Symmetric and Asymmetric Fission

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Fission yields have been calculated assuming as a first approximation that they are proportional to the product of the level densities of a pair of binary fission products. The level densities have been calculated with the simplified shell-model methods of Newson and Duncan. The calculations predict a single symmetric peak when the mass of the fissioning nucleus, $A_0<N_1+Z_1+N_2+Z_2=50+28+82+50=210$ (in agreement with the observed fission yields for bismuth and lighter elements), and three maxima in the fission yield curves for heavier compound nuclei. The peak corresponding to approximately equal-size binary fission products is very much higher than is observed experimentally. This is undoubtedly due to the fact that in asymmetric fission a core corresponding to 82 neutrons and 50 protons remains intact in the heavier fission product, whereas for symmetric fission this core is disrupted at the cost of several Mev. Since correction for this energy effect involves a number of unknown factors, the calculated yields for symmetric fission have been reduced by the same empirical factor in all calculations. An additional parameter, n, is introduced in correcting for excitation energy of the fission products and for possible departures from equilibrium. These calculations, which involve only two free parameters, explain most of the fission yield data for all five known cases where the compound nucleus is within an Mev or so of the fission threshold: Pu^{239}, U^{238}, U^{235}, U^{233}, and Th^{232}, but it is necessary to treat n as a free parameter for each curve to fit the small steep regions on each side of mass number $\frac{1}{2}A_0$. The calculated fission yields of the more highly excited compound nucleus, Ac^{227}, predict three equally prominent maxima in qualitative agreement with observation.

IT has been pointed out by Fong[1] that the fission yield of a particular mass number should be proportional to the level density of the fission product of the same or slightly greater mass number multiplied by the level density of the accompanying fission product. Starting from this assumption and inferring level densities from low-energy neutron resonances, Fong calculated a remarkably good fit to the experimentally measured thermal fission yields for U^{235}. Cameron[2] also found an excellent fit starting from the same general idea. However, neither of these authors has extended his calculations to other fissionable nuclides. This is probably because the neutron resonance data gives a level density at a very different (several Mev) excitation energy from that of the fission products. Level densities are so sensitive to excitation energy that this correction tends to become an empirical fitting of experimental data with the use of many free parameters.[2] On the other hand, Fong and Cameron achieved enough success to make it seem very likely that the fundamental assumption of their calculations was at least approximately correct.

The shell model suggests two principal types of fission[3]: (1) We define (binary) *asymmetric fission* by the relations: $50<N_1$, $N_2>82$, $28<Z_1$, and $Z_2>50$, or after neutron emission, $77\lesssim A_1$ and $A_2\gtrsim 132$. (2) We define (binary) *symmetric fission* by: $28<Z_1<Z_2<50$ and $50<N_1<N_2\leq 82$ so that $A_2\lesssim 132$; this second mode is less favorable energetically because of the breakup of the "double magic core" ${}_{50}Sn_{82}{}^{132}$ of the heavier fragment. (3) The remaining cases, ternary and *very asymmetric* binary fission (where $A_1=N_1+Z_1<77$) have extremely small yields and will not be discussed or plotted in Fig. 1. The subscripts 0, 1, and 2 on the mass, charge, and neutron numbers (A, Z, and N) refer to the fissioning nucleus, the light fragment, and the heavy fragment, respectively.

We first calculate the relative level density of both binary asymmetric fission fragments by the method of Newson and Duncan,[4] plot the product of the relative level densities of each pair of fragments against A, and normalize the resultant curve so that the sum of the fission yields of all masses is 200%. This yield curve, which is shown in dashed curve (b) of Fig. 1(A) for the light fission fragment, does not fit the experimental points very well,[5,6] but the low calculated yields below $A_2=140$ would be greatly increased, and the fit of the calculated curve improved if a correction like Fong's were made for the very high excitation energy of these primary fission products compared to neutron escape energy. However, a much simpler calculation suggests itself. The considerations of Fong and Cameron were based on the assumption that after the saddle point, the motion of the fission fragments is so slow that there is always equilibrium, and the relative yield of a pair of fission fragments can be calculated exactly from their level densities. However, this equilibrium condition will not hold just before the instant of scission and a certain amount of smearing of curves[7] such as (b) in Fig. 1(A) will result. The neutrons which boil off the fission fragments after scission, have a similar but smaller smearing effect.

This statistical approach calculates the charge and mass of the potential fission fragments just before scission, while the yields are measured for the separated

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fission products after neutron and β-ray emission. The following conventions were adopted to bridge this gap. (1) The average number of neutrons ν emitted per fragment was assumed to be 2 for $N_2 \geq 84$, i.e., $A_2 \gtrsim 134$, and $\nu = 0.5$, for the accompanying fragment and for $N_0/2 < N_2 < 82$. Except near $N_2 = 82$ where the above assumption is reasonable, the fit is not very sensitive to ν which has been taken as 2.5 for *every* pair of fission products. A 50% probability for an odd value of ν must be assumed since the Newson-Duncan procedure predicts only even-even primary fragments. (2) All possible primary fission products which are β^- emitters were considered equally probable for each *even value* of A_1 or A_2. For each pair of products this procedure in itself weights the "equal length chains" drastically enough for practical purposes. (3) An average, over all possible sizes of the neck (which we would expect from the liquid drop model—see p. 304 of reference 3) has also been made. Clearly, from our point of view, the neck may vary between zero and $(A_0 - 78 - 132)$ mass units. This average (performed before the smearing calculation) changes the dashed curve (b) Fig. 1(A) into the reflection of curve (a) in that figure; after the smearing operation has been carried out, the difference does not affect our ability to fit the data but a small effect on the smearing parameter n is apparent.

Since the long-range electrical forces acting alone favor symmetric fission, any last-minute smearing before scission should be in the direction to reduce the mass difference between the two fragments. In order to calculate the light fragment asymmetric fission yield for Pu^{239}, six curves with smaller abscissas but congruent to (a) [Fig. 1(A)] were added to it; the ordinates of all the maxima were the same but each curve was displaced two mass units from its nearest neighbors. Thus the calculated distribution curve (a) was smeared toward the left over an interval, $n = 12$ mass units. The yield of the other fragment was calculated similarly. Actually the single free parameter $n = 10$ gives a very good fit to five of the six curves in Fig. 1 except for the relatively few points between $A_0 - 130 < A_1$ and $A_2 < 130$. The sensitivity of the calculated curves to the value of n is indicated in Fig. 1(B).

The smearing process introduces more nearly equal mass fission fragments than our first definition of asymmetric fission permitted. The process also increases the predicted relative yield of fragments with $A_2 < 132$. An excitation energy correction would have had the same qualitative effect. Thus it is not surprising that when n is used as a free parameter the smearing operation replaces the corrections both for departure from equilibrium and for excitation energy.

The calculated asymmetric fission yields in the two upper curves [Figs. 1(A) and 1(B)] account satisfactorily for the experimental data, but a third maximum, corresponding to symmetric fission, may be seen in each of the two lower curves [Figs. 1(E) and 1(F)]. The symmetric fission has been calculated according to our

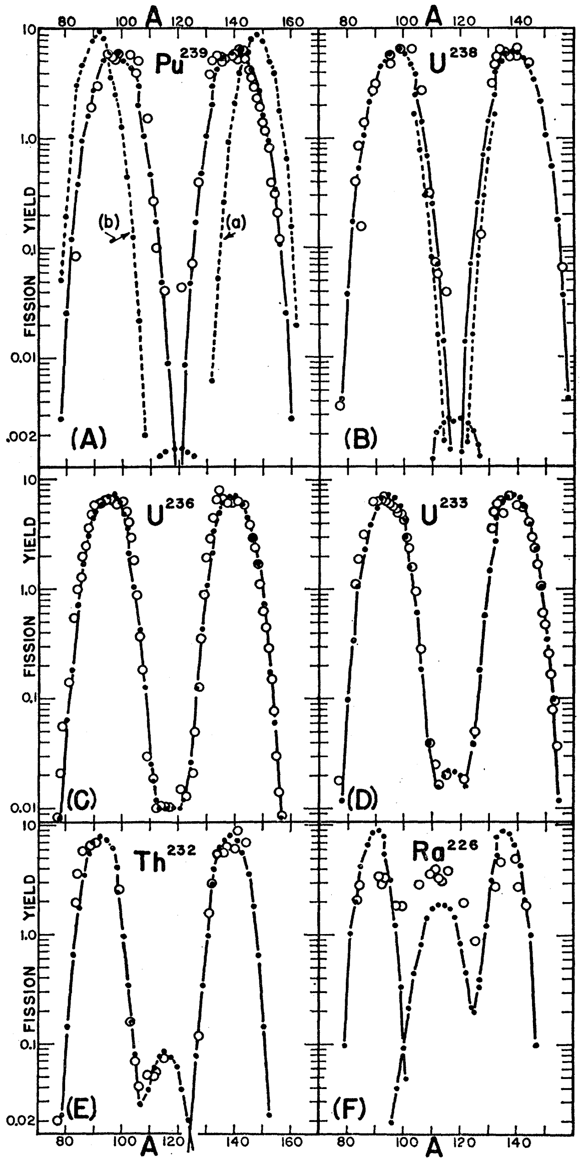

FIG. 1(A) The experimental fission yields (in percent) from $Pu^{239}(n,f)$ for thermal neutrons are plotted as circles (o) against mass number A. The calculated yields are plotted as solid dots which are connected (where practical) by straight lines. For this calculation the resolution width n was 12 mass units. The calculated points at the very bottom of the plot are the predictions for symmetric fission with $n = 12$. The dashed calculated curves, (*a*) and (*b*), assume $n = 0$. (B) $U^{238}(n,f)$ for 2.8-Mev neutrons. The solid calculated curve assumes $n = 12$ and the dashed curve $n = 10$. The two calculated curves do not differ appreciably except where indicated. (C) $U^{235}(n,f)$ for thermal neutrons. The calculated points ($n = 10$) near $A = 116$ are the *sums* of the calculated yields of symmetric and asymmetric fission. (D) $U^{233}(n,f)$ for thermal neutrons. Otherwise the plot is the same as Fig. 1(C). (E) $Th^{232}(n,f)$ for 2.8-Mev neutrons. The calculated points ($n = 8$) show yields for the two types of fission separately near the right minimum and the sum near the left minimum. The experimental points for Figs. 1(A) to 1(E) are from the compilation of Katcoff[6] where references to the original work may be found. (F) $Ra^{226}(p,f)$ for 11-Mev protons. The experimental data are those of Jensen and Fairhall.[5] In Fig. 1(C) read U^{235} instead of U^{236}.

previous definition and added to Fig. 1 as indicated in the caption. The prescription of Newson and Duncan predicts a ratio of integrated symmetric to asymmetric fission which is much larger than found experimentally. The predicted ratio has been reduced by the *same factor* for all six parts of Fig. 1. This factor corresponds to a correction for the relatively low excitation energy of the symmetric fragments[3] since they are formed at the expense of the energy (>4 Mev) to break up the double magic core ($_{50}Sn_{82}{}^{132}$) of the heavier asymmetric fragment. It seems reasonable to find the same factor for compound nuclei with about the same energy relative to fission threshold.

Only two parameters are needed to explain most of the fission yield data for all five cases where the compound nucleus is within one Mev or so of the fission threshold, but it is necessary to treat n as a free parameter for each curve to fit the small steep regions on each side of mass number $\frac{1}{2}A_0$. The qualitative agreement [Fig. 1(F)] for Ra^{226} is also interesting. One would expect a better fit in this case if the proton bombarding energy were closer to the threshold value. It is interesting to note that, according to our definition, asymmetric fission becomes impossible for $A_0<78+132=210$, which is in agreement with the fact that single maxima are always observed for the fission of Bi^{209} and lighter nuclides.[3] The interpretation of underthreshold (spontaneous) and over-threshold ($E_n>5$ Mev) fission will be discussed in a later paper.

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Gamma-Gamma Directional Correlations in Nd^{147}†*

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Directional correlation measurements have been made on the 320- to 92-kev and 280-to 320-kev gamma-ray cascades in Pm^{147} following the decay of 11.1-day Nd^{147} with a coincidence scintillation spectrometer using NaI detectors. The observed correlation functions are: $W(\theta)=1-(0.1030\pm0.0298)\ P_2(\cos\theta)+(0.0107\pm0.0099)\ P_4(\cos\theta)$, and $W(\theta)=1+(0.0710\pm0.0162)\ P_2(\cos\theta)-(0.0126\pm0.0103)\ P_4(\cos\theta)$, respectively, for the two cascades. The energy levels of Pm^{147} at ground state, 92 kev, 410 kev, and 690 kev were found to be $\frac{7}{2}+$, $\frac{7}{2}+$, $\frac{7}{2}+$, and $\frac{5}{2}+$, respectively. It was found that the 92-kev gamma ray has a mixture of $(95\pm2)\%$ $M1$ and $(5\pm2)\%$ $E2$ with $\delta_{92}=+(0.229\pm0.143)$, the 320-kev gamma ray has a mixture of 1% $M1$ and 99% $E2$ with $\delta_{320}=+9.95\pm0.11$, and the 280-kev gamma ray has a mixture of 99% $M1$ and 1% $E2$ with $\delta_{280}=-0.11\pm0.11$.

INTRODUCTION

THE decay scheme of Nd^{147}, 11.1 day half-life, has been investigated by different authors[1–9] and is shown in Fig. 1. There is complete agreement in the decay schemes as proposed by Hans *et al.*[8] and Mitchell[6] on the one hand, and Cork[7] on the other hand, except for the level at 289 kev, the transition β_5^-, the gamma rays of energies 160 kev and 198 kev and the position of the gamma ray of energy 410 kev. But none of these discrepancies are involved in the present correlation studies. The gamma rays and the energy levels of interest are shown by boldface lines. The spin of the ground-state of Nd^{147} has been measured by Abraham[10] to be $\frac{5}{2}-$. Very recently the ground-state spin of Pm^{147} has been measured by Klinkenberg and Tomkins,[11] and by Cabezas *et al.*,[12] and is found to be $\frac{7}{2}$. All the β^- transitions from the ground state of Nd^{147} and feeding the excited states of Pm^{147} have been classified as first forbidden with a spin change of zero or one and with a change of parity.[2,3,5] Thus each excited level in Pm^{147} must have one of the following values: $\frac{3}{2}+$, $\frac{5}{2}+$, $\frac{7}{2}+$.

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