PHYSICAL REVIEW VOLUME 122, NUMBER 3 MAY 1, 1961

Interpretation of Elastic π^+–p Scattering at 1.1 Bev

B. J. MALENKA
Northeastern University, Boston, Massachusetts

AND

H. S. VALK
University of Nebraska, Lincoln, Nebraska

(Received December 27, 1960)

Using a high-energy approximation, it is shown that the π^+-p elastic scattering data at 1.1 Bev can be interpreted in terms of coherent scattering produced by an absorptive Gaussian well having a root-mean-square range of the size of the proton charge radius plus a short-range interaction whose principal effect is represented phenomenologically as a contribution to the S wave.

I. INTRODUCTION

THE experimental data obtained by Roellig and Glaser for the angular distribution for π^+-p elastic scattering at 1.1 Bev exhibits a large forward peak and relatively isotropic scattering at backward angles.[1] The peak has been interpreted in terms of optical model diffraction scattering, while the isotropic angular distribution has been attributed,[1] as in the interpretation of π^--p scattering,[2] to incoherent elastic scattering arising from the formation of a compound pion-proton system which decays isotropically. To the present authors, the hypothesis of the formation of a compound system which decays isotropically at high energies seems somewhat improbable since it would appear to require that the energy of the incident pion be shared with the virtual pion cloud in the extremely short collision time associated with a high-energy scattering.

In this paper, we will show by way of a simple application of the procedure of our previous note[3] that the observed elastic π^+-p scattering at 1.1 Bev can also be interpreted as principally coherent. In particular, it can be described by an optical model potential having a radius determined by the Stanford electron scattering experiments plus a short-range interaction which will be treated phenomenologically at present. In so doing, we also assume that any spin-flip contribution to the cross section can be neglected. Our justification is a gain in simplicity for our calculation and the fit of our final results.

II. ELASTIC π^+-p SCATTERING AT 1.1 BEV

Neglecting Coulomb scattering and assuming that in addition to a long-range optical potential $V(r)$, the incident pion sees a short-range interaction of range a such that we may then use Eq. (9) of the preceding paper[3]:

$$f(\theta)=\frac{k}{i}\int_0^\infty\left[J_0(2kb\sin\tfrac{1}{2}\theta)-\frac{1}{kb}J_1(2kb)\right]\{e^{i\chi(b)}-1\}bdb+k^{-1}\Delta,\quad(1)$$

where

$$\Delta=e^{i\delta}\sin\delta,\quad(2)$$

and

$$\chi(b)=-\frac{m}{\hbar^2k}\int_{-\infty}^{\infty}V(b,z)dz.\quad(3)$$

The scattering angle θ is measured in the center-of-mass system, and the notation employed is essentially[4] that of reference 3. In this approximation, it will be recalled that all the effects of the short-range interaction are included in the S-wave phase shift.

If we take the optical potential $V(r)$ to be a Gaussian well with real and absorptive parts, then

$$V(r)=-(V_0+iW_0)\exp[-r^2/R^2],\quad(4)$$

and using $r^2=b^2+z^2$, we have

$$\chi(b)=\alpha(V_0+iW_0)\exp[-b^2/R^2],\quad(5)$$

where

$$\alpha=mkR\pi^{\frac{1}{2}}/\hbar^2k^2.\quad(6)$$

Expanding the exponential of Eq. (1) and substituting Eq. (5) into Eq. (1), we find

$$f(\theta)=\frac{k^2R^2}{i}\int_0^\infty\left[J_0(2kRx\sin\tfrac{1}{2}\theta)-\frac{1}{kRx}J_1(2kRx)\right]\times\sum_{n=1}^{\infty}\frac{[i\alpha(V_0+iW_0)]^n}{n!}\exp[-nx^2]xdx+k^{-1}\Delta,\quad(7)$$

where $x=b/r$. In this form, we may then carry through the integration using the formula of Weber and Sonine[5]

[1] L. O. Roellig and D. A. Glaser, Phys. Rev. **116**, 1001 (1959).
[2] L. M. Eisberg, W. B. Fowler, R. M. Lea, W. D. Shephard, R. P. Shutt, A. M. Thorndike, and W. L. Whittemore, Phys. Rev. **97**, 797 (1955).
[3] B. J. Malenka and H. S. Valk, preceding paper [Phys. Rev. **122**, 931 (1961)]. A preliminary report of this work will also be found in B. J. Malenka and H. S. Valk, Bull. Am. Phys. Soc. **5**, 269 (1950).
[4] See Appendix for further comments on the interpretation of m, k, and V.
[5] The expressions for the definite integrals involving J_0 and J_1 may be obtained or directly deduced from W. Magnus and F. Oberhettinger, *Special Functions of Mathematical Physics* (Chelsea Publishing Company, New York, 1949), p. 35.

to obtain[6]

$$f(\theta)=\frac{kR^2}{i}\sum_{n=1}^{\infty}\frac{[i\alpha(V_0+iW_0)]^n}{n!}\left\{\frac{1}{2n}\exp(-\beta^2/4n)-\frac{1}{2k^2R^2}[1-\exp(-k^2R^2/n)]\right\}+k^{-1}\Delta, \quad (8)$$

where

$$\beta=2kR\sin\tfrac{1}{2}\theta. \quad (9)$$

For the purpose of comparison with experiment, considerable simplification in calculation is gained by making use of the dispersion-relation calculations of Sternheimer[7] which indicate that at 1.1 Bev, the real part of the π^+-p forward scattering amplitude is fortuitously zero. Actually, the more recent calculations of Cronin[8] indicate a small value of 0.15×10^{-13} cm. However, this value is below his estimated error at 1.1 Bev, so that, for simplicity, we will take the real part of the forward scattering to be zero in this paper. Using the condition $\mathrm{Re}f(0)\approx0$, then permits us to make the additional simplifying assumptions that $\mathrm{Re}\Delta\approx0$ and $V_0\approx0$. With $V_0\approx0$, the long-range interaction can be thought of as representing a diffuse absorbing sphere. Under these conditions, Eq. (8) becomes

$$f(\theta)=\frac{kR^2}{i}\sum_{n=1}^{\infty}\frac{(-\alpha W_0)^n}{n!}\left\{\frac{1}{2n}\exp(-\beta^2/4n)-\frac{1}{2k^2R^2}[1-\exp(-k^2R^2/n)]\right\}+ik^{-1}\Delta_i, \quad (10)$$

where

$$\Delta_i=\mathrm{Im}\Delta=\mathrm{Im}e^{i\delta}\sin\delta. \quad (11)$$

As usual, the differential cross section $d\sigma/d\Omega$ is found by taking the absolute square of Eq. (10), and the total cross section is obtained from

$$\sigma_T=(4\pi/k)\,\mathrm{Im}f(0). \quad (12)$$

To find parameters that would fit the experimental points shown in Fig. 1, a trial and error procedure was used. First $d\sigma/d\Omega$ and σ_T were evaluated only to second order in αW_0. When rough agreement with experiment was found, smaller ranges of the parameters were examined for $d\sigma/d\Omega$ and σ_T calculated to fourth order in αW_0. The fourth-order results were not significantly different from the second order, thereby justifying to some extent our use of Eq. (10) as an expansion in αW_0.

Our best fit to the data is shown by the solid line in Fig. 1. The parameters are $(kR)^2\approx4.8$, $\alpha W_0\approx0.88$, and $\Delta_i\approx0.9$. This gives $R\approx0.67\times10^{-13}$ cm which corresponds to a root-mean-square radius of 0.82×10^{-13} cm for the Gaussian well. This result falls within the range of the charge radius of the proton $(\langle r^2\rangle_{\mathrm{av}})^{\frac{1}{2}}\approx(0.77\pm0.10)\times10^{-13}$ cm found in the Stanford electron scattering experiments.[9] The depth of the absorbing part of the well is $W_0\approx0.23$ Bev. From Eq. (12) evaluated to fourth order in αW_0 with the above parameters, we find $\sigma_T\approx27$ mb which is consistent with the experimental value of $\sigma_T=28.8$ mb$\pm10\%$ found by Cool, Piccioni, and Clark.[10]

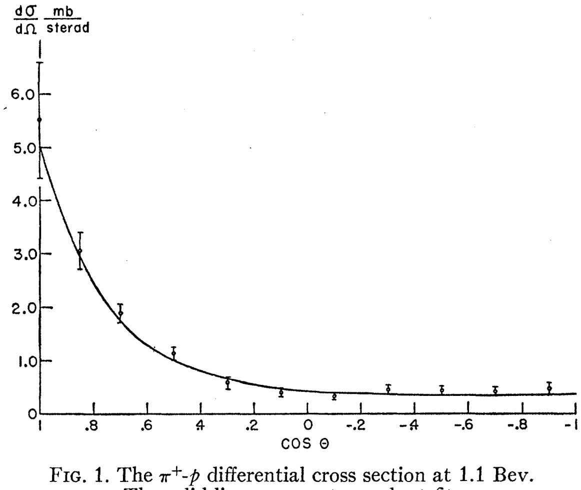

FIG. 1. The π^+-p differential cross section at 1.1 Bev. The solid line represents our best fit.

In order to test the sensitivity of the results to well-shape, an attempt was also made to fit the data with a square well. For

$$\begin{aligned}V(r)&=-(V_0+iW_0), & r&\leqslant R,\\ V(r)&=0, & r&>R,\end{aligned} \quad (13)$$

we find from Eqs. (1) and (2) that

$$f(\theta)=ikR^2\frac{J_1(2kR\sin\frac{1}{2}\theta)}{2kR\sin\frac{1}{2}\theta}-ikR^2\int_0^1 xdx\times\exp[(i2mR/\hbar^2k)(V_0+iW_0)(1-x^2)^{\frac{1}{2}}]J_0(2kRx\sin\tfrac{1}{2}\theta)+\frac{i}{2k}[J_0(2kR)-1]+iR\int_0^1 dx\exp[(i2mR/\hbar^2k)\times(V_0+iW_0)(1-x^2)^{\frac{1}{2}}]J_1(2kRx)+k^{-1}\Delta. \quad (14)$$

If the exponentials are expanded, the integrals appearing above may be evaluated[11] yielding a series expansion for $f(\theta)$ somewhat analogous to Eq. (8). In practice, since we confined our attention to values of R giving root-mean-square values in the range of the Stanford proton charge radius, the use of numerical integration was not too difficult. Again, the vanishing of the real

[6] A somewhat similar expansion without the S-wave subtraction has also been obtained by I. I. Shapiro, thesis, Harvard University, 1955 (unpublished).
[7] R. M. Sternheimer, Phys. Rev. **101**, 384 (1956).
[8] J. W. Cronin, Phys. Rev. **118**, 824 (1960).
[9] E. E. Chambers and R. Hofstadter, Phys. Rev. **103**, 1454 (1956).
[10] R. Cool, O. Piccioni, and D. Clark, Phys. Rev. **103**, 1082 (1956).
[11] See reference 5, p. 30.

part of the forward scattering amplitude was obtained most simply by taking $\mathrm{Re}\Delta \approx 0$ and $V_0 \approx 0$.

Our square-well results corroborate some of the statements of Roellig and Glaser[1] in that we are unable to find a good fit to their data for coherent scattering by a sharp-edged absorbing sphere without also introducing an incoherent contribution. The difficulty seems to be in the deep diffraction minimum in the differential cross section produced by a square well. In particular, we are unable to fit $d\sigma/d\Omega$ in the region of this minimum (which corresponds to $0.8 > \cos\theta > 0.2$) and still preserve a reasonable fit to the rest of the angular distribution.

III. DISCUSSION AND INTERPRETATION OF RESULTS

The relatively good agreement with experiment shown in Fig. 1 indicates that the π^+-p elastic scattering data at 1.1 Bev can be interpreted in terms of coherent scattering by a diffuse absorbing sphere represented by a Gaussian well having an imaginary depth of about 0.23 Bev and a root-mean-square range of about 0.82×10^{-13} cm plus a short-range interaction whose effect is felt principally in the S wave. Our examination of the square well case would seem to indicate that the diffuseness of the edge of the well is necessary for a coherent scattering interpretation of elastic π^+-p scattering at this energy, since no strong dips are exhibited in the data after the large forward peak. Since this is contrary to the data at lower energies,[12] it would be of considerable interest to determine experimentally if the suppression of relative dips and nonforward maxima persists at higher energies.

Referring to the short-range contribution to the S-wave phase shift, from Eq. (20) of reference 3, we have

$$\delta_a \approx -\tfrac{1}{2} i \ln(2i\Delta+1) + \frac{m}{\hbar^2 k} \int_0^\infty V(r) dr. \qquad (15)$$

Using $\Delta = i\Delta_i \approx 0.9i$ and the other parameters determined above for our Gaussian well, we find

$$\delta_a = \pm \pi/2 - 0.33i. \qquad (16)$$

To attempt to give an explicit interpretation of this value from a fundamental theory is beyond the intent of this paper. However, it is of some interest to examine the consequence of simply assuming that δ_a results from a short-range square well interaction. The resulting range and well depths calculated using Eq. (16) are not unique. However, a reasonable interpretation, namely that the height of the imaginary part of this well should not exceed 0.23 Bev, roughly requires that $a/R \geq 0.5$. Since the minimum value is most consistent with the approximations used above to obtain δ_a, we take $a/R \approx 0.5$. This corresponds to assuming a ratio of the root-mean-square radii which is approximately 0.25. We then have $a \approx 0.33 \times 10^{-13}$ cm. The depth of the real part of the well turns out to be approximately 1.8 Bev. Its imaginary part is positive and has a magnitude of about 0.23 Bev. The long- and short-range parts are now combined into a single potential which interprets the entire elastic scattering as resulting from a potential hole of approximate depth 1.8 Bev and range $a \approx 0.33 \times 10^{-13}$ cm surrounded by a diffuse absorbing region of the Gaussian shape described above. One difficulty with this simple interpretation is that the condition $ka < 1$ is violated to the extent that $ka \approx 1.1$, so that the effect of the short-range potential on the P wave, although probably small compared to the S wave, should also be considered if this interpretation is taken more seriously.[13]

ACKNOWLEDGMENTS

We would like to thank Professor R. J. Glauber, Professor P. C. Martin, and Dr. I. I. Shapiro for their constructive comments. We are also grateful to Professor M. S. Livingston for use of some of the Harvard-M.I.T. Cambridge Electron Accelerator facilities during the summer.

APPENDIX

The meaning of a potential well when applied to high-energy scattering is not entirely clear, particularly as applied in the high-energy approximation of Eq. (1) and (2) for the center-of-mass system. The interpretation used here is as follows:

In the center-of-mass system, we have the total energy E for the pion of mass μ_π and proton of mass μ_p (with $c=1$) given by

$$E = (p^2+\mu_\pi^2)^{\frac{1}{2}} + (p^2+\mu_p^2)^{\frac{1}{2}} + V. \qquad (\mathrm{A.1})$$

Solving for p^2, this gives

$$p^2 = \frac{(E-V)^4 + (\mu_p^2-\mu_\pi^2)^2}{4(E-V)^2} - \frac{\mu_\pi^2+\mu_p^2}{2}. \qquad (\mathrm{A.2})$$

If, in keeping with our high-energy approximation, we expand Eq. (A.2) in terms of V/E, we find to first order

$$p^2 \approx \left[\frac{E^4+(\mu_p^2-\mu_\pi^2)^2}{4E^2} - \frac{\mu_\pi^2+\mu_p^2}{2}\right] - \frac{1}{2E^2}[E^4-(\mu_p^2-\mu_\pi^2)^2]\frac{V}{E}. \qquad (\mathrm{A.3})$$

[12] O. Piccioni, *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 65.

[13] A somewhat similar possible interpretation for elastic π^--p scattering at 1.4 Bev has been mentioned by Eisberg *et al.* in reference 2. It should be noted here that the depth of the potential hole found above may be decreased if we allow the range a to increase. It is also anticipated that including the short-range contribution to the P wave would enable us to reduce the extremely deep potential hole found in this simple model.

This yields an approximate center-of-mass Schrödinger equation of the usual form:

$$\nabla^2\psi + k^2\psi - (2/\hbar^2)mV\psi = 0, \tag{A.4}$$

where

$$k^2 = \frac{1}{\hbar^2}\left[\frac{E^4+(\mu_p^2-\mu_\pi^2)^2}{4E^2} - \frac{\mu_\pi^2+\mu_p^2}{2}\right], \tag{A.5}$$

and

$$mV = \frac{1}{4E^3}[E^4-(\mu_p^2-\mu_\pi^2]V, \tag{A.6}$$

are interpreted as the values to be used in Eqs. (1) and (2).

PHYSICAL REVIEW VOLUME 122, NUMBER 3 MAY 1, 1961

Scattering of 2-Bev/c Muons in Carbon and Lead*

G. E. Masek, L. D. Heggie, Y. B. Kim, and R. W. Williams
Department of Physics, University of Washington, Seattle, Washington
(Received November 23, 1960)

The scattering cross section of high-energy μ mesons in carbon and lead has been measured, using a pure, monoenergetic beam of muons obtained with the Bevatron at the Lawrence Radiation Laboratory. Preparation, purification, and measured properties of the beam are described. The median momentum was 2.00 ± 0.03 Bev/c, the spread in momentum was not more than $\pm3.5\%$, and the effective contamination due to pions was 4.9×10^{-6}. During the experiment the total number of muons incident on the apparatus was 2.5×10^7. Counter hodoscopes recorded the muons scattered from 14.4 g/cm^2 of lead and from 27 g/cm^2 of carbon. Inelastic as well as elastic processes were accepted. Scattered particles were observed at angles up to 12° (momentum transfer $\sim$400 Mev/c). The lead data cover the same range as those cosmic-ray experiments which have appeared to indicate an anomalously large scattering. No anomaly is found; the lead scattering agrees closely with the distribution calculated by Cooper and Rainwater for purely electromagnetic interactions. The carbon data permit a better comparison with theoretical expectations, since one is measuring the single-scattering cross section directly, and one can account for the effects of nuclear structure rather accurately, using electron-scattering data and a detailed theoretical analysis of Drell and Schwartz. The carbon scattering results, based on 300 events in the region 70 Mev/c–400 Mev/c momentum transfer, agree closely with the Drell-Schwartz theory. The upper limits which this result places on a nonelectromagnetic scattering cross section and on a muon form factor are discussed.

I. INTRODUCTION

Low-energy experiments concerned with muons have indicated that they behave as "heavy electrons."[1,2] However, some experiments on the nuclear scattering of *high-energy* muons[1] have indicated that the experimental cross section at large angles is in excess of that of the electromagnetic prediction. Lloyd and Wolfendale,[3] using cosmic-ray muons of energies between 0.8 Bev and 10 Bev, find that their experimental scattering distributions in Pb tend to follow the Molière *point*-charge multiple-scattering distribution instead of the distribution expected from a realistic consideration of nuclear structure effects. Similarly, McDiarmid,[4] Whittemore and Shutt,[5] further experiments by Lloyd and Wolfendale,[6] and others[1] find the same general behavior for muon energies above 1 Bev, and for scattering angles such that the elastic momentum transfer [$q_0=2k_0\sin(\theta/2)$, where k_0 is the incident momentum and θ the scattering angle in the laboratory system] is between 100 and 200 Mev/c.[7] However, this anomaly is not seen by all such nuclear scattering experiments. The experiments of Watase *et al.*[8] and Amaldi *et al.*[9] (although sensitive to slightly lower incident energies, 0.3 to 1 Bev/c, and slightly lower elastic momentum transfers, 20–100 Mev/c) see no such behavior. Hence, the past experiments on nuclear scattering are not in agreement with each

* This work has been supported by the Office of Naval Research. An account of it will be given in the doctoral thesis of the junior author: L. D. Heggie, thesis, University of Washington, 1961 (unpublished).

[1] For a summary of muon experiments up to 1958, see G. N. Fowler and A. W. Wolfendale, *Progress in Elementary Particle and Cosmic-Ray Physics* (North-Holland Publishing Company, Amsterdam, 1958), Vol. 4, p. 123. Currently, the experiments on the g factor of the muon[2] have shown no differences from the predicted value and hence give further support to the above statement.

[2] R. L. Garwin, D. P. Hutchinson, S. Penman, and G. Shapiro, Phys. Rev. **118**, 271 (1960).

[3] J. L. Lloyd and A. W. Wolfendale, Proc. Phys. Soc. (London) **A68**, 1045 (1955).

[4] I. B. McDiarmid, Phil. Mag. **46**, 177 (1955).

[5] W. L. Whittemore and R. P. Shutt, Phys. Rev. **88**, 1312 (1952).

[6] J. L. Lloyd, E. Rossle, and A. W. Wolfendale, Proc. Phys. Soc. (London) **A70**, 421 (1957).

[7] In addition to the anomalies indicated by these nuclear scattering experiments, the results of R. F. Deery and S. H. Neddermeyer on the high-energy interaction of muons with electrons [Phys. Rev. **121**, 1803 (1961)] indicate a possible deviation from the expected electromagnetic behavior which could be interpreted in terms of a fundamental difference between the muon and electron.

[8] S. Fukui, T. Kitamura, and Y. Watase, Phys. Rev. **113**, 315 (1959).

[9] E. Amaldi and G. Fidecaro, Nuovo cimento **7**, 535 (1950).