Cluster Nature of Li^7 and Be^7†

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(Received November 17, 1960)

Measurements of the capture γ-radiation processes, mass $3+\alpha \rightarrow$ mass $7+\gamma$ and nucleon$+Li^6 \rightarrow$ mass $7+\gamma$, give information about the cluster structure of the mirror nuclei Li^7 and Be^7. The cluster model predicts that the ground state and low excited states of these nuclei should have large reduced widths θ_{3+4}^2 for the configuration mass $3+\alpha$ particle and small reduced widths θ_{1+6}^2 for the configuration nucleon $+Li^6$. Scattering experiments provide accurate initial, capturing, wave functions, and an assumption of the cluster nature of the final, bound, states allows the electromagnetic capture cross sections to be calculated and compared to experiment. The reduced widths deduced show that θ_{3+4}^2 is large, θ_{1+6}^2 is small, and that the ground states and first excited states of Li^7 and Be^7 are primarily of the two-body cluster form mass $3+\alpha$ particle.

INTRODUCTION

IN the past two years considerable new work has been done on a description of nuclei in terms of two-body clusters.[1,2] This renewed interest may be traced primarily to the hope that the "cluster model" might serve to unify the other nuclear models and to overcome some of their deficiencies. Although the basic ideas of the cluster model are not new and may be traced back to the α-particle model and the resonating-group formulation of Wheeler,[3] the new developments have extended the model's range of usefulness and have been fairly successful in explaining many dynamical features of nuclear phenomena.

The success or failure of the cluster model depends upon the degree to which the stable and semistable states of a complex nucleus may be described as resonances of pairs of complex clusters. The Li^7-Be^7 system is the lightest mirror pair for which enough data are presently available to provide a check on the validity of the cluster model predictions for the bound states of the system. The "cluster picture" of these nuclei for low excitation energies is that of an α particle and a mass 3 particle in a relative P state. While this description is similar to that of the shell model, the cluster model additionally supposes that the residual forces between the three P-shell particles serve to produce a semistable cluster structure similar to a triton or He^3. The low-energy excited states of such a system are also expected to show evidence of this structure; measurement of elastic scattering for $He^3(\alpha,\alpha)He^3$ has shown that the $\frac{7}{2}-$ state in Be^7 has the expected cluster form.[4] This scattering data has also been interpreted as giving indirect evidence that the ground and first excited states of the Li^7-Be^7 are of the $\alpha+$ mass 3 cluster form.

The purpose of this paper is to show that electromagnetic transitions reveal this same cluster structure for the bound states of these nuclei. This is accomplished by calculating the capture cross sections for the mass $3(\alpha,\gamma)$mass 7 and nucleon(Li^6,γ)mass 7 reactions. This experimental example also serves to emphasize the extent that it is possible to make assertions about the spatial localization of the nucleons within the nucleus. By the uncertainty principle, any determination of spatial ordering must be accompanied by a suitable dispersion in the energy. In the examples considered in this paper, the results of measurements of scattering and capture *over a range of energies* allow definite details of the capturing, continuum wave functions to be determined. For example, the S- and P-wave scattering phase shifts derived from He^3+He^4 scattering[4] determine these partial wave functions rather well for all distances of He^3-He^4 separation greater than the range of the strong nuclear forces, and also determine the range. It should be noted that the only way that phase shifts and radii can be determined with any uniqueness is by measurements over a range of energies that start at low energies, and by also knowing the number of bound states of each partial wave.

The final, captured state wave function is, of course, unknown. In this paper a final-state wave function will be assumed so as to be appropriate to the model to be investigated.

The comparison of the calculated capture γ-ray transition rates with the capture data provides a means of obtaining the partial reduced widths of the ground states and first excited states for the configurations mass $3+\alpha$ and nucleon$+Li^6$, respectively. The magnitudes of these reduced widths, θ_{1+6}^2 for the Li^6+nucleon configuration, and θ_{3+4}^2 for the He^4+mass 3 particle configuration, show, respectively, the extent to which the system is described by an extreme independent-particle model (i.e., $\theta_{1+6}^2\approx 1$) or by the cluster model (i.e., $\theta_{3+4}^2\approx 1$).

EXPERIMENTAL DATA

The levels in the Li^7-Be^7 system are shown in Fig. 1.[5] (The absence of a state at ≈6.5 Mev will be discussed

† Supported in part by the U. S. Atomic Energy Commission.
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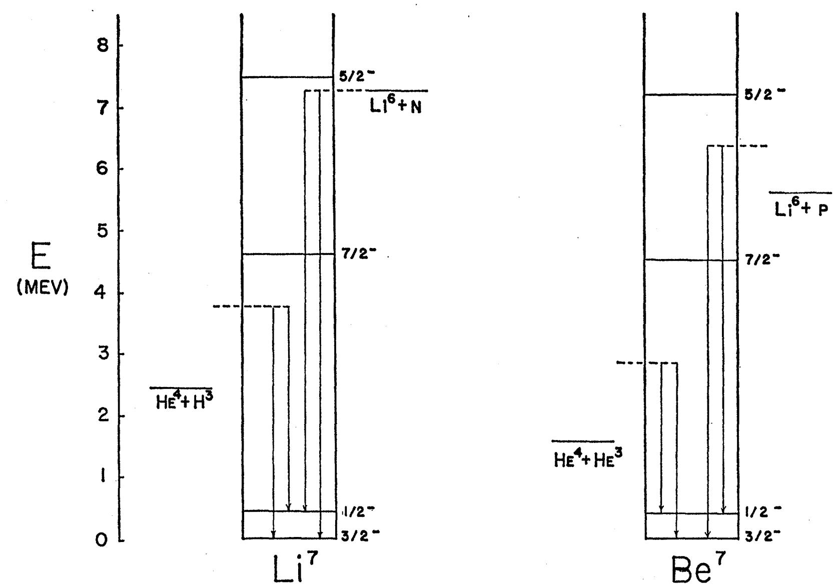

FIG. 1. The energy level diagrams of the mirror nuclei Li^7 and Be^7. The electromagnetic capture transitions, mass 3(α,γ)mass 7 and nucleon(Li^6,γ)mass 7, that yield information about the cluster structure of the ground and first excited states are shown.

later.) The angular momenta and energies of these states are in general well explained on the basis of the intermediate coupling calculations of Inglis.[6] In addition, Kurath[7] was able to fit the correct ordering of these states and, with the exception of the $\frac{5}{2}^-$ state, was able to give the correct relative spacing of the levels. An equally good fit has also been accomplished in the cluster model calculations of Wildermuth.[2] The experimental data that are relevant to our calculations will be discussed briefly.

(1) The capture reactions $He^3(\alpha,\gamma)Be^7$ and $H^3(\alpha,\gamma)$-Li^7 have been investigated experimentally for α-particle energies between 0.5 Mev and 1.3 Mev.[8,9] The data show that the reaction proceeds by nonresonant capture and that the angular distribution of radiation is approximately isotropic. These data indicate that the radiation is chiefly electric dipole radiation ($E1$) with, perhaps, a small amount of magnetic dipole ($M1$) radiation appearing at higher bombarding energies.

(2) The phase-shift analysis of $He^4(He^3,He^3)He^4$ by Miller and Phillips shows that the S-wave scattering phase shift is well described by a hard-sphere interaction, while the P-wave scattering phase shifts can be fitted with a bound-state plus hard-sphere interaction.[4] The energy variation of the phase shifts δ_0, δ_1^-, and δ_1^+ yield hard-sphere radii, respectively, of 2.8×10^{-13} cm, 3.5×10^{-13} cm, and 4.4×10^{-13} cm. A similar phase shift analysis of the existing $He^4(t,t)He^4$ data[10] reproduced these hard-sphere radii.

(3) The capture of protons by Li^6 has been determined by Warren *et al.* at energies below 1 Mev.[11] The results show that the $Li^6(p,\gamma)Be^7$ reaction proceeds by nonresonant capture, while the angular distribution of the radiation rules out the possibility of S-wave capture. Only P-wave capture is found to be consistent with the data and the radiation is either magnetic dipole ($M1$), or electric quadrupole ($E2$), or a mixture of both. This fact in itself tends to preclude the existence of a virtual S state of Be^7 in this energy region. In addition to this information, recent work has shown that the earlier reports of this state arose from a contaminant.[12]

CALCULATIONS

The Hamiltonian for the interaction of a γ ray and the nucleus in the capture process is in general given by

$$H_1=(\mathbf{j}\cdot\mathbf{A}+\boldsymbol{\mu}\cdot\mathbf{B})^*,$$

where $\mathbf{j}$ is the current operator, $\mathbf{A}$ is the vector potential of the radiation field, $\boldsymbol{\mu}$ is the magnetic dipole operator, and $\mathbf{B}=\nabla\times\mathbf{A}$.

If the system is now considered as being composed of two clusters of mass m_1 and m_2, charge Z_1 and Z_2, and magnetic moment μ_1 and μ_2, and assuming that the total Hamiltonian for the two-cluster system contains only central forces, the matrix element for the capture interaction may be obtained. For electric multipole capture of order λ, where the effects of the motion of the magnetic dipole moments may be neglected, and where the wavelength of the γ ray is long compared to nuclear dimensions, this matrix element is

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[12] E. W. Hamburger and J. R. Cameron, Phys. Rev. **117**, 781 (1960).

$$M_{E-\lambda}=i^{\lambda}(-1)^{\lambda-1}\left(\frac{\mu_0\hbar c^2}{2\omega}\right)^{\frac{1}{2}}\frac{(k_\gamma)^\lambda c}{(2\lambda-1)!!}\left[\frac{2\pi\lambda(\lambda+1)}{2\lambda-1}\right]^{\frac{1}{2}}\times\left[Z_1\left(\frac{\mu}{m_1}\right)^{\lambda}+\left(-\frac{\mu}{m_2}\right)^{\lambda}Z_2\right]\times\sum_\kappa D_{\kappa p}{}^{\lambda}(\theta_\gamma,\phi_\gamma)\langle\phi_f|r^\lambda Y_\lambda{}^\kappa(\theta,\phi)|\phi_i\rangle.$$

In this expression, μ_0 is the permeability of free space, $\hbar$ is Planck's constant, c is the velocity of light, and ω is the angular frequency of the γ ray. $D_{\kappa p}{}^{\lambda}$ is the element of the rotation matrix, θ_γ and ϕ_γ the polar and azimuthal angles of the γ radiation, and p is the state of circular polarization of the γ ray whose wave number is k_γ. The initial and final wave functions of the system are denoted by ϕ_i and ϕ_f. $(2\lambda-1)!!\equiv(2\lambda-1)(2\lambda-3)\cdots 5\times3\times1$.

For magnetic dipole capture in the long-wavelength approximation, the absolute value of the matrix element is

$$|M_{M-1}|=k_\gamma\left(\frac{\mu_0\hbar c^2}{2\omega}\right)^{\frac{1}{2}}\times|\langle\phi_f|\mu_1\sum_\kappa\sigma_1{}^{*\kappa}D_{\kappa p}{}^1+\mu_2\sum_\kappa\sigma_2{}^{*\kappa}D_{\kappa p}{}^1|\phi_i\rangle|,$$

where $\sigma_i{}^\kappa$ is the κth spherical component of the spin operator for the ith particle.

Outside the range of nuclear forces between the colliding clusters, the wave function for the initial state is provided by the solution of the particular two-body scattering problem and in principle may be deduced from the analysis of the scattering data. This solution is usually given in the form of a partial wave expansion in the orbital angular momentum of a Coulomb distorted incoming plane wave plus an outgoing Coulomb distorted spherical wave.

In this same exterior region the final-state wave function is a rapidly decreasing function of the distance from the nuclear center, and its magnitude at the nuclear surface is related to the reduced width of the final state for that particular two-body form.

For the numerical evaluation of the radial integral occurring in the matrix element it is necessary to generate both the initial- and final-state wave functions as functions of the radial coordinate, r. The JWKB approximation was used to obtain a closed form for the final-state wave function[13]:

$$u_f(k,\eta,l)=\left(\frac{\rho}{[\rho^2+2\eta\rho+l(l+1)]^{\frac{1}{2}}}\right)^{\frac{1}{2}}\times\{[\rho^2+2\eta\rho+l(l+1)]^{\frac{1}{2}}+\rho+\eta\}^{-\eta}\times\left[\frac{1}{\rho}\{[\rho^2+2\rho\eta+l(l+1)]^{\frac{1}{2}}+[l(l+1)]^{\frac{1}{2}}\}+\frac{\eta}{[l(l+1)]^{\frac{1}{2}}}\right]^{[l(l+1)]^{\frac{1}{2}}}\times\exp\{-[\rho^2+2\rho\eta+l(l+1)]^{\frac{1}{2}}\},$$

where $\rho=kr$, $\eta=\mu Z_1Z_2e^2/\hbar^2k$, l is the orbital angular momentum of the final state, and u_f is subject to the normalization condition that[14]

$$\frac{Ru_f{}^2(R)}{3\theta^2}+\int_R^\infty u_f{}^2(R)dr=1.$$

In this expression, R is the nuclear radius and $\theta^2=(2\mu R^2/3\hbar^2)\gamma_f{}^2$, $\gamma_f{}^2$ being the reduced width of the state.

For the generation of the initial state wave function it was necessary to evaluate both the regular and irregular Coulomb wave functions. A modified expression for the JWKB wave function has been found[15] that is correct to about 1% of tabulated values. The expression is valid to this accuracy for $\rho>\rho_{\max}$, where the first maximum of the regular Coulomb wave function occurs at $\rho_{\max}$.[16] Using this expression, one obtains

$$\mathcal{G}_l=F_l(kr)\cos\delta_l+G_l(kr)\sin\delta_l\simeq\{\rho/[\rho^2-2\eta\rho-l(l+1)]^{\frac{1}{2}}\}^{\frac{1}{2}}\sin(\phi+\sigma_l+\delta_l-\tfrac{1}{2}l\pi),$$

where δ_l is the scattering phase shift for the lth partial wave, σ_l is the Coulomb phase shift of the lth partial wave, F_l and G_l are the regular and irregular Coulomb wave functions, and

$$\phi=\eta+[\rho^2-2\eta\rho-l(l+1)]^{\frac{1}{2}}-\eta\ln\{\rho-\eta+[\rho^2-2\rho\eta-l(l+1)]^{\frac{1}{2}}\}-[l(l+1)]^{\frac{1}{2}}\tan^{-1}\left\{\frac{[l(l+1)]^{\frac{1}{2}}[\rho^2-2\eta\rho-l(l+1)]^{\frac{1}{2}}}{\rho\eta+l(l+1)}\right\}+[l(l+1)]^{\frac{1}{2}}\tan^{-1}\left\{\frac{[l(l+1)]^{\frac{1}{2}}}{\eta}\right\}.$$

This expression was used to calculate a pair of starting values for the initial-state wave function. The wave function was then continued back to the nuclear surface by using the approximate relation:

$$\mathcal{G}_l(r-h)=-\mathcal{G}_l(r+h)+\mathcal{G}_l(r)\left\{2-\frac{h^2}{r^2}[\rho^2-2\eta\rho-l(l+1)]\right\}.$$

The numerical integration of the radial integral was done step-by-step by the trapezoidal rule as each new value of $\mathcal{G}_l$ was generated. An interval of about 0.15 for ρ was used.

The accuracy of both the continuation process and the generation of initial values has been checked against tables and has been found to be quite satisfactory. The error due to the use of the approximate final-state wave function is difficult to estimate, but it is doubtful that the final answers would be very sensitive to small changes in its form.

[13] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1955), p. 184.

[14] R. G. Thomas, Phys. Rev. 84, 1061 (1951).

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[16] Arnold Tubis, Los Alamos Scientific Laboratory Report LA-2150, 1957 (unpublished).

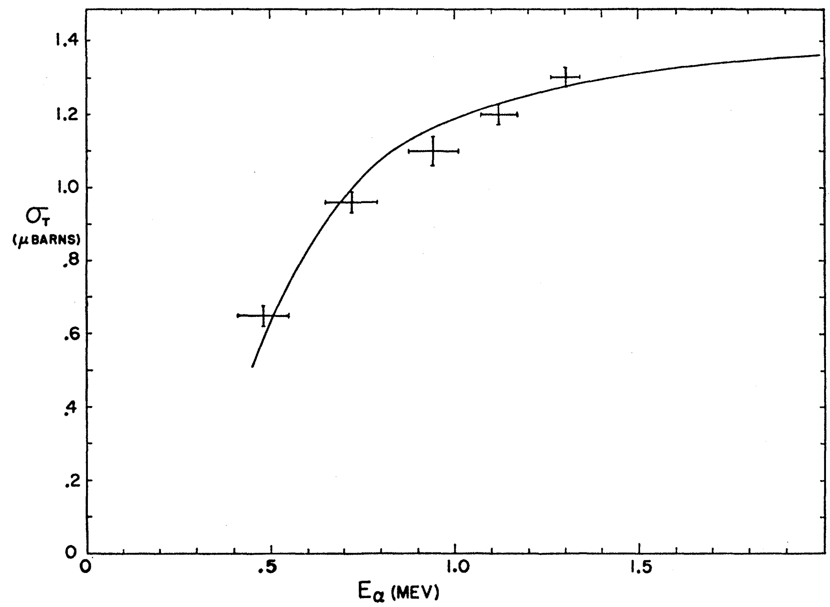

FIG. 2. Capture γ-radiation excitation curve for the $T+\alpha \rightarrow Li^7+\gamma$ reaction. The experimental points are those of Holmgren and Johnson.[8] The curve of $E1$ radiation, discussed in the text, was calculated for an assumed S-wave capture to both final P states (ground and first excited states). A hard-sphere, S-wave radius of 2.8×10^{-13} cm was employed.[4] The calculated branching ratio of ground state to first excited state was 2.35. The measured ratio of Riley *et al.*[9] is 2.5. A reduced width of $\theta_{3+4}{}^2=0.06$ was determined from the data of Holmgren and Johnson while the data of Riley *et al.* give $\theta_{3+4}{}^2=0.12$.

The only major approximation involved in this treatment is the neglecting of that portion of the matrix element that is due to the integration over the nuclear volume. The consequences of this approximation will be investigated separately for all the cases considered.

RESULTS

A. Li^7

Since the scattering of S- and P-wave tritons from helium is well described in terms of a hard sphere interaction, the initial state wave function for the reaction $T(\alpha,\gamma)Li^7$ is approximately zero at the nuclear surface. This fact makes it possible to neglect the interior portion of the radial integral without introducing a significant error. The results of this calculation for an $E1$ transition following S-wave capture are shown in Fig. 2. The contribution due to the $M1$ radiation following P-wave capture has been calculated, and it was found that $\sigma_{M1}\leqslant 0.02$ μb at $E_\alpha=1$ Mev and $\sigma_{M1}\leqslant 0.05$ μb at $E_\alpha=2$ Mev. For this reason the effect due to the $M1$ transition has been ignored.

The experimental data shown in Fig. 2 are those of Holmgren[7]; the adjustment of the calculated to the measured values of the cross section yields a value of $\theta_{3+4}{}^2\simeq 0.06$ for both the ground state and the first excited state. The data of Riley *et al.* at 1.6 Mev yield $\theta_{3+4}{}^2\simeq 0.12$ for both states. The observed branching ratio σ(ground state)/σ(first excited state)$=2.5$ is to be compared with the calculated value of 2.35.

A rough calculation of the thermal neutron capture by Li^6 was also made and compared to the experimental data.[17] The values of $\theta_{1+6}{}^2$ for both bound states were shown to be less than 0.003.

[17] G. A. Bartholomew and P. J. Campion, Can. J. Phys. **35**, 1347 (1957).

B. Be^7

The experimental data for the reaction $He^3(\alpha,\gamma)Be^7$ was examined in a similar manner. As in the previous case neglecting the portion of the matrix element corresponding to the integration over the nuclear volume introduces no serious source of error. The results are shown in Fig. 3. Since the S-wave hard-sphere phase shift $\delta_0\simeq -0.6°$ at $E_\alpha=1$ Mev, it is of interest to note the sensitivity of the capture cross section to small changes in the phase shift. It is possible that a slightly better fit to the data could be obtained by making δ_0 positive. Since δ_0 would only have to be of the order of 0.5°, this certainly cannot be ruled out from the results of the phase-shift analysis. As in the previous case the contribution due to the $M1$ radiation was negligible.

Since the branching ratio is not well known for this reaction, it was assumed that $\theta_{3+4}{}^2$ was the same for both bound states. The resulting fit to the data gave $\theta_{3+4}{}^2\simeq 0.17$.

Again a rough calculation for the $Li^6(p,\gamma)Be^7$ reaction was made. It was found that it was possible to set an upper limit on $\theta_{1+6}{}^2$ for the two states of 0.006. This value was obtained after calculating both the $E1$ and $M1$ cross sections. $\theta_{1+6}{}^2$ had to be small enough that the $E1$ radiation could not be observed, but large enough that it was possible to fit the data by assuming a reasonable value of the $\delta_{l=1,\, j=\frac{3}{2}}$ phase shift. The value of 0.006 requires only that $\delta_{1,3/2}\approx 15°$ and that δ_0 was described by a hard-sphere interaction. It is to be emphasized that even though this is only an approximate treatment and neglects the possibility of an $E2$ transition, it represents an extreme upper limit on the value of $\theta_{1+6}{}^2$.

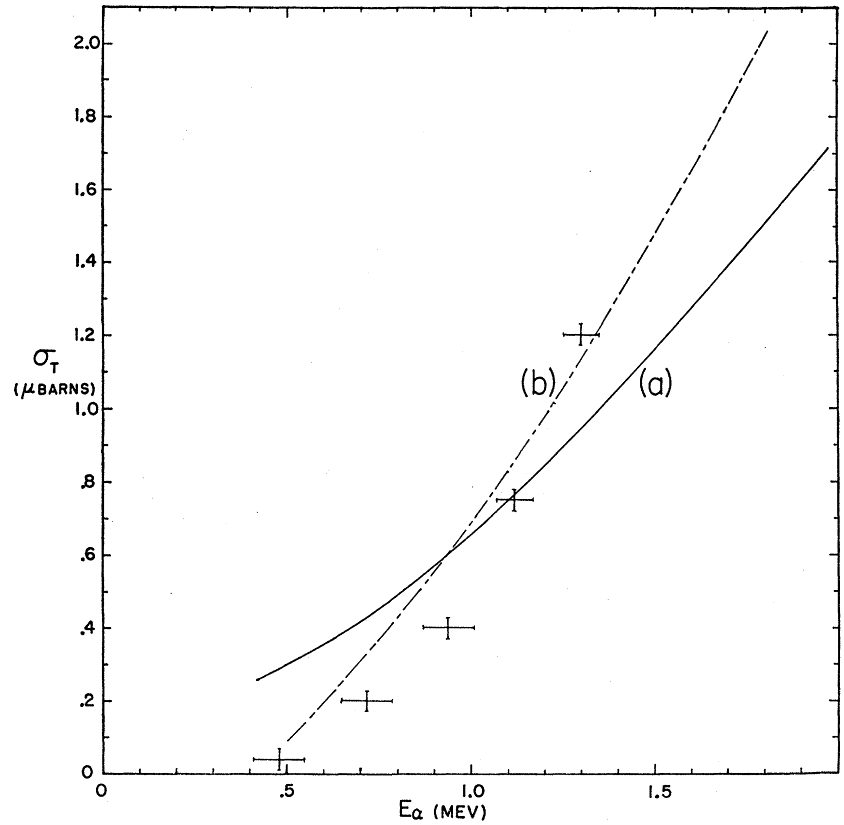

FIG. 3. Capture γ-radiation curve for the $He^3+\alpha \rightarrow Be^7+\gamma$ reaction. The experimental points are those of Holmgren and Johnson.[8] Two curves of $E1$ radiation, discussed in the text, were calculated employing S-wave phase shifts with (*a*) a hard-sphere radius 2.8×10^{-13} cm, and (*b*) no interaction in the initial states, i.e., $\delta(E)\equiv 0$. The calculated branching ratio of ground to first excited state radiation was 2.5. The reduced width θ_{3+4}^2 was determined to be 0.23 for curve (*a*) and 0.17 for curve (*b*).

CONCLUSIONS

The calculations described have given the consistent result that $\theta_{3+4}^2 \approx 20\theta_{1+6}^2$ for the ground states and first excited states of Li^7 and Be^7. This result is, of course, exactly what would be expected from the cluster model picture of these nuclei. The results are summarized in Table I.

The values of the tabulated reduced widths θ^2 refer to both the ground states and the first excited states of the nuclei. The mass-3+α reduced width θ_{3+4}^2(Li^7, ground state) has been shown to be approximately equal to the θ_{3+4}^2(Li^7, first excited state) by a consideration of the observed branching ratio. Because of the experimental uncertainty in the branching ratio, a comparison of θ_{3+4}^2 for the states of Be^7 was impossible; by analogy with Li^7 the two θ_{3+4}^2 for Be^7 were thus taken to be equal. Nevertheless, the branching ratio is known well enough for Be^7 to show that θ_{3+4}^2(Be^7, ground state) must be of the same order of magnitude as θ_{3+4}^2(Be^7, first excited state). The reduced widths θ_{1+6}^2 for the configurations nucleon+Li^6 are small in all cases.

In addition to this confirmation of the cluster structure of the bound states of these nuclei, these results also show the applicability of such calculations and capture experiments to the detailed study of nuclear structure.

TABLE I. Reduced widths for the ground states and first excited states of Li^7 and Be^7.

Nucleus	θ_{3+4}^2	θ_{1+6}^2
Li^7	0.06–0.12	$\leqslant 0.003$
Be^7	0.17–0.23	$\leqslant 0.006$

ACKNOWLEDGMENTS

The authors wish to thank the Shell Development Company and the University of Houston for the use of their computational facilities. They also wish to thank Dr. Harry Holmgren, Dr. W. Tobocman, and T. A. Griffy for helpful discussions.