E,\epsilon)$ is the average cross section in an energy interval $\Delta\epsilon$ for the particle b with an incident energy ϵ relative to the residual nucleus at an excitation energy E in the c.m. system (the energy interval $\Delta\epsilon$ is defined by the experimental equipment); and $P(E)$ is the number of levels in the energy interval $\Delta\epsilon$ at an excitation energy E of the residual nucleus.

We are using primarily only the basic Bohr assumption of the theory of the compound nucleus: independence of the formation and decay of the compound nucleus. It is not necessary for the present purpose to invoke the statistical model to predict the energy level spacing of the residual nucleus, nor to assume any interference between energy levels.

The excitation energy region of interest varies from 0 to E_m, where E_m is the energy necessary for the emission of particle b plus another particle. E_m is normally about 7 Mev or greater.

Two energy distributions are measured for incident energies α_1 and α_2, where $\alpha_2=\alpha_1+\Delta\epsilon$ and $\Delta\epsilon$ is a small energy increment. The experimental resolution is $\Delta\epsilon$ or less. The energy distribution of particle b is given by Eq. (2). We then have the set of equations representing the energy distribution according to compound nucleus theory for an incident energy, α_1:

$$I(\alpha_1,\epsilon_1)=K(\alpha_1)\sigma(\alpha_1+Q-\epsilon_1,\ \epsilon_1)P(\alpha_1+Q-\epsilon_1), \tag{3}$$

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[1] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 337.

[2] Reference 1, p. 367.

where

$$\alpha_1+Q-E_m<\epsilon_1<\alpha_1+Q,$$

and ϵ_1 varies from α_1+Q-E_m to α_1+Q in $\Delta\epsilon$ steps. There are $E_m/\Delta\epsilon$ equations. Similarly, for the incident energy α_2 we have the set of equations:

$$I(\alpha_2,\epsilon_2)=K(\alpha_2)\sigma(\alpha_2+Q-\epsilon_2,\ \epsilon_2)P(\alpha_2+Q-\epsilon_2), \quad (4)$$

where

$$\alpha_2+Q-E_m<\epsilon_2<\alpha_2+Q,$$

and ϵ_2 varies from α_2+Q-E_m to α_2+Q in energy intervals of $\Delta\epsilon$.

Two reaction cross sections are measured for incident energies γ_1 and γ_2, where $\gamma_2=\gamma_1+\Delta\epsilon$ and γ_1 is an arbitrary energy. The cross sections $\sigma_0(\gamma_1)$ and $\sigma_0(\gamma_2)$ for the residual nucleus in its ground state are thus known.

For the present, let us assume that the reaction cross sections are independent of excitation energy. In order to eliminate the constants $K_{\alpha 1}$ and $K_{\alpha 2}$, Eq. (3) is divided by its value at energy γ_1 and Eq. (4) is divided by its value at energy γ_2. This gives

$$\frac{I(\alpha_1,\epsilon_1)}{I(\alpha_1,\gamma_1)}=\frac{\sigma(\epsilon_1)P(\alpha_1+Q-\epsilon_1)}{\sigma(\alpha_1)P(\alpha_1+Q-\gamma_1)}, \quad (5)$$

$$\frac{I(\alpha_2,\epsilon_2)}{I(\alpha_2,\gamma_2)}=\frac{\sigma(\epsilon_2)P(\alpha_2+Q-\epsilon_2)}{\sigma(\gamma_2)P(\alpha_2+Q-\gamma_2)}. \quad (6)$$

If we remember that $\alpha_2=\alpha_1+\Delta\epsilon$ and $\gamma_2=\gamma_1+\Delta\epsilon$, it is easy to see that the $P(E)$'s are eliminated in the ratio obtained by dividing Eq. (5) by Eq. (6) when ϵ_1 and ϵ_2 are related by the relation, $\epsilon_2=\epsilon_1+\Delta\epsilon$.

The following general relation may be obtained:

$$\sigma(\gamma_1+(n+1)\Delta\epsilon) = f\sigma(\gamma_1+n\Delta\epsilon)\frac{I(\alpha_2,\ \gamma_1+(n+1)\Delta\epsilon)}{I(\alpha_1,\ \gamma_1+n\Delta\epsilon)}, \quad (7)$$

where

$$f=\frac{I(\alpha_1,\gamma_1)}{I(\alpha_2,\gamma_2)}\frac{\sigma(\gamma_2)}{\sigma(\gamma_1)}.$$

When $n=1$, this becomes

$$\sigma(\gamma_1+2\Delta\epsilon)=f\sigma(\gamma_2)I(\alpha_2,\ \gamma_2+\Delta\epsilon)/I(\alpha_1,\gamma_2).$$

n is varied from $n=1$ up to its maximum positive value determined by the following limits:

$$\gamma_2<\gamma_1+(n+1)\Delta\epsilon<\alpha_1.$$

The remaining cross sections are obtained by varying n from $n=-1$ to its maximum negative value determined by the following limits:

$$\alpha_1+Q-E_m<\gamma_1-n\Delta\epsilon<\gamma_1.$$

Thus it is only necessary to know two reaction cross sections at energies γ_1 and $\gamma_1+\Delta\epsilon$ and two energy distributions at α_1 and $\alpha_1+\Delta\epsilon$ to obtain the reaction cross section as a function of energy. The energy γ_1 should be chosen as close to α_1 as possible since this then corresponds to the nucleus near its ground state which is compatible with the ground-state measured cross sections. It is not possible to take $\gamma_1=\alpha_1$ because of limitations introduced by the elastic scattering peak and low excited level structure.

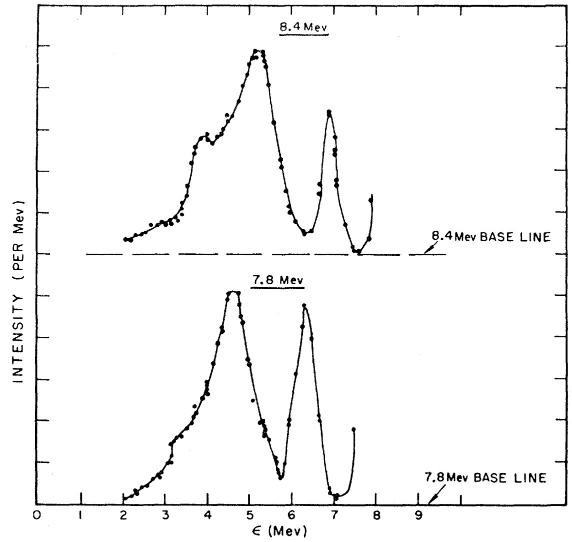

FIG. 1. Relative intensity distributions of emitted protons as a function of ϵ for incident protons of 7.8 and 8.4 Mev on Ni at 135°.

Each reaction cross section thus obtained will correspond to a nucleus having an excitation energy given by $(\alpha_1+Q-\epsilon)$. By varying α_1 and γ_1, a complete family of cross-section curves plotted against particle energy with excitation energy as a parameter may be obtained.

By the use of this procedure it is also possible to obtain information about the compound elastic cross section.

The compound elastic differential scattering cross section is normally difficult to obtain, since experiments measure the total elastic cross section which is the sum of compound elastic and shape elastic cross sections. However, at high energy there is little compound elastic cross section present due to competition of the large number of channels available for inelastic scattering. Reaction cross sections and energy spectra are measured at high energies where compound elastic scattering is small. By the use of these energy spectra, the measured cross sections, and Eq. (7), reaction cross sections are obtained for low energies. The inelastic scattering cross section excluding compound elastic scattering, plus the cross section for the emission of all other particles, is measured at low energies where compound elastic scattering is large. The compound elastic scattering cross section is obtained by subtracting these contributions from the reaction cross section.

AN EXAMPLE

An example using Eq. (7) will now be described. The energy distribution of the emitted protons at 135° from Ni^{58} has been measured at incident proton energies of 7.8 and 8.4 Mev.[3] The energy spectra are shown in Fig. 1 and are plotted against ϵ (the kinetic energy of the relative motion of the emitted proton and the residual nucleus in the c.m. system). The distributions are corrected for the variation in solid angle in the c.m. system as a function of the emitted proton energy. The incident energies are given in the laboratory system.

The energy dependence of the (p,n) cross section on Ni was measured in the energy region of 4.5 to 5.1 Mev.[4] We assume for this example that the proton reaction cross section in this energy region is proportional to the (p,n) cross section due to the similarity in nuclear structure of the different Ni isotopes. In Eq. (7), we let $\gamma_1=4.5$ and $\gamma_2=5.1$.

The Coulomb proton reaction cross sections were calculated from the expression[5]

$$\sigma_c=\pi\lambda\!\!\!^{-2}\sum_{l=0}^{\infty}(2l+1)\frac{4s_lKR}{\Delta_l^2+(KR+s_l)^2}, \tag{8}$$

where $\lambda\!\!\!^{-}$ is the wavelength of the incident proton, K is the wavelength of the proton inside the nucleus, R is the nuclear radius,

$$\Delta_l=R\left[\frac{G_l\left(\dfrac{dG_l}{dr}\right)+F_l\left(\dfrac{dF_l}{dr}\right)}{G_l^2+F_l^2}\right]_{r=R},$$

and

$$s_l=R\left[\frac{G_l\left(\dfrac{dF_l}{dr}\right)-F_l\left(\dfrac{dG_l}{dr}\right)}{F_l^2+G_l^2}\right]_{r=R}.$$

[3] R. Fox and R. D. Albert, Phys. Rev. **121**, 597 (1961).
[4] R. D. Albert, Phys. Rev. **115**, 925 (1959).
[5] Reference 1, p. 354.

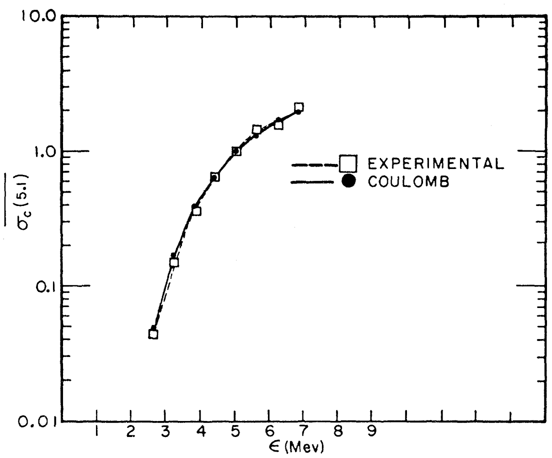

FIG. 2. Energy dependence of the proton reaction cross section of Ni and the comparison with the energy dependence of the Coulomb cross section.

F_l and G_l, the regular and irregular solutions of the Coulomb wave equation, were generated by means of an IBM 650 coded program. In calculating K, a nuclear potential of 45 Mev was used. In our particular example, we used R equal to 5.81×10^{-13} cm.

Using Eq. (7), the ratio of the reaction cross section to $\sigma_{5.1}$ as a function of energy for our data is shown as the solid curve in Fig. 2. The ratio of the Coulomb cross section to $\sigma_{5.1}$ as a function of energy is shown as the dashed curve in Fig. 2. The two curves overlap one another. Let us assume that the reaction cross section does not vary with excitation of the target nucleus and that a compound nucleus is formed for incident energies of 7.8 and 8.4 Mev. Then to the extent that we have used the approximation of a sharp-edged black nucleus model, it appears that the proton reaction cross section is Coulomb in the energy region 2.7–7.0 Mev.

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