transformation

$$M_{l+} \leftrightarrow -M_{l-}$$

M_{l+} and M_{l-} correspond both to magnetic $2l$ pole radiation [parity $(-1)^{l+1}$] but the total angular momentum is $l+\frac{1}{2}$ for M_{l+} and $l-\frac{1}{2}$ for M_{l-}. These invariance properties can be checked in the tables.

We also notice that under the transformation

$$M_{l+} \leftrightarrow -E_{(l+1)-}, \quad M_{l-} \leftrightarrow E_{(l-1)+},$$

the angular distribution for $\varphi=0°$ goes over into the angular distribution for $\varphi=90°$ and vice versa. This does not occur with the polarization. As a consequence, the angular distribution, but not the polarization for unpolarized photons, is invariant under this transformation. This is the well-known Minami invariance.[7] The relevance of this ambiguity to high-energy photoproduction experiments has been emphasized by Sakurai[8] and Moravcsik.[9]

ACKNOWLEDGMENT

We wish to give thanks to Professor J. J. Sakurai for his guidance and criticism while working on this article.

[7] See, for instance, Hayakawa *et al.*, reference 4. The discrepancy between the sign of E_{l-} in this article and in ours is due to a different definition of these amplitudes.
[8] J. J. Sakurai, Phys. Rev. Letters **1**, 258 (1958).
[9] M. J. Moravcsik, Phys. Rev. Letters **2**, 171 (1959).

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Range of Proton-Antiproton Annihilation Near 1.0 Bev

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The range of the proton-antiproton annihilation was calculated for antiproton with energy near 1 Bev. The point is to get the range of pure annihilation interaction, separating the effect of pion production. It was found that the root mean square of this range is given by $(1.19\pm0.07)\times10^{-13}$ cm almost independently of the energy.

THE range of proton-antiproton annihilation is of special interest in connection with the problem of nucleon structure, since it gives in some sense direct information about the core size. Lévy estimated this to be 1.43×10^{-13} cm.[1] His result is, however, subject to an ambiguity, since in his calculation the effect of pion production is not separated. Recently, the pion production cross section in p-$\bar{p}$ collision was found to be (5 ± 1) mb at 940 Mev.[2] It seems possible to use this new information to eliminate to some extent the ambiguity in Lévy's calculation.

If we denote the phase shift due to pure annihilation as η_l, and the correction to η_l due to pion production as $\delta\eta_l$, the total inelastic cross section and the scattering amplitude are given by

$$\sigma_{\text{inel}}=\pi\lambda\!\!\!^{-2} \sum (2l+1)(1-e^{4i(\eta_l+\delta\eta_l)}), \tag{1}$$

and

$$f(\theta)=\tfrac{1}{2}i\lambda\!\!\!^{-} \sum (2l+1)(1-e^{2i(\eta_l+\delta\eta_l)})P_l(\cos\theta), \tag{2}$$

respectively, where $\lambda\!\!\!^{-}$ is the wavelength of the incident particle in the c.m. system, and the spins of both particles were neglected. η_l and $\delta\eta_l$ are assumed to be imaginary.

Equation (1) can be written as

$$\sigma_{\text{inel}}=\pi\lambda\!\!\!^{-2}\{ \sum (2l+1)(1-e^{4i\eta_l}) + \langle e^{4i\eta}\rangle \sum (2l+1)(1-e^{4i\delta\eta_l})\}, \tag{3}$$

where $\langle e^{4i\eta}\rangle$ is the average value of $e^{4i\eta_l}$ over the range of l in which $\delta\eta_l$ is appreciably different from zero. The first term on the right-hand side of (3) is the pure annihilation cross section. To relate the second term to observed quantities, we assume first that the phase shifts due to annihilation and pion production can be defined separately and are additive, and second that the mechanism of pion production is essentially the same for p-p and p-$\bar{p}$ collisions. Then $\delta\eta_l$ can be identified with the phase shift for p-p collision with the same energy, and (3) can be written as

$$\sigma_{\text{inel}}=\sigma_{\text{an}}+\langle e^{4i\eta}\rangle\sigma_{\text{pro}}(pp),$$

where $\sigma_{\text{pro}}(pp)$ is the pion production cross section for p-p collision with the same energy. The second term in the right-hand side of (3) is the pion production cross section in p-$\bar{p}$ collision, and from this we see that $\langle e^{4i\eta}\rangle$ can be expressed as

$$\langle e^{4i\eta}\rangle=\sigma_{\text{pro}}(p\bar{p})/\sigma_{\text{pro}}(pp), \tag{4}$$

where $\sigma_{\text{pro}}(p\bar{p})$ is the pion production cross section in p-$\bar{p}$ collision.

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[1] M. Lévy, Phys. Rev. Letters **5**, 377 (1960).
[2] O. Chamberlain, *Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960* (Interscience Publishers, New York, 1960).

In the same way, (2) can be transformed into

$$f(\theta)=\tfrac{1}{2}i\bar{\lambda}\{\sum(2l+1)(1-e^{2i\eta_l})P_l(\cos\theta)+\langle e^{2i\eta}\rangle\sum(2l+1)(1-e^{2i\delta\eta_l})P_l(\cos\theta)\},\quad(5)$$

where $\langle e^{2i\eta}\rangle$ is the average value of $e^{2i\eta_l}$ taken in the same way as in $\langle e^{4i\eta}\rangle$. The first term on the right-hand side of (5) is the amplitude for the diffraction scattering due to annihilation, and the second term is, with the assumption that $\delta\eta_l$ is equal to the phase shift for p-p collision with the same energy, equal to the amplitude for the diffraction scattering for p-p collision except for the factor $\langle e^{2i\eta}\rangle$.[3] Writing these amplitudes as $f_{\rm an}(\theta)$ and $f_{\rm pro}(pp,\theta)$, respectively, we get

$$f(\theta)=f_{\rm an}(\theta)+\langle e^{2i\eta}\rangle f_{\rm pro}(pp,\theta).\quad(6)$$

To express $f_{\rm an}(\theta)$ in terms of observed quantities only, it is necessary to assume further that

$$\langle e^{2i\eta}\rangle=(\langle e^{4i\eta}\rangle)^{\frac{1}{2}}.\quad(7)$$

Then, we get from (6)

$$f_{\rm an}(\theta)=i\left\{\left(\frac{d\sigma_{\rm el}(p\bar{p})}{d\omega}\right)^{\frac{1}{2}}-\left(\frac{\sigma_{\rm pro}(p\bar{p})}{\sigma_{\rm pro}(pp)}\right)^{\frac{1}{2}}\left(\frac{d\sigma_{\rm el}(pp)}{d\omega}\right)^{\frac{1}{2}}\right\},\quad(8)$$

where $d\sigma_{\rm el}(p\bar{p})/d\omega$ and $d\sigma_{\rm el}(pp)/d\omega$ are differential cross sections for the elastic parts in p-$\bar{p}$ and p-p collisions, and we have used a well-known relation

$$d\sigma/d\omega=|f(\theta)|^2.$$

The assumptions made to obtain (8) are: (i) Phase shifts due to annihilation and pion production can be defined separately, and they are additive; (ii) $\delta\eta_l$ are equal to the phase shift for p-p collision with the same energy; (iii) $\langle e^{2i\eta}\rangle=(\langle e^{4i\eta}\rangle)^{\frac{1}{2}}$; (iv) the real part of the phase shift can be neglected. If we describe annihilation and pion production by some imaginary potentials, (i) is correct in the Born approximation, but it can not be proved rigorously. (ii) would be reasonable if (i) is correct, and moreover the wave function of the incident antiproton in the pion production region is not so much altered by the annihilation process, since the pion cloud would be the same for proton and antiproton. The validity of (iii) can be checked to some extent by using the value of the parameters determined in the following. (iv) should be reasonable in the high-energy region ($\sim$1 Bev) which is considered here. So the validity of (8) rests mainly upon whether the annihilation and pion production interaction is regarded as weak or not. Unfortunately, we can not expect that this condition is well satisfied, and in this sense our analysis is also not completely free from difficulty. It is expected, however, that $f(\theta)$ is not so sensitive to the small variations of the phase shifts with small l because of the factor $(2l+1)$ and also because $e^{2i\eta_l}$ or $e^{2i\delta\eta_l}$ itself is small for small l. For large l, our assumptions should be pretty good, since impact parameter is large there. Thus, it may be hoped that this way of analysis can be used at least as the first approximation.

If the effect of the pion production is subtracted, we can determine the annihilation range assuming a formula given by Belenkii[4]:

$$\alpha_l=1-e^{2i\eta_l}=\alpha\exp(-l^2\bar{\lambda}^2/R^2).\quad(9)$$

Substituting (9) for η_l and replacing the sum over l by an integral, we get, neglecting terms of order $(\bar{\lambda}/R)$,

$$\sigma_{\rm an}=\tfrac{1}{2}\pi\alpha^2R^2\left(\frac{4}{\alpha}-1\right),\quad(10)$$

$$\sigma_d=\tfrac{1}{2}\pi\alpha^2R^2,\quad(11)$$

and

$$f_{\rm an}(\theta)=\frac{\alpha R^2}{2\bar{\lambda}}\exp(-\theta^2R^2/4\bar{\lambda}^2),\quad(12)$$

respectively, where σ_d is the diffraction cross section due to annihilation.

From (10)–(12), the annihilation range can be determined in two different ways. The first way is to use the total cross sections. From (10) and (11) we get

$$\alpha=4(\sigma_d/\sigma_t),\quad(13)$$

and

$$R=(\sigma_t/2\pi\alpha)^{\frac{1}{2}},\quad(14)$$

where $\sigma_t=\sigma_{\rm an}+\sigma_d$. σ_d is obtained by integrating numerically $|f_{\rm an}(\theta)|^2$ given by (8), and $\sigma_{\rm an}=\sigma_{\rm inel}-\sigma_{\rm pro}(p\bar{p})$. The second way is to use the angular distributions. From (12) we get

$$\ln(f_{\rm an}(\theta)/\bar{\lambda})=\ln(\alpha R^2/2\bar{\lambda}^2)-\theta^2R^2/4\bar{\lambda}^2.\quad(15)$$

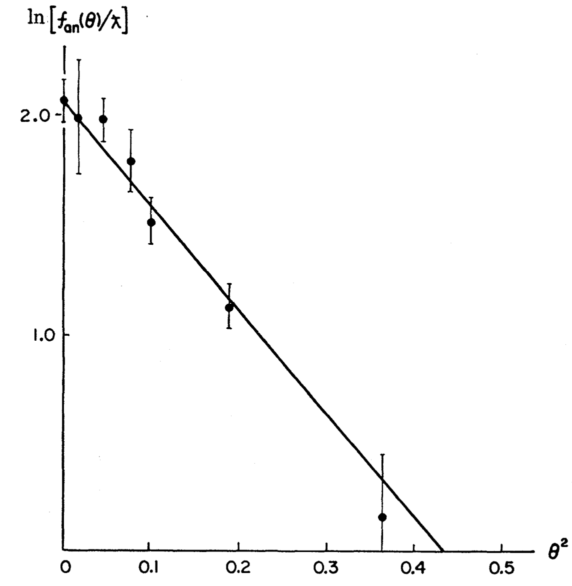

FIG. 1. Plot of $\ln[f_{\rm an}(\theta)/\bar{\lambda}]$ versus θ^2 at 1.25 Bev.

[3] For simplicity, we assume that $\langle e^{2i\eta}\rangle$ is independent of θ.

[4] S. Belenkii, J. Exptl. Theoret. Phys. (U. S. S. R.) **30**, 983 (1956) [translation: Soviet Phys.-JETP **3**, 813 (1956)].

TABLE I. Determination of R and α.

	R (in 10^{-13} cm)		α	
Energy (in Bev)	From total cross sections	From angular distribution	From total cross sections	From angular distribution
0.95	...	1.12±0.03	...	0.89±0.08
1.00	1.13±0.13	1.15±0.06	0.92±0.15	0.94±0.11
1.07	...	1.12±0.06	...	0.88±0.13
1.25	1.11±0.13	1.16±0.06	0.85±0.14	0.84±0.10

Therefore, if $\ln(f_{\rm an}(\theta)/\lambdabar)$ is plotted against θ^2, it should be linear, and its slope gives $(R^2/4\lambdabar^2)$ and the intersection with the vertical axis gives $\ln(\alpha R^2/2\lambdabar^2)$.

The calculation was made at 0.95 Bev, 1.00 Bev, 1.07 Bev, and 1.25 Bev, where the necessary data are available now.[5,6] For $\sigma_{\rm pro}(p\bar{p})$ we used Chamberlain's value at 0.94 Bev, and for $\sigma_{\rm pro}(pp)$ and $d\sigma_{\rm el}(pp)/d\omega$ we use the values at 1.0 Bev of Smith *et al.* and Duke *et al.*[7] As an example, the plot of $\ln(f_{\rm an}(\theta)/\lambdabar)$ versus θ^2 at 1.25 Bev is given in Fig. 1. It shows a remarkable linearity as expected. At 0.95 Bev and 1.07 Bev, the measured distribution is not so accurate as to allow the numerical integration of $|f_{\rm an}(\theta)|^2$, and only the determination from the plot of $\ln(f_{\rm an}(\theta)/\lambdabar)$ versus θ^2 was possible. At 1.00 Bev and 1.25 Bev where both methods are applicable, the agreement between two determinations was satisfactory. The results are given in Table I.

From Table I we see that R and α are given by

$$R=(1.13\pm0.06)\times10^{-13}\text{ cm},\quad\text{and}\quad \alpha=0.89\pm0.10,$$

almost independently of the energy at least near 1.0 Bev.

Applying the same methods, we get $R=(0.77\pm0.26)\times10^{-13}$ cm and $\alpha=0.93\pm0.14$ from the 1.4-Bev pion-nucleon collision,[8] and $R=(0.70\pm0.24)\times10^{-13}$ cm, and $\alpha=1.07\pm0.20$ from the 6.2-Bev p-p collisions.[9] Thus, the range of proton-antiproton annihilation is really large, even larger than the range of pion production. As emphasized by Lévy, this is rather a puzzling result, since following the familiar argument of pion physics, this range is expected to be of order of $2\times$(Compton wavelength of the nucleon)$\simeq0.4\times10^{-13}$ cm.

The root mean squares of these ranges are given by[4]

$$\langle r^2\rangle_{\rm av}^{\frac{1}{2}}=\left(\frac{1-\alpha/8}{1-\alpha/4}\right)^{\frac{1}{2}}R. \tag{16}$$

From this we get

$$\langle r_{\rm an}^2\rangle_{\rm av}^{\frac{1}{2}}=(1.19\pm0.07)\times10^{-13}\text{ cm}.$$

This shows that, if we assume that Lévy's value corresponds to our $\langle r_{\rm an}^2\rangle_{\rm av}^{\frac{1}{2}}$, about 20% of his value should be due to pion production.[10]

ADDED NOTE

Recently, the total cross section for p-$\bar{p}$ collision was measured to about 30 Bev at CERN.[11] At 12 Bev, the total inelastic cross section was found to be about 37 mb. CERN workers ascribe 29 mb of this cross section to pion production process, assuming that the pion production cross section is the same for p-$\bar{p}$ and nucleon-nucleon collisions. It seems to the author that this assumption is not justified, because even if the mechanism of pion production is the same for p-$\bar{p}$ and nucleon-nucleon collisions, the pion production cross section in p-$\bar{p}$ collision should be reduced due to the fact that the incident antiproton flux in the pion production region is strongly damped by the annihilation process. This effect is expressed by the factor $\langle e^{4i\eta}\rangle$ in Eq. (3).

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[9] B. Cork, W. A. Wenzel, and C. W. Causey, Phys. Rev. **107**, 859 (1957).

[10] Using (16), we get $\langle r_{\rm pro}^2\rangle_{\rm av}^{\frac{1}{2}}=(0.81\pm0.27)\times10^{-13}$ from the 1.4-Bev pion-nucleon collisions, $=(0.74\pm0.25)\times10^{-13}$ cm from the 6.2-Bev p-p collisions, as the root mean square of the range of pion production. This agrees well with the results given by other authors [D. Blohincev, V. Barašenkov, and V. Grišin, Nuovo cimento **9**, 249 (1958)].

[11] A. M. Wetherell, *Report on the Conference on Strong Interaction*, Berkeley, December, 1960 [Bull. Am. Phys. Soc. **5**, 516 (1960)].