Optical Constants of Germanium in the Region 0–27 ev*

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The reflectivity R has been measured for Ge crystal over the energy range 7.6–18.0 ev and the transmissivity T for an evaporated Ge film over the range 16.6–27.4 ev. The reflectivity data near normal incidence, combined with existing data at lower energies, were used in the Kramers-Krönig dispersion relation to evaluate the index of refraction n and extinction coefficient k of Ge over the range 0–18 ev. Electron characteristic losses are predicted at 10 and 15 ev, consistent with those observed at 11 and 16 ev. The plasma frequency of bulk Ge as determined by the optical data is $\hbar\omega=10$ ev.

I. INTRODUCTION

COMPUTATIONS of the optical constants of Ge, based on a technique developed by Robinson,[1] have recently been carried out by Philipp and Taft[2] over the range 0–10 ev. These computations have been subsequently repeated by Rimmer and Dexter.[3] Although certain extrapolation procedures used in these two analyses were somewhat different and arbitrary, there was general agreement in the final results, minor differences occurring in the fine structure.

The purpose of the present work is to extend the previous results to energies beyond 10 ev. The optical constants at higher energies are particularly significant in that they can be used to determine the plasma frequency of Ge.

The reflectivity R has been measured for a Ge crystal over the range 7.6–18 ev, and the transmissivity T has been obtained for an evaporated Ge film over the range 16.6–27.4 ev. The data on reflectivity, combined with those of Philipp and Taft,[2] were used in a dispersion relation to evaluate the real and imaginary parts of the complex index of refraction, $\bar{n}=n-ik$, over the range 0–18 ev. These in turn were used to predict electron energy characteristic losses and the plasma frequency of bulk Ge.

II. EXPERIMENTAL

The reflectivity R at room temperature, shown in Fig. 1, has been measured inside a vacuum monochromator[4] at angles of incidence of 15°, 25°, 35°, 45°, and 55° over the range 7.6–18.0 ev. The polished n-type Ge crystal which was used had a thickness of $\frac{1}{8}$ in., a resistivity of 22 ohm cm, and was cut along the (111) plane. Damage caused by polishing was estimated to be up to 6μ in depth. A point of inflection at 14.7 ev in the reflectivity curve at 15° turned into a peak at higher angles of incidence, and in addition, another broad peak was observed near 16.5 ev. The peaks at 14.7 and 16.5 ev are not due to interference between front and back surface reflections because their positions should have changed with angle of incidence.

In the region 7.6–11.3 ev, the values of the reflectivity at 15° incidence obtained here fall somewhat below those obtained by Philipp and Taft[2] from an etched crystal of Ge at a small angle of incidence. Measurements on an evaporated film[5] of Ge (not shown) gave higher values of reflectivity in the region 7.6–11.3 ev, but the general form of the curve was unchanged.

Attempts were made to prepare unbacked films of Ge, but the films could not be floated off in water and broke into too many small pieces. Also, an attempt was made to evaporate Ge on single crystals of stilbene, but this was not successful, probably because of the high vapor pressure of stilbene, since the glass bell jar was coated with Ge.

The transmissivity T at room temperature, which is shown in Fig. 1, was obtained for a film of Ge, 800 A thick, which was evaporated on a glass slide coated with sodium salicylate. It is seen that T rises from 0.33% at 16.6 ev to a maximum of 4.4% at the limit of measurement near 27.4 ev.

The values of the absorption coefficient μ and the extinction coefficient k can be calculated directly from the transmissivity and are given in Table I. It should be emphasized that these results apply to a thin evaporated film of Ge and should not be compared quantitatively with the results for μ and k obtained in the next section. The latter are based on reflectivity data obtained for a Ge crystal and refer to the bulk material.

III. DISPERSION ANALYSIS

The real and imaginary parts of the complex refractive index $\bar{n}=n-ik$ for Ge were obtained[6] by making use of the Kramers-Krönig relation[7]

$$\theta(E)=\frac{E}{\pi}\int_{E'=0}^{\infty}\frac{\ln R(E')}{E'^2-E^2}dE', \tag{1}$$

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[5] See also S. Robin-Kandare and M. B. Vodar, J. recherches centre natl. recherche sci. Labs. Bellevue (Paris) **10**, 311 (1959).

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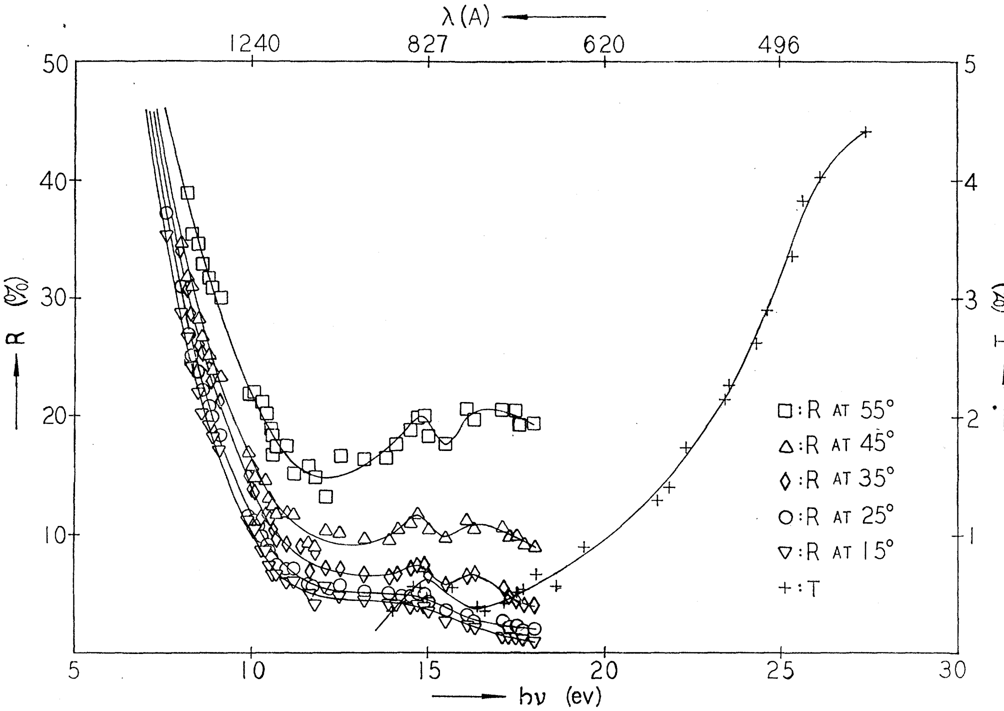

FIG. 1. Reflectivity R of Ge crystal at various angles of incidence and transmissivity T of an evaporated Ge film 800 A thick.

together with the Fresnel equation

$$r=(n-ik-1)/(n-ik+1)=|r|e^{i\theta}. \quad (2)$$

Here $|r|$ denotes the amplitude of the reflected intensity at normal incidence and the reflectivity is $R=|r|^2$.

In the actual computation, Eq. (1) is replaced by the approximate relation

$$\theta(E)=\frac{E}{\pi}\int_{E'=0}^{E_1}\frac{\ln R(E')}{E'^2-E^2}dE'+\frac{E}{\pi}\int_{E'=E_1}^{E_2}\frac{\ln R(E')}{E'^2-E^2}dE'. \quad (3)$$

Here E_1 is the energy up to which the reflectivity R has been measured; thus the integrand of the first term on the right-hand side of Eq. (3) is known. The integrand of the second term on the right-hand side of Eq. (3) is obtained by extrapolating R beyond E_1 in some appropriate manner. The method of extrapolation is somewhat arbitrary and the cutoff energy E_2 will depend upon the method chosen. In general, the results obtained using the approximate relation (3) are not expected to be reliable for energies much larger than E_1.

The extrapolation procedure used here is similar to that of Rimmer and Dexter.[3] The functional form $\ln R(E)=aE+b$ was assumed in the region $E>E_1$, the constants a and b being chosen so that the extrapolated portion of the reflectivity curve joins on smoothly to the measured portion, i.e., in such a way as to make $R(E)$ and dR/dE continuous at $E_1=18.0$ ev. This requirement was satisfied with the choice $a=-0.39$ ev^{-1} and $b=2.49$. The one remaining parameter, the cutoff energy E_2, was then determined by requiring that the calculated values of the extinction coefficient k (or equivalently the absorption coefficient $\mu=4\pi k/\lambda$) should agree as well as possible with the measured

TABLE I. Absorption coefficient μ and extinction coefficient k for a Ge film, 800 A thick, as calculated from the transmissivity.

λ (A)	$\hbar\omega$ (ev)	$10^{-5}\mu$ (cm^{-1})	k
452	27.40	3.90	0.14
475	26.10	4.00	0.15
482	25.70	4.08	0.16
490	25.30	4.24	0.17
503	24.60	4.43	0.18
510	24.30	4.55	0.18
525	23.60	4.73	0.20
530	23.40	4.80	0.20
555	22.30	5.06	0.22
568	21.83	5.34	0.24
575	21.56	5.43	0.25
637	19.40	5.89	0.30
665	18.60	6.46	0.34
686	18.05	6.26	0.34
700	17.70	6.53	0.36
706	17.55	6.58	0.37
717	17.30	6.60	0.38
723	17.15	6.82	0.39
746	16.60	7.05	0.42

values of Dash and Newman[8] in the range 0.6–1.7 ev, and with those of Archer[9] in the range 1.9–3.4 ev. The best fit to these data, shown in Fig. 2, was obtained with a value of $E_2=34.5$ ev. As can be seen in Fig. 2, this choice of the parameters a, b, and E_2 also yields values of n which agree quite well with the measured values of Archer[9] in the region 1.9–3.4 ev and with those of Salzberg and Villa,[10] and Briggs[11] below 0.7 ev.

IV. RESULTS AND DISCUSSION

The values of the index of refraction n and the extinction coefficient k calculated from the dispersion relation are shown in Fig. 2, and the corresponding absorption coefficient, μ, in Fig. 3. In the region 0–10 ev the values of n and k obtained here are substantially the same as those obtained by Philipp and Taft[2] and by Rimmer and Dexter.[3] Beyond 6 or 7 ev the general behavior of the n and k curves is more or less the same as that which would be expected on the basis of the free electron gas model with damping.[12] Any detailed analysis in terms of such a model would, however, be a gross oversimplification.

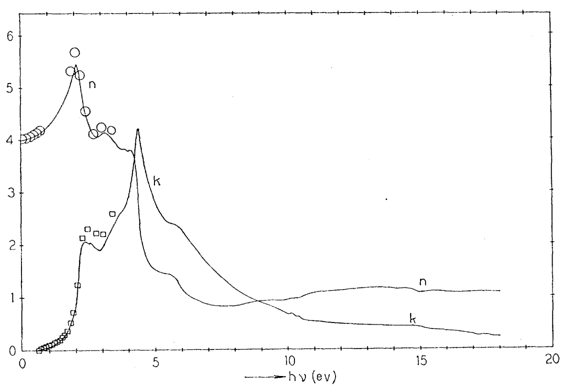

FIG. 2. Index of refraction n and extinction coefficient k for Ge as calculated from dispersion relations. Experimental data are indicated by circles and squares and are discussed in the text.

One expects the plasma frequency[13] to be close to the frequency at which n and k become equal, in this case close to $\hbar\omega=9$ ev. In this connection it is useful to examine the behavior of the imaginary part of $1/\epsilon$, which is given by

$$\mathrm{Im}(1/\epsilon)=2nk/(n^2+k^2)^2.$$

This is shown in Fig. 4. The probability $P(\omega)d\omega$ that an electron (with high energy) will lose energy between $\hbar\omega$ and $\hbar(\omega+d\omega)$ in transversing a medium characterized by a complex dielectric constant ϵ is proportional to

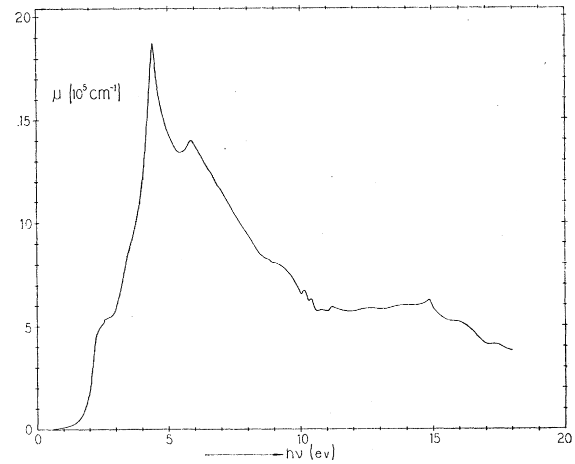

FIG. 3. Absorption coefficient μ for Ge as calculated from dispersion relations.

$\mathrm{Im}(1/\epsilon)$.[14] Thus this quantity should exhibit a sharp maximum at the plasma frequency. From Fig. 4, it is seen that this maximum occurs at $\hbar\omega=10$ ev; hence we conclude that $\hbar\omega=10$ ev is the plasma frequency of bulk germanium. The abrupt change in slope of the absorption coefficient at 10 ev (see Fig. 3) is also consistent with this conclusion.

It should be noted that this value is considerably different from the free electron plasma frequency ω_p given by the well-known relation $\omega_p^2=4\pi ne^2/m$. This latter quantity has the value $\hbar\omega_p=16$ ev for Ge (if one assumes that all four valence electrons contribute to the electron density n), and would be the plasma frequency only if the effects of interband electronic transitions and polarizability of the ion core were negligible.

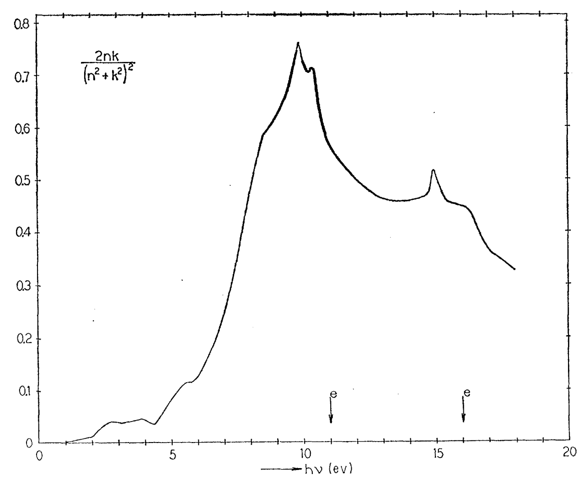

FIG. 4. Imaginary part of the reciprocal of the complex dielectric constant for Ge as calculated from dispersion relations. Vertical lines marked e denote positions of observed electron energy characteristic losses.

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It is, however, possible to extract the parameter ω_p, as defined above, from the n and k curves shown in Fig. 2, and the value thus obtained is in fact in good agreement with the value $\hbar\omega_p=16$ ev. This will be discussed elsewhere.[15]

In addition to the peak at 10 ev, there seems to be another small but sharp peak in the $2nk/(n^2+k^2)^2$ curve at 15 ev. It is not altogether certain that this second peak is real; it may be an extraneous result introduced by the extrapolation procedure used in the dispersion analysis or by small errors in the reflectivity data. Certainly a visual inspection of the n and k curves shown in Fig. 2 would not lead one to expect this peak. In any case the two peaks at 10 and 15 ev are consistent with the electron characteristic losses in Ge observed by Powell[16] at 11.1 and 16.0 ev, and also by Marton *et al.*,[17] at 16.0 ev.

A more complete discussion of the results obtained in this work will be presented later.[15]

V. SUMMARY

The reflectivity has been measured for an n-type Ge crystal over the range 7.6–18.0 ev, and the transmissivity for a thin film of Ge over the range 16.6–27.4 ev. The reflectivity data were used in a dispersion relation to evaluate the optical constants n and k. Analysis of the quantity, $\mathrm{Im}(1/\epsilon)$, indicates that the plasma frequency for bulk Ge is $\hbar\omega=10$ ev. This analysis also predicts electron characteristic losses at 10 and 15 ev consistent with those observed at 11 and 16 ev.

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