can have no turning points. However, when anomalous thresholds are present turning points may occur at contacts with the anomalous thresholds. Thus, for example, when there are anomalous thresholds in two directions (but no complex singularities on the physical sheet) the boundary of the region for ρ_1 for the fourth-order contribution is the arc γ, which includes parts of positive slope as well as of negative slope [Fig. 1(b)].

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Effects of K-Meson Interactions on the Nucleon Anomalous Magnetic Moment

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A previous calculation of the physical nucleon wave function in static source pion theory is extended by including the pseudoscalar K-meson interactions, the motivation being that this should increase the scalar part of the nucleon anomalous magnetic moment, thus improving the result of previous calculations. In constructing a trial function for the physical nucleon, the method of moments was used and terms containing up to three mesons were included. Calculation shows that a too strong K-meson coupling is detrimental to the vector part, and that the scalar part can be increased approximately by 10% if the K-meson interaction is made moderately low.

INTRODUCTION

IN a recent paper[1] (hereafter referred to as I), the method of moments was applied to the problem of the physical nucleon in pion theory, using the Chew-Low-Wick static source Hamiltonian.[2] An approximate ground-state nucleon wave function was constructed, with terms containing as many as five virtual pions. One of the important results was that the average number of pions in the cloud is more than might be suggested by the success of the one-meson approximation. When the electromagnetic properties of the nucleon were calculated from the wave functions, however, hardly any significant improvement was achieved over the results obtained by the one-meson approximation.

On the other hand, we are aware of the possible contributions of the strange particles to the physical nucleon state, since these particles are known to interact strongly with nucleons and mesons. In particular, conservation of strangeness allows nucleons to emit a virtual K^+ but not K^-. Thus the inclusion of interactions involving pseudoscalar K mesons should have the consequence of increasing μ_S, the scalar part of the nucleon anomalous magnetic moment.[3] This implies an improvement because pseudoscalar pion theories invariably give too large a negative value for μ_S. This motivates us to extend the work of I by including strange particle interactions, even though a fixed-source approximation is less justified for K mesons than for pions.

Another difficulty involved here is that we still do not know definitely the relative parities or intrinsic spins of these particles. In this paper we assume that different baryons are different states of the same fermion, all having spin $\frac{1}{2}$ and the same intrinsic parity, and that the K mesons are pseudoscalar with respect to the baryons. The static version of the relativistic interactions[4] consistent with the Gell-Mann-Nishijima isospin assignments will be used.[5] In this scheme, the interaction Hamiltonian H_I contains eight coupling constants and as many cutoff functions. Even though there is no good reason to keep these parameters from changing for different processes, simplicity is of great importance in the present work since the number of terms contributing to the wave function increases quite rapidly as the order of approximation increases. In this spirit, the same cutoff function will be used uniformly for the pionic and K-mesonic contributions for all the baryons. Throughout this paper, we employ a square cutoff at $K=6$ ($\hbar=c=\mu=1$, where μ is the pion mass) so that comparison with I will be possible. In the same spirit, the square of the unrationalized, unrenormalized $NN\pi$ coupling constant will take the values 0.2, 0.4, and 0.6. The other coupling constants are assumed to satisfy the following relations;

$$(f_{NN\pi}^0)^2=(f_{\Sigma\Lambda\pi}^0)^2=(f_{\Sigma\Sigma\pi}^0)^2=(f_{\Xi\Xi\pi}^0)^2\equiv(f_\pi^0)^2, \quad (1a)$$

and

$$(f_{N\Lambda K}^0)^2=(f_{N\Sigma K}^0)^2=(f_{\Lambda\Xi K}^0)^2=(f_{\Sigma\Xi K}^0)^2\equiv(f_K^0)^2. \quad (1b)$$

One of the consequences of these simplifications is that invariance under rotations in this representation

[1] F. R. Halpern, L. Sartori, K. Nishimura, and R. Spitzer, Ann. Phys. 7, 154 (1959).
[2] G. F. Chew and F. E. Low, Phys. Rev. 101, 1570 (1956); G. C. Wick, Revs. Modern Phys. 27, 339 (1955).
[3] G. Sandri, Phys. Rev. 101, 1616 (1956).
[4] M. Gell-Mann, Phys. Rev. 106, 1296 (1957).
[5] W. G. Holladay, Phys. Rev. 115, 1331 (1959).

space which mix, for example, a nucleon and Ξ, or Σ and Λ, leads to the conclusion that the forces between baryons having the same hypercharge are equal to those between baryons having different hypercharges, which is in disagreement with experiment. Another well-known consequence is that all baryons will have the same observed mass, which again is in disagreement. In the present work, however, we ignore these disagreements in favor of keeping the calculations within a reasonable length.

NUCLEON WAVE FUNCTION

The Hamiltonian for the static model may be written as

$$H=H_0+H_I, \tag{2}$$

where the unperturbed part has the form

$$H_0=\sum_{k,i,\lambda}\{\omega_\pi(k)a_{i\lambda}^*(k)a_{i\lambda}(k) +\omega_K(k)[b_{i\lambda}^*(k)b_{i\lambda}(k)+c_{i\lambda}^*(k)c_{i\lambda}(k)]\}. \tag{3}$$

Here, $a_{i\lambda}(k)$, $b_{i\lambda}(k)$, and $c_{i\lambda}(k)$ are annihilation operators for a pion, a K meson, and a $\bar{K}$ meson, respectively, with linear momentum k, charge i, and z component of angular momentum λ; $\omega_\pi(k)=(1+k^2)^{\frac{1}{2}}$ and $\omega_K(k)=(m^2+k^2)^{\frac{1}{2}}$, where m is the K-meson mass.

The baryon wave function is described by a spinor

$$\psi=\begin{bmatrix}\psi_p\\ \psi_n\\ \psi_{\Sigma^+}\\ \psi_{Y^0}\\ \psi_{Z^0}\\ \psi_{\Sigma^-}\\ \psi_{\Xi^0}\\ \psi_{\Xi^-}\end{bmatrix}, \tag{4}$$

where ψ_p, ψ_n, etc., are, respectively, two component spinors for p, n, etc.: $Y^0=(\Lambda^0-\Sigma^0)/\sqrt{2}$, $Z^0=(\Lambda^0+\Sigma^0)/\sqrt{2}$.

The basis vectors in this representation space are

$$\phi_1\equiv\psi(p\uparrow)=\begin{bmatrix}1\\0\\ \vdots\\0\end{bmatrix},\quad \phi_2\equiv\psi(p\downarrow)=\begin{bmatrix}0\\1\\0\\ \vdots\\0\end{bmatrix},\quad \cdots,\quad \phi_{16}\equiv\psi(\Xi^-\downarrow)=\begin{bmatrix}0\\0\\ \vdots\\0\\1\end{bmatrix}, \tag{5}$$

and they represent the states of a bare baryon with a definite spin orientation as indicated by the arrow.

With the assumptions described in the introduction, the interaction Hamiltonian H_I can be written in the following form;

$$H_I=\sum_k(V_kA_k+U_kB_k+U_kC_k+\text{H.c.}), \tag{6}$$

where

$$V_k=\frac{f_\pi^0}{\sqrt{3}}\frac{kv(k)}{[2\omega_\pi(k)]^{\frac{1}{2}}}, \tag{6a}$$

and

$$U_k=\frac{f_K^0}{\sqrt{3}}\frac{kv(k)}{[2\omega_K(k)]^{\frac{1}{2}}}. \tag{6b}$$

The matrices A_k, B_k, and C_k are all 16×16 and they are given by

$$A_k=\begin{bmatrix}A_k' & 0 & 0 & 0\\ 0 & A_k' & 0 & 0\\ 0 & 0 & A_k' & 0\\ 0 & 0 & 0 & A_k'\end{bmatrix},\quad B_k=\begin{bmatrix}0 & B_k' & B_k'' & 0\\ 0 & 0 & 0 & B_k''\\ 0 & 0 & 0 & -B_k'\\ 0 & 0 & 0 & 0\end{bmatrix},\quad C_k=\begin{bmatrix}0 & 0 & 0 & 0\\ C_k' & 0 & 0 & 0\\ C_k'' & 0 & 0 & 0\\ 0 & C_k'' & -C_k' & 0\end{bmatrix}, \tag{7}$$

where

$$A_k'=\begin{bmatrix}a_{00}(k) & \sqrt{2}a_{0+}(k) & \sqrt{2}a_{+0}(k) & 2a_{++}(k)\\ \sqrt{2}a_{0-}(k) & -a_{00}(k) & 2a_{+-}(k) & -\sqrt{2}a_{+0}(k)\\ \sqrt{2}a_{-0}(k) & 2a_{-+}(k) & -a_{00}(k) & -\sqrt{2}a_{0+}(k)\\ 2a_{--}(k) & -\sqrt{2}a_{-0}(k) & -\sqrt{2}a_{0-}(k) & a_{00}(k)\end{bmatrix}, \tag{7a}$$

$$B_k'=\begin{bmatrix}\sqrt{2}b_{00}(k) & 2b_{0+}(k) & 0 & 0\\ 2b_{0-}(k) & -\sqrt{2}b_{00}(k) & 0 & 0\\ 0 & 0 & \sqrt{2}b_{00}(k) & 2b_{0+}(k)\\ 0 & 0 & 2b_{0-}(k) & -\sqrt{2}b_{00}(k)\end{bmatrix}, \tag{7b}$$

$$C_k'=\begin{bmatrix}\sqrt{2}c_{00}(k) & 2c_{0+}(k) & 0 & 0\\ 2c_{0-}(k) & -\sqrt{2}c_{00}(k) & 0 & 0\\ 0 & 0 & \sqrt{2}c_{00}(k) & 2c_{0+}(k)\\ 0 & 0 & 2c_{0-}(k) & -\sqrt{2}c_{00}(k)\end{bmatrix}, \tag{7c}$$

TABLE I. The self-energy of the nucleon, as a measure of convergence, as function of the coupling constants and order of approximation n.

$(f_\pi^0)^2$	$(f_K^0)^2$	$n=0$	1	2	3
0.2	0.02	0	-6.0	-7.7	-8.4
0.4	0.04	0	-9.3	-12.2	-13.0
0.6	0.06	0	-11.9	-15.8	-17.0

and B_k'' is equal to B_k' except that the first subscripts of the annihilation operators are changed from 0 to $+$ and, similarly, C_k'' is obtained from C_k' by changing the first subscripts of the annihilation operators from 0 to $-$.

The state vector for the physical nucleon is calculated with the method of moments.[6] A trial vector of the form $P_n(H)\phi_1$ is used, where P_n is a polynomial of degree n. Its coefficients are determined variationally so as to minimize the energy. The method of moments gives the following formula

$$P_n(H)=f_n(H)/(H-\bar{E}), \tag{8}$$

where

$$f_n(\xi)=\begin{vmatrix} 1 & \xi & \xi^2 & \cdots & \xi^n \\ H_0 & H_1 & H_2 & \cdots & H_n \\ H_1 & H_2 & H_3 & \cdots & \cdot \\ \vdots & & & & \vdots \\ H_{n-1} & \cdot & \cdot & \cdots & H_{2n-1} \end{vmatrix}, \tag{9}$$

and $\bar{E}$ is the lowest root of the equation $f_n(\xi)=0$ and this quantity represents the self-energy of the nucleon. This also gives a measure of convergence of our series. The self-energy of the nucleon is tabulated in Table I. In all cases, the convergence is no slower than those in I up to the third order and it is reasonable to assume an equally quick convergence for higher order wave functions. The symbols H_i used in the above determinant are defined by

$$H_i=\langle\phi_1|H^i|\phi_1\rangle, \tag{10}$$

and these are called the moments.

The moments are calculated in the manner described in I. The moments in all orders can be expressed in terms of the integrals I_m and J_m defined by

$$I_m=\sum|V_k|^2\omega_\pi^{m-2}(k)\to \frac{1}{3\pi}(f_\pi^0)^2\int k^4\omega_\pi^{m-3}|v(k)|^2dk, \tag{11a}$$

$$J_m=\sum|U_k|^2\omega_K^{m-2}(k)\to \frac{1}{3\pi}(f_K^0)^2\int k^4\omega_K^{m-3}|v(k)|^2dk. \tag{11b}$$

The first few moments are

$$\begin{aligned} &H_0=1, \qquad H_1=0,\\ &H_2=9I_2+12J_2, \quad H_3=9I_3+12J_3,\\ &H_4=171I_2^2+360I_2J_2+312J_2^2+12J_4+9I_4. \end{aligned} \tag{12}$$

It is evident that the number of terms contained is much greater than that in the corresponding moment in I. Already H_8 contains as many as 30 terms, out of which only 7 would remain if the K-meson interaction were switched off. This is why those simplifications regarding to the coupling constants and cutoff functions were given importance.

Once the trial function (in nth order) is found,

$$\psi_n=(\sum_{i=1}^n a_iH^i)\phi_1, \tag{13}$$

we can calculate $P(x;y,z)\equiv$probability of the core being

TABLE II. The probability (in %) of the physical proton core being in the 16 different states. Calculation was done with the third order trial functions.

$(f_\pi^0)^2$ $(f_K^0)^2$	0.2 0.02	0.4 0.04	0.6 0.06	0.2 0.1	0.4 0.2	0.6 0.3
$P(p\uparrow)$	47.85	43.16	40.81	52.41	45.78	43.15
$P(p\downarrow)$	11.87	12.77	13.33	7.62	8.40	8.74
$P(n\uparrow)$	11.85	12.74	13.30	7.34	8.02	8.31
$P(n\downarrow)$	23.68	24.40	26.58	14.67	16.05	16.61
$P(\Sigma^+\uparrow)$	0.78	0.97	0.99	2.85	3.24	3.42
$P(\Sigma^+\downarrow)$	0.97	1.24	1.16	4.66	5.08	5.25
$P(Y^0\uparrow)$	0.59	0.79	0.81	1.05	1.39	1.58
$P(Y^0\downarrow)$	0.0015	0.0007	0.0006	1.8×10^{-6}	4.0×10^{-7}	1.2×10^{-7}
$P(Z^0\uparrow)$	0.78	0.97	0.99	2.85	3.24	3.42
$P(Z^0\downarrow)$	0.97	1.24	1.16	4.66	5.08	5.25
$P(\Sigma^-\uparrow)$	0.59	0.79	0.81	1.05	1.39	1.58
$P(\Sigma^-\downarrow)$	0.0015	0.0007	0.0006	1.8×10^{-6}	4.0×10^{-7}	1.2×10^{-7}
$P(\Xi^0\uparrow)$	0.028	0.035	0.040	0.28	0.38	0.43
$P(\Xi^0\downarrow)$	0.050	0.067	0.074	0.56	0.76	0.86
$P(\Xi^-\uparrow)$	0.0035	0.0035	0.0047	2.3×10^{-5}	9.7×10^{-6}	4.3×10^{-6}
$P(\Xi^-\downarrow)$	0	0	0	0	0	0

[6] F. R. Halpern, Phys. Rev. **107**, 1145 (1957).

TABLE III. Probability (in %) of y pions and z K mesons surrounding the core of the physical nucleon in the nth order trial function.

			(y,z)									
$(f_\pi{}^0)^2$	$(f_K{}^0)^2$	n	(0,0)	(1,0)	(0,1)	(2,0)	(1,1)	(0,2)	(3,0)	(2,1)	(1,2)	(0,3)
0.2	0.02	1	64.74	31.82	3.45							
		2	48.09	38.75	3.67	8.31	1.08	0.10				
		3	38.19	38.31	2.66	15.54	1.70	0.13	3.12	0.30	0.027	0.0039
0.4	0.04	1	60.65	35.50	3.84							
		2	41.14	41.82	4.07	11.33	1.47	0.14				
		3	32.68	40.91	3.48	17.35	2.01	0.18	3.10	0.30	0.027	0.0039
0.6	0.06	1	58.76	37.21	4.03							
		2	38.00	42.96	4.25	12.95	1.68	0.16				
		3	29.36	40.58	3.17	19.90	2.34	0.20	4.06	0.40	0.037	0.0051
0.2	0.1	1	63.32	23.80	12.88							
		2	47.41	29.00	14.30	4.83	3.13	1.51				
		3	47.26	29.12	13.98	4.86	3.14	1.48	8.1×10^{-4}	4.0×10^{-4}	1.8×10^{-4}	1.3×10^{-4}
0.4	0.2	1	59.60	25.21	14.19							
		2	41.19	30.92	15.26	6.45	4.19	1.99				
		3	40.00	30.93	15.25	6.46	4.19	1.98	3.5×10^{-4}	1.7×10^{-4}	7.7×10^{-5}	5.4×10^{-5}
0.6	0.3	1	57.88	27.32	14.79							
		2	38.37	31.55	15.78	7.31	4.75	2.25				
		3	37.01	31.53	15.76	7.31	4.74	2.25	1.5×10^{-4}	7.6×10^{-5}	3.4×10^{-5}	1.2×10^{-5}

in the state ϕ_x, surrounded by y pions and z K mesons (and $\bar{K}$ mesons). Table II shows the probability $P(x)$ of the core being in the state ϕ_x, i.e.,

$$P(x)\equiv\sum_{y,z}P(x;y,z). \tag{14}$$

For this table, the third order trial functions were used. Notice that $P(p\downarrow)$ is no longer equal to $P(n\uparrow)$ because of the K-meson interactions, and the difference is larger for the stronger K-meson couplings. Table III shows the probability

$$P(y,z)\equiv\sum_x P(x;y,z), \tag{15}$$

of y pions and z K mesons (and $\bar{K}$ mesons) surrounding the core.

We can make observations similar to those made in I. First, the probability, when regarded as a function of the order of approximation, seems to have a single maximum and it decreases uniformly after passing it. A second observation is that the average number of mesons surrounding the core is again too large to justify the one-meson approximation. As the order of approximation is increased still further, states with larger number of mesons will definitely make significant contributions to the wave function.

NUCLEON ANOMALOUS MAGNETIC MOMENT

The contribution of the core to the magnetic moment of the physical proton is, in units of the nuclear magneton,

$$\mu_{pc}=P(p\uparrow)-P(p\downarrow)+P(\Sigma^+\uparrow)-P(\Sigma^+\downarrow)$$
$$-P(\Sigma^-\uparrow)+P(\Sigma^-\downarrow)-P(\Xi^-\uparrow)+P(\Xi^-\downarrow). \tag{16}$$

That to the magnetic moment of the physical neutron is

$$\mu_{nc}=P(n\uparrow)-P(n\downarrow)+P(Y^0\uparrow)-P(Y^0\downarrow)$$
$$-P(Z^0\uparrow)+P(Z^0\downarrow)-P(\Xi^0\uparrow)+P(\Xi^0\downarrow). \tag{17}$$

This is so because our Hamiltonian is invariant under the transformation

$$p\leftrightarrow n,\ \Sigma^+\leftrightarrow Y^0,\ \Sigma^-\leftrightarrow Z^0,\ \Xi^0\leftrightarrow\Xi^-,\ \pi^0\leftrightarrow -\pi^0,$$
$$\pi^\pm\leftrightarrow\pi^\mp,\ K^\pm\leftrightarrow K^\pm,\ K^0\leftrightarrow K^0,\ \bar{K}^0\leftrightarrow\bar{K}^0. \tag{18}$$

This invariance property shows that pions contribute only to the vector part and K mesons only to the scalar part of the anomalous magnetic moment. The operators for their contributions are,[7] respectively,

$$\sum_k\frac{M}{\omega_\pi(k)}[a_{++}{}^*a_{++}-a_{+-}{}^*a_{+-}-a_{-+}{}^*a_{-+}+a_{--}{}^*a_{--}$$
$$+a_{--}{}^*a_{++}{}^*+a_{--}a_{++}-a_{+-}{}^*a_{-+}{}^*-a_{+-}a_{-+}], \tag{19}$$

and

$$\sum_k\frac{M}{\omega_K(k)}[b_{++}{}^*b_{++}-b_{+-}{}^*b_{+-}+c_{--}{}^*c_{--}-c_{-+}{}^*c_{-+}$$
$$+b_{++}{}^*b_{+-}{}^*+b_{++}b_{+-}-c_{--}{}^*c_{-+}{}^*-c_{--}c_{-+}], \tag{20}$$

TABLE IV. Calculated and extrapolated values of the scalar part of the nucleon anomalous magnetic moment.

$(f_\pi{}^0)^2$	$(f_K{}^0)^2$	$n=1$	2	3	Extrap.
0.2	0.02	−0.22	−0.30	−0.35	−0.38
0.4	0.04	−0.24	−0.34	−0.38	−0.40
0.6	0.06	−0.25	−0.35	−0.40	−0.43
0.2	0.1	−0.18	−0.19	−0.19	−0.19
0.4	0.2	−0.19	−0.22	−0.22	−0.23
0.6	0.3	−0.20	−0.23	−0.23	−0.24

[7] R. G. Sachs, Phys. Rev. 87, 1100 (1952).

TABLE V. Calculated and extrapolated values of the vector part of the nucleon anomalous magnetic moment.

$(f_\pi^0)^2$	$(f_K^0)^2$	$n=1$	2	3	Extrap.
0.2	0.02	0.06	0.33	0.59	1.2
0.4	0.04	0.04	0.36	0.62	1.6
0.6	0.06	0.04	0.37	0.64	2.0
0.2	0.1	−0.04	0.14	0.14	0.14
0.4	0.2	−0.04	0.14	0.14	0.14
0.6	0.3	−0.04	0.14	0.14	0.14

where M is the nucleon mass and the arguments of the creation and annihilation operators are omitted. If μ_π and μ_K are the matrix elements of these operators in the trial state, the scalar and vector parts of the nucleon anomalous magnetic moment are

$$\mu_S=\tfrac{1}{2}(\mu_{pc}-1+\mu_{nc})+\mu_K, \tag{21}$$

$$\mu_V=\tfrac{1}{2}(\mu_{pc}-1-\mu_{nc})+\mu_\pi. \tag{22}$$

Our calculation shows, as in I, that the matrix elements of the meson operators are quite slowly convergent while those of the core operators converge fairly quickly. Since calculation was stopped at the third order in this present work, a freehand extrapolation is not trustworthy. But the speed of convergence is very much like that which occurred in I. The extrapolated values of μ_S and μ_V in Tables IV and V are obtained by comparison with I.

A curious thing that we observe is that the convergence becomes very fast as the K-meson coupling constant becomes large with respect to the pion interaction. In this case, not only the scalar part but the vector part becomes much too small. Experimentally, $\mu_V=1.85$.

A comparison with I shows that the inclusion of the K-meson interaction causes a change in the scalar part by approximately 10% and this seems to be the best that one can expect to do with the static-source meson theory.

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