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Cyclotron Emission from Plasmas with Non-Maxwellian Distributions*

G. Bekefi, Jay L. Hirshfield, and Sanborn C. Brown
Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts

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Cyclotron emission from high-energy plasmas is calculated for two classes of electron distribution functions: (a) those that decrease monotonically with increasing electron energy, and (b) those that have one or more maxima displaced from zero energy. In (a) the emission does not differ greatly compared with the emission from a Maxwellian plasma of the same energy. In (b) the emission can grow exponentially with distance traversed in the plasma, resulting in a greatly enhanced loss of radiant power.

I. INTRODUCTION

The success of proposed thermonuclear reactors rests, among other things, on the balance between the gain of energy from thermonuclear reactions and the loss of energy by electromagnetic radiation. Loss by radiation has received increasing attention[1–4] since it was pointed out that cyclotron emission from plasma electrons, gyrating in a magnetic field, could outweigh bremsstrahlung.

Previous calculations considered an idealized plasma in the form of a homogeneous slab or cylinder, immersed in a uniform static magnetic field B and with plasma electrons in a Maxwellian distribution of velocities. Here we remove the latter restriction and compute the cyclotron emission from plasmas with arbitrary distributions, f. We use a method of computing the emission that was outlined in a previous paper.[5]

A single electron of speed v radiates in an infinite set of harmonics n of its orbital frequency. The radiant energy propagates through the plasma at an angle θ to the magnetic field in two characteristic modes of propagation, sometimes called the ordinary and extraordinary, that will be denoted by letters o and x. The rate of emission at the radian frequency ω is

$$j_\omega{}^{(o,x)}(\beta,\theta)=\frac{e^2\omega^2}{8\pi^2\epsilon_0 c}\sum_{n=1}^{\infty}A_n{}^{(o,x)}(\beta,\theta)\times\delta[n\omega_b(1-\beta^2)^{\frac{1}{2}}-\omega].\quad(1)$$

Here c and ϵ_0 are the velocity of light and permittivity of free space, respectively, and $\beta=v/c$. The dimensionless parameter A_n determines the strength of the radiation and ω_b is the orbital frequency eB/m reduced to the rest mass m of the electron.

The radiation intensity $I_\omega(\theta)$ that escapes from unit area of plasma surface, per unit solid angle, is obtained by tracing the energy degradation along a ray within the plasma. For a homogeneous, uniform plasma, in the absence of reflections at the plasma boundaries,

$$I_\omega{}^{(o,x)}(\theta)=B^{(o,x)}(\omega,T_r)\{1-\exp[-\alpha_\omega{}^{(o,x)}(\theta)l]\}.\quad(2)$$

$B(\omega,T_r)$ is the equilibrium "blackbody" intensity, α_ω the absorption coefficient of the plasma, and l the length of the ray within the plasma.

The blackbody intensity, generalized to an arbitrary distribution of electron energies $f(\epsilon)$, is[5]

$$B^{(o,x)}(\omega,T_r)=\frac{\omega^2}{8\pi^3c^2}\left[-\frac{\int j_\omega{}^{(o,x)}(\epsilon)f(\epsilon)p^2dp}{\int j_\omega{}^{(o,x)}(\epsilon)[\partial f(\epsilon)/\partial\epsilon]p^2dp}\right],\quad(3)$$

where p is the momentum of the electron, $p=\epsilon\beta/c$, and ϵ the total energy (rest plus kinetic), $\epsilon=mc^2(1-\beta^2)^{-\frac{1}{2}}$. For simplicity of computation, the distribution function is assumed to be spherically symmetric in velocity space and is normalized so that $\int f(\epsilon)4\pi p^2dp=1$. The bracketed term of Eq. (3) is equal to kT_r and it thus defines the radiation temperature T_r of a non-Maxwellian plasma. When the distribution is Maxwellian, the bracketed term becomes kT, where T is the electron temperature, and Eq. (3) reduces to the Rayleigh-Jeans limit of Planck's blackbody intensity, $B(\omega,T)=kT\omega^2/8\pi^3c^2$. The absorption coefficient α_ω, which can be deduced from 3 and from Kirchhoff's law, is,

$$\alpha_\omega{}^{(o,x)}=-[32\pi^4c^2N/\omega^2]\int j_\omega{}^{(o,x)}(\epsilon)[\partial f(\epsilon)/\partial\epsilon]p^2dp,\quad(4)$$

where N is the electron density.

Equations (1) through (4) allow us to evaluate the radiation intensity, $I_\omega=I_\omega{}^{(o)}+I_\omega{}^{(x)}$, that leaves the

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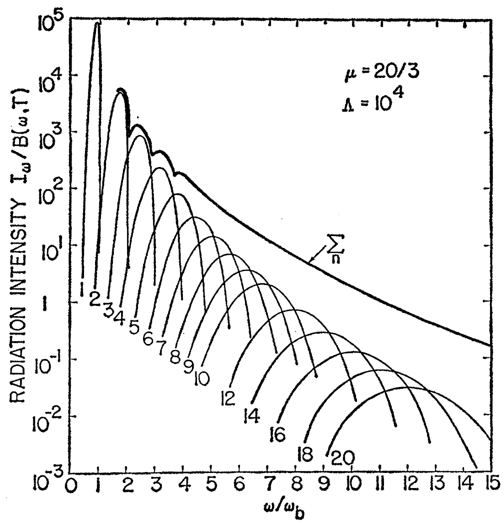

FIG. 1. The radiation intensity for various harmonics ($n=1, 2, 3\ldots$) and the total intensity (Σ_n), as a function of frequency, with self-absorption neglected. Maxwellian plasma with $\mu\equiv mc^2/\bar{u}=20/3$ and $\Lambda\equiv\omega_p{}^2L/\omega_b c=10^4$.

plasma surface. For purposes of calculation we make the following assumptions: The plasma is in the form of a homogeneous slab of thickness L, and the uniform magnetic field is oriented parallel to the faces of the slab. Since the emission takes place mainly in a narrow cone inclined at 90° to the direction of the magnetic field, we compute only the intensity I_ω at right angles to B (and hence at right angles to the slab surfaces). By considering emission only at an angle $\theta=\pi/2$, some error is incurred in the estimate of the absolute, total cyclotron emission from the plasma. However, we are mainly concerned in estimating the relative emission from non-Maxwellian plasmas compared with a Maxwellian one, and hence our errors are not appreciable.

Difficulties arise unless the complicated strength function A_n of Eq. (1) is simplified by suitable approximations.[1,6] For electron energies less than 100 kev the following approximations were used:

(a) At low harmonic numbers ($n\doteqdot\omega/\omega_b<3$), the ordinary wave ($o$) which is polarized with its electric vector along B, carries little energy and most of the radiation is carried by the extraordinary wave polarized at right angles to B. Under these conditions,

$$A_n{}^{(o)}=0,\qquad A_n{}^{(x)}=[n^{2n}/(2n+1)!]\beta^{2n}\quad\text{for}\quad n<3. \tag{5}$$

Since there is little overlapping of the radiation from these harmonics (for energies less than 50 kev) the radiation from each harmonic can be considered as a separate entity, and the summation in Eq. (1) need not be carried out.

(b) For harmonic numbers greater than approximately 3, a good approximation for A_n is

$$A_n{}^{(o)}=0,$$
$$A_n{}^{(x)}=(mc^2/\epsilon)^{\frac{1}{2}}[(\epsilon-mc^2)/(\epsilon+mc^2)]^n\times\frac{\exp[2nmc^2/\epsilon]}{4(\pi n^3)^{\frac{1}{2}}}. \tag{6}$$

Here, overlapping of the radiation from successive harmonics can be large and the net emission generally becomes a monotonically varying function of frequency.

II. RESULTS

The magnitude and spectrum of the cyclotron emission fall into two distinct classes, given by the dependence of the distribution function $f(\epsilon)$ on the electron energy ϵ. When $f(\epsilon)$ is Maxwell-like, that is when $f(\epsilon)$ decreases monotonically with increasing ϵ, $[\partial f(\epsilon)/\partial\epsilon<0]$, the general characteristics of I_ω do not differ appreciably from those for a Maxwellian distribution of the same mean electron energy. The total emission is also insensitive to small perturbations from the Maxwell distribution. However, when the distribution function has one or more maxima displaced from zero energy $[\partial f(\epsilon)/\partial\epsilon>0$ in some range of $\epsilon]$, the characteristics of the cyclotron emission are altered greatly: At certain frequencies the self-absorption α_ω becomes negative with the result that the radiation, instead of being attenuated in its passage through the plasma, grows exponentially, and I_ω can exceed black-body emission by many orders of magnitude. If the growth is too excessive, nonlinear effects can set in and the plasma can become unstable. It has been shown[5] that this amplification process is not confined to cyclotron emission alone. However, in the case of cyclotron emission, it is a relativistic effect and will not take place in the limit when terms of order $(v/c)^2$ are neglected. Hence, first-order Doppler broadening (which is of order v/c) will not give rise to amplification.

A. Emission for Maxwell-like Distributions, $\partial f/\partial\varepsilon<0$

We assume distribution functions of the form

$$f(\beta)\propto\exp(-b\beta^l), \tag{7}$$

where l and b are positive constants; $l=2$ represents a Maxwellian distribution of a given mean kinetic energy $\bar{u}$, whose magnitude determines the value of b; $l<2$ implies that there is an excess of energetic electrons in the tail of the distribution function, compared to a Maxwellian distribution of the same mean energy $\bar{u}$. The opposite is true when $l>2$.

The absorption coefficient of the plasma, obtained from Eqs. (1) and (4), is

$$\alpha_\omega{}^{(o,x)}L=-(4\pi^2m^4c^5\Lambda)/\Omega^3\times\sum_{n\geq\Omega}^{\infty}n^2[(n/\Omega)^2-1]^{\frac{1}{2}}\times[A_n{}^{(o,x)}(\epsilon)[\partial f(\epsilon)/\partial\epsilon]]_{\epsilon=mc^2n/\Omega}. \tag{8}$$

[6] H. Rosner, Republic Aviation Corporation Report AFSWC-TR-58-47, 1958 (unpublished).

Here $\Omega \equiv \omega/\omega_b$. The dimensionless parameter $\Lambda = \omega_p^2 L/\omega_b c$ (where ω_p is the plasma frequency $Ne^2/m\epsilon_0$) specifies the electron density, size of plasma, and the strength of the magnetic field.

The characteristics of the spectrum of cyclotron emission from a plasma with a Maxwellian distribution of electrons ($l=2$), with a mean energy of 75 kev $[\mu \equiv (mc^2/\bar{u}) = 20/3]$, are illustrated in Fig. 1. Λ was chosen to be 10^4. This means that when $L=1$ meter and $B=10^4$ gauss, the electron density is approximately 10^{14} cm^{-3}, and the outward kinetic pressure, $2NkT$, of the charged particles is approximately balanced by the inward magnetic pressure $B^2/2\mu_0$.

The various curves of Fig. 1 labeled 1, 2, 3. . . represent the emission from the individual harmonics, with self-absorption neglected. They were obtained from the individual terms of Eq. (8) and from $I_\omega = B(\omega,T)\alpha_\omega L$, which is the limit of Eq. (2), as $\alpha_\omega L \to 0$. The total emission, obtained by summing over all the harmonics, is shown by the upper curve of Fig. 1. At frequencies $\omega/\omega_b > 3$, the harmonics overlap so strongly that the net emission forms a monotonically decreasing function of frequency. However, since the emission cannot exceed the blackbody limit $[I_\omega/B(\omega,T)=1$ on the ordinate of Fig. 1], most of the harmonics are "trapped" in the blackbody continuum. The plasma will thus radiate almost as a blackbody from $\omega=0$ to some characteristic frequency ω^*. This characteristic frequency (equal to $\omega^*=11.2\omega_b$ in our case) is a function of the electron energy, density, and the plasma dimensions. At frequencies greater than ω^*, the cyclotron emission can escape almost freely from the plasma. The dividing line between the two regimes of emission is conveniently defined by

$$\alpha(\omega=\omega^*)L=1. \tag{9}$$

Thus the total cyclotron emission $I(\theta=\pi/2)$ from unit area of plasma surface is

$$\begin{aligned} I &= \int_0^\infty B(\omega,T)[1-\exp(-\alpha_\omega L)]d\omega \\ &\approx \int_0^{\omega^*} B(\omega,T)d\omega + L\int_{\omega^*}^\infty B(\omega,T)\alpha_\omega d\omega. \end{aligned} \tag{10}$$

The second term of Eq. (10) represents the free emission of cyclotron radiation for frequencies $\omega > \omega^*$. This radiation decreases nearly exponentially with increasing frequency and we shall neglect it. Performing the integration in Eq. (10) leads to an energy loss of $kT(\omega^*)^3/24\pi^3c^2$ watt meter^{-2} [for $f(\epsilon)$ Maxwellian], which, when added to the bremsstrahlung loss, should not exceed the power produced by thermonuclear reactions.

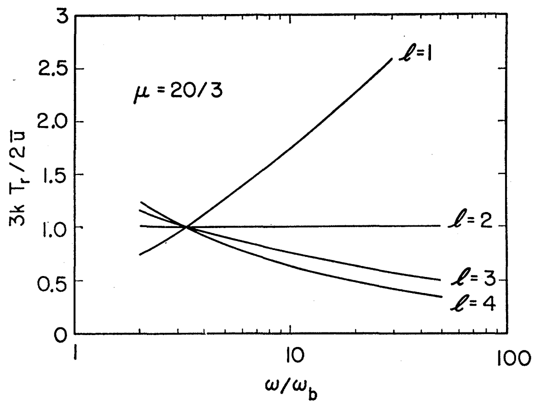

FIG. 2. The radiation temperature T_r as a function of frequency, for various distribution functions $f \propto \exp(-b\beta^l)$.

To offset cyclotron radiation, we see that the dimensions of the plasma must be increased, since the cyclotron effect is largely proportional to the surface area, while the energy production is a volume effect. For the plasma we have chosen and for a D-D thermonuclear reaction, the minimum plasma size (L) is several meters.[1]

The approach outlined above, and Eqs. (2)–(4), are applicable only for tenuous plasmas, $\omega_p/\omega < 1$, and our results would be in error if ω_p were near ω^*. However, for $\omega_p < \omega^*$ (as was the case considered here) the refractive index of the plasma differs only slightly from unity near ω^*, and the equations were correctly applied in estimating the value of ω^* and of the total energy loss.[3,4]

A procedure similar to that used above in the case of a Maxwellian distribution was used for the more general distribution functions given by Eq. (7). To calculate ω^* and the total emission, one must first find the spectral distribution of $B(\omega,T_r)$. We substitute Eq. (1) into Eq. (3) and evaluate the integral, with the result that

$$B(\omega,T_r) = \frac{\omega^2}{8\pi^3c^2}\left\{ -\frac{\sum_{n\geqslant\Omega}^{\infty} n^2[(n/\Omega)^2-1]^{\frac{1}{2}}[A_n(\epsilon)f(\epsilon)]_{\epsilon=mc^2n/\Omega}}{\sum_{n\geqslant\Omega}^{\infty} n^2[(n/\Omega)^2-1]^{\frac{1}{2}}[A_n(\epsilon)[\partial f(\epsilon)/\partial\epsilon]]_{\epsilon=mc^2n/\Omega}} \right\}, \tag{11}$$

where $\Omega=\omega/\omega_b$. Figure 2 shows the variation of the radiation temperature T_r [defined by the bracketed term of Eq. (11)] with frequency, for different distribution functions l. The absorption coefficient is obtained from Eq. (8), and ω^* from Eq. (9). The results are plotted in Fig. 3. The total emission between frequen-

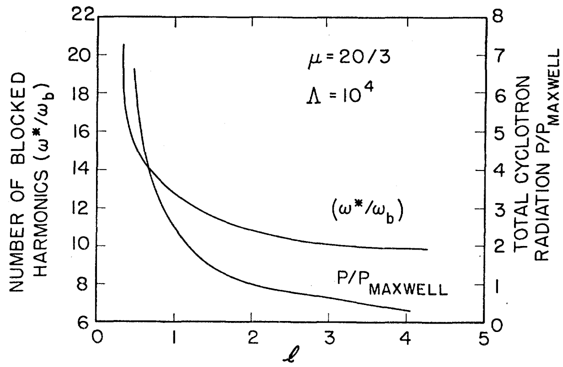

FIG. 3. The number of harmonics ω^* trapped in the blackbody continuum, and the total cyclotron emission (normalized to the emission from a Maxwellian plasma), as a function of the distribution of electron velocities, $f\propto\exp(-b\beta^l)$.

cies $\omega=0$ and $\omega=\omega^*$ was calculated from Eq. (10) using the results of Fig. 2. This emission, normalized to that of a Maxwellian plasma of the same mean energy $\bar{u}$, is illustrated in Fig. 3.

From this figure we conclude: (1) Cyclotron emission is not greatly diminished by inducing the plasma to have an excess of slow electrons ($l>2$); (2) a large excess of fast electrons in the tail of the distribution ($l<2$) causes a large energy loss and is very detrimental to the operation of a thermonuclear device; (3) small perturbations of the Maxwellian distribution ($l\approx2$) do not affect the energy loss.

B. Emission in the Presence of Peaked Distributions, $\partial f/\partial\varepsilon>0$

Here we assume distribution functions of the form

$$f(\beta)\propto\beta^p\exp(-b\beta^2);\quad p\geqslant0. \qquad (12)$$

When $p=0$, the distribution is Maxwellian; when $p\neq0$, the distribution function is peaked at some electron velocity $\beta\neq0$. For a fixed mean electron energy $\bar{u}$ the spike in the distribution function becomes narrower, the larger the value of p.

The peaked distribution function implies the existence of an excess of energetic electrons (maintained there by some external agency) in some energy range, as compared with the population in neighboring energy ranges. This excess of electrons can cause the stimulated emission of photons to exceed the spontaneous absorption of photons, with the result that the total effective absorption, [Eq. (4)], becomes negative. However, a peaked distribution function, though necessary, is not sufficient to ensure that the integral of Eq. (4) remain positive. The second condition is that[5]

$$\partial\{j_\omega(\epsilon)\epsilon[\epsilon^2-(mc^2)^2]^{\frac{1}{2}}\}/\partial\epsilon<0. \qquad (13)$$

[To use this inequality one must first remove the δ-function dependence of $j_\omega(\epsilon)$. This can be done by allowing the electron to make collisions with atoms or ions.]

Equation (13) is satisfied near the peaks of the various harmonics of the cyclotron emission. Substituting for A_n from Eq. (5) into Eq. (8) and making use of the distribution function given by Eq. (12), we obtain the following expression for the absorption coefficient:

$$\alpha_\omega L=-2\pi\Lambda\mu^{2-n}\frac{[(p+3)/4]^{2-n}}{[(p+1)/2]!}\times\sum_{n\geqslant\Omega}^{\infty}\frac{n^{2n-1}}{(2n+1)!}(1-Q)^{-2}X^{(2n+p-1)/2}\times[p-2X]\exp(-X). \qquad (14)$$

$X=[(p+3)/4]\mu[1-(\Omega/n)^2]$ is the frequency variable, and $Q=X\mu^{-1}[(p+3)/4]^{-1}$ is a parameter which is generally small compared with unity. When $X<p/2$, α_ω is negative and when $X>p/2$, it is positive. Thus, in narrow frequency ranges near the maxima of the harmonics, α_ω becomes negative (the radiation is amplified), and outside these ranges the radiation attenuates in the normal way. Figure 4 illustrates this effect for the first two harmonics. The calculations are for a distribution function with $p=0.2$. Despite the fact that this represents a relatively small perturbation of the Maxwellian distribution, the amplification of the radiation $(-\alpha_\omega L)$ is appreciable. For a magnetic field of 10^4 gauss, a mean electron energy $\bar{u}$ of 75 kev and an electron density of 2×10^{12} cm^{-3}, the peak value of $(-\alpha_\omega)$ for the first harmonic, as found from Fig. 4,

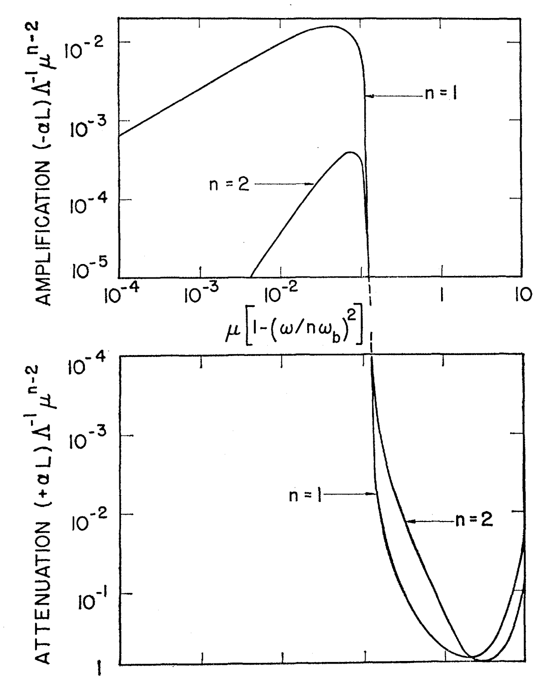

FIG. 4. The transition from a positive to a negative absorption coefficient, with varying frequency, for the first two harmonics. The plot is for a distribution function $f\propto\beta^p\exp(-b\beta^2)$, with $p=0.2$.

is $\alpha_\omega \approx 0.11$ cm^{-1}, and the wave amplifies at a rate of 0.5 db per cm path length. When $\bar{u}=7.5$ kev, the amplification is 5 db/cm.

Figure 5 shows the variation of the intensity of emission I_ω as function of frequency, for the first two harmonics, as calculated from Eqs. (2) and (11) and from the data of Fig. 4. Along the ordinate is plotted the intensity I_ω normalized to what the blackbody intensity $B(\omega,T)$ would be, had the distribution function been Maxwellian $[B(\omega,T)=2\bar{u}\omega^2/24\pi^3c^2]$. We note the large peak of intensity, $[I_\omega/B(\omega,T)\gg 1]$, superimposed on the blackbody emission. This peak occurs in the region of the first harmonic where the amplification is very large, but does not occur at higher harmonics for the plasma considered here ($\mu=mc^2/\bar{u}=20/3$, $p=0.2$ and $\Lambda=100$). Had we taken a larger value for Λ (or p), amplification would have taken place also at higher harmonic numbers. However, we have purposely chosen Λ to be as small as 100. This means that when $L=1$ m and $B=10^4$ gauss, $\omega_b=1.8\times 10^{11}$ rad sec^{-1}, and $\omega_p=7.3\times 10^{10}$ rad sec^{-1}. Therefore, at the frequencies of interest ($\omega\approx\omega_b$, $2\omega_b$, etc.), the plasma frequency is less than ω, and only then (when the plasma is tenuous), is the theory strictly applicable.

Figure 6 shows a complete view of the emission spectrum. Except for the peak at $\omega\approx\omega_b$, the emission follows closely the trend of events of the Maxwellian plasma discussed in connection with Fig. 1: The radiation intensity is blackbody up to a frequency $\omega^*=4.3\omega_b$, and at frequencies greater than ω^* the cyclotron emission leaves the plasma with negligible self-absorption.

Despite the narrowness of the emission spike at $\omega=\omega_b$ (the half-power width $\Delta\omega$ is $3\times 10^{-3}\omega_b$), the total emission under the spike exceeds the blackbody emission summed between frequencies $\omega=0$ and $\omega=\omega^*$.

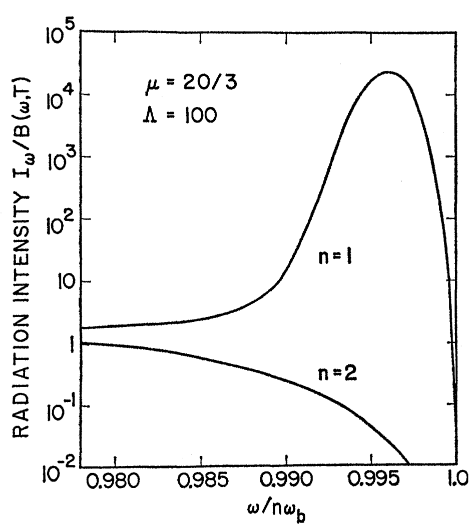

FIG. 5. The radiation intensity, normalized to $B(\omega,T) =(2\bar{u}\omega^2/24\pi^3c^2)$, as a function of frequency, showing amplificatoin for the first harmonic $n=1$, and no amplification for $n=2$. ($p=0.2$).

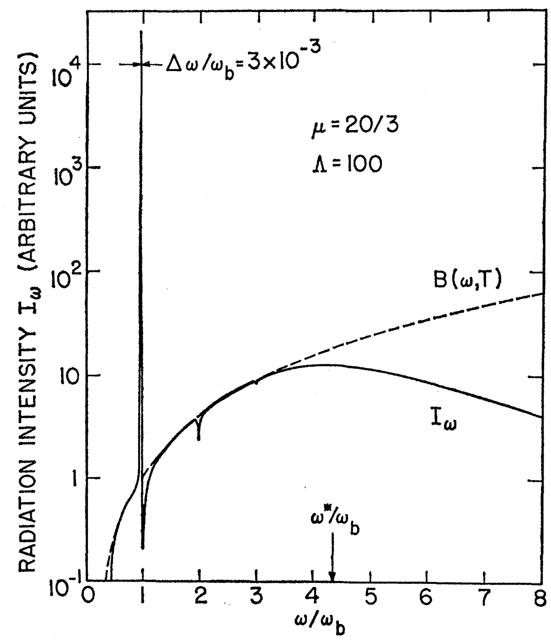

FIG. 6. Complete view of the emission spectrum, showing the large peak at $\omega=\omega_b$. ($p=0.2$).

Therefore, the amplification process plays an important part in the total radiation loss from thermonuclear reactors, since in these devices we have no reason to assume that the electron distribution is strictly Maxwellian. In fact, the form of the perturbation of the Maxwell distribution we have been considering, $[f\propto\beta^{0.2}\exp(-b\beta^2)]$, is observed[7] in existing mirror devices in which scattering depletes the plasma of slow electrons.

As the electron distribution departs more and more from a Maxwellian, (p increases), the magnitude of the amplification ($-\alpha L$) rises rapidly. Figure 7 shows the maximum amplification attainable in each of the first three harmonics, for progressively larger values of p.

The calculations of the amplification were made for each harmonic separately [the summation over n in Eqs. (1) and (14) was neglected]. This can be justified only for the low harmonics, when overlapping of the radiation from successive harmonics is negligible; that is, when the separation between harmonics, $(\Delta\omega)_s =mc^2\omega_b/\epsilon$, is large compared with the second-order Doppler broadening of the radiation, $(\Delta\omega)_D\approx n\omega_b(mc^2/\epsilon) \times(\Delta\epsilon/\epsilon)$, where $\Delta\epsilon$ is the spread of the distribution of electron energies. It follows that the condition for negligible overlapping is approximately

$$n\Delta\epsilon/\epsilon<1. \tag{15}$$

For mildly relativistic electrons ($\beta^2\ll 1$) with a Maxwellian distribution, $(\Delta\omega)_D\approx n^{\frac{3}{2}}[2\bar{u}/3mc^2]\omega_b$; when $\bar{u}=75$ kev, overlapping becomes important beginning with the third or fourth harmonic (see Fig. 1).

[7] R. F. Post, Lawrence Radiation Laboratory, University of California (private communication).

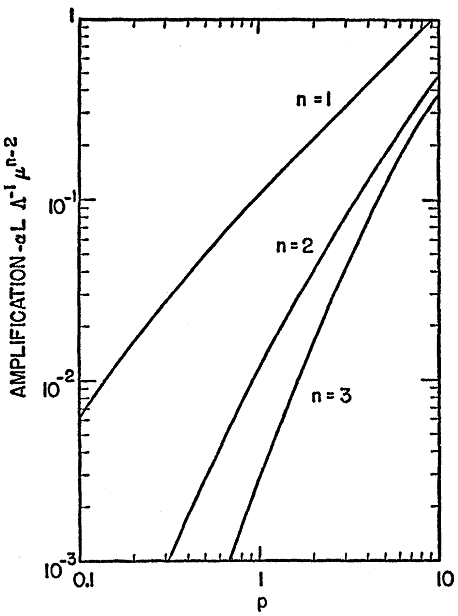

FIG. 7. Maximum value of the amplification coefficient α_ω with increasing perturbation (p) of the Maxwell distribution.

It is found that amplification of cyclotron emission does not occur (inequality 13 is not satisfied) when overlapping of the radiation is so pronounced that the emission becomes a smoothly varying function of frequency. For instance, for highly relativistic electrons, ($\epsilon \gg mc^2$), the harmonics are spaced very near to each other, the summation in Eq. (1) over n can be replaced by an integration, and the emission j_ω per electron becomes[1]:

$$j_\omega^{(o,x)} \propto \alpha\left[\int_\alpha^\infty K_{5/3}(t)dt \mp K_{2/3}(\alpha)\right]. \qquad (16)$$

Here $\alpha = (2/3)(\omega/\omega_b)[mc^2/\epsilon]^2$ and K_ν is the modified Hankel function of order ν. The integral of Eq. (16) has been tabulated.[8] The emission j_ω of Eq. (16) increases with energy ϵ for low energies, reaches a maximum at $\alpha \approx 1$, and then decreases with increasing ϵ. However, $[\epsilon^2 j_\omega(\epsilon)]$ is a monotonically increasing function of energy. Therefore, Eq. (13) is not satisfied and amplification does not occur.

Amplification of cyclotron emission by highly relativistic electrons has been suggested[9] as a possible mechanism leading to the intense nonthermal radiation from certain extraterrestrial radio sources. Whether the almost monoenergetic electron distribution functions necessary for amplification, as required by Eq. (15), exist in interstellar space, is not known.

Note added in proof. Since the time of writing this paper, the following related work has been brought to our attention: J. Schneider, Phys. Rev. Letters **2**, 504 (1959); Z. Naturforsch. **15a**, 484 (1960).

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[8] V. V. Vladimirsky, J. Exptl. Theoret. Phys. U.S.S.R. **18**, 392 (1948).

[9] R. Q. Twiss, Australian J. Physics **11**, 564 (1958). Here Twiss concludes (in disagreement with us) that amplification can occur even if the harmonics overlap. The basic difference is between his Eq. (47) and our Eq. (4) for the absorption coefficient. We integrate over momentum space $d^3p \propto \epsilon[\epsilon^2 - (mc^2)^2]^{\frac{1}{2}} d\epsilon$, while Twiss integrates over energy $d\epsilon$.