point of the order of 5.24 Mev would be more compatible with the present data. In fact, recognizing the problems inherent in measuring short-range particles, even this value could be interpreted as only a lower limit. It is conceivable, for instance, that tracks of shorter range than the peak p were heavily discriminated against because of difficulty in recognition. Further, both latent-image fading and incomplete development, when effecting a track of the order of, say, 5 μ, would severely reduce the observation efficiency. Thus group p might actually only represent the distorted high-energy tail of a more intense group with a range several microns shorter. Two microns, for example, would imply an error of about 100 kev. Considering this possibility, most of the plates involved were scanned by four different observers. In the absence of any systematic disagreement, it was concluded that the effect probably did not occur, and no account of it was taken in the error estimates.

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Core Excitations in Nondeformed, Odd-A, Nuclei*

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The possibility of describing some excited states of odd-A nuclei in terms of excitations of the even-even core is investigated. No assumption is made on the nature of the core excitation, but certain relations involving electromagnetic transitions and moments are deduced. These seem to fit well some data available on Ag^{107}, Ag^{109}, Au^{197}, Hg^{199}, Tl^{203}, and Tl^{205}. More experimental data are required to test the validity of this picture in other cases.

INTRODUCTION

THERE are several known ways of exciting a nucleus from its ground state. The simplest of them is described, in the approximation of independent particle motion, by the elevation of a single particle from one state to another. These single particle excited states show up especially well[1] in stripping, pickup, and possibly other reactions. A slightly more complex excitation is that in which the ground-state configuration remains unchanged but the nucleons in this configuration change the relative orientation of their orbits. A typical case is offered by ${}_{23}V_{28}{}^{51}$ whose neutron shell is a closed one. Its ground state has $J=\frac{7}{2}$ and is believed to be the $J=\frac{7}{2}$ state of the configuration $(1f_{7/2})^3$. The first excited state has $J=\frac{5}{2}$ and is believed to be the $J=\frac{5}{2}$ state of the same configuration $(1f_{7/2})^3$. A slightly different example[2] is that of ${}_{17}Cl_{21}{}^{38}$ whose ground state and three lowest excited states are believed to be the four states of the configuration $(1d_{\frac{3}{2}};1f_{7/2})$, i.e., the configuration of one proton in $1d_{\frac{3}{2}}$ and one neutron in $1f_{7/2}$. A third class of excitations is that due to the collective motion of many nucleons.[3] These well-known modes of excitation include collective rotations, vibrations, etc.

The above modes of excitations may combine in characteristic ways. Thus it is well known that in the regions of large deformations, where collective rotations generally represent the lowest excitations, it is possible to excite a single nucleon from one orbit in the deformed-potential to another. This excited single-particle state then forms the basis for a new rotational band.

Another interesting "combined" excitation is suggested by the jj-coupling shell model, as was stressed by Lawson and Uretsky.[4] Let the ground-state configuration of an odd-even nucleus be described by $|(j_p{}^2)_{J_p=0}j_n\rangle$, i.e., by a pair of protons in j_p coupled to $J_p=0$ and a neutron in j_n. Then in addition to the single-particle excitation, which will be described by the configurations $|(j_p{}^2)_{J_p=0}j_n'\rangle$, one should expect also excitations described by $[(j_p{}^2)_{J_p\neq 0}j_n]_J$. In such states the neutron remains in its lowest state, while the proton pair is decoupled and excited to a state with $J_p\neq 0$, J_p and j_n then being coupled to the total angular momentum J.

This mode of excitation can be generalized slightly. In fact, consider an even-even nucleus A. Its ground state has $J=0$ followed by various excited states. Let us now

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[1] See, for instance, N. H. McFarlane and J. B. French, Revs. Modern Phys. **32**, 567 (1960).
[2] S. Goldstein and I. Talmi, Phys. Rev. **102**, 589 (1956); S. P. Pandya, *ibid.* **105**, 956 (1956).
[3] K. Alder, N. Bohr, T. Huus, B. Mottelson, and A. Winther, Revs. Modern Phys. **28**, 432 (1956).
[4] R. D. Lawson and J. L. Uretsky, Phys. Rev. **108**, 1300 (1957).

add a nucleon to this nucleus. The ground state will be obtained by putting the odd nucleon in the lowest allowed orbit of the average potential created by the core of the A nucleons. Suppose now that the next single-particle state in this average potential is high compared to the lowest excitation energy of the even-even core. It is then reasonable to assume that the lowest excitations of the nucleus $A+1$ will be described by the odd nucleon staying in its lowest orbit, and the core excited to its first excited state.

An example may clarify the point further. Consider ${}_{80}\mathrm{Hg}_{122}{}^{202}$, whose decay scheme is shown[5] in Fig. 1. In the average field created by Hg^{202} the lowest allowed proton orbit is presumably an $s_{\frac{1}{2}}$ orbit; in fact ${}_{81}\mathrm{Tl}_{122}{}^{203}$ has a measured $\frac{1}{2}^+$ ground state. Let us now consider the excited states in Tl^{203}, as shown in Fig. 1(b). The usual interpretation is to say that these states are obtained by promoting the proton (or rather a proton hole), from the single-particle $s_{\frac{1}{2}}$ state to the single-particle $d_{\frac{3}{2}}$ and $d_{\frac{5}{2}}$ states. We note, however, that the excitation energy of these states is comparable to that of the first excited state in Hg^{202}. If the $\frac{3}{2}^+$ and the $\frac{5}{2}^+$ states in Tl^{203} are in fact hole excitations, one may ask where are the $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states which would result from the coupling of the $s_{\frac{1}{2}}$ proton to the 2^+ excited state of the Hg^{202} core. The fact that such states are not known around an excitation energy of 500 kev in Tl^{203} suggests that the two observed levels are indeed those obtained from coupling the $s_{\frac{1}{2}}$ proton to the 2^+ state of the Hg^{202} core. The single-hole excitations will then have to be looked for at higher energies.

The clarification of the nature of these excited states in Tl^{203}, as well as of similar states in other nuclei, is interesting not only for its own sake. If it turns out, as seems to be the case, that these states are indeed core excitations coupled to the odd nucleon in its lowest state, we could use measured data on these levels to deduce further properties of the core. We shall also have to modify our picture of the spacings between single-particle levels in such nuclei.

The present paper is devoted to the empirical study of this question. We first develop some simple relations concerning such modes of excitations. Then, using these relations, we see to what extent some empirical evidence can be interpreted as indicative of core excitations in odd-even nuclei. We shall also discuss some information that can be deduced on the core states from the available data on the excited states of some odd-even nuclei.

BASIC RELATIONS

To investigate core excitations in odd-even nuclei, we shall describe the zeroth-order wave functions of such nuclei by the ket $|J_c j,JM\rangle$. Here J_c stands for the core angular momentum, j for that of the odd particle, and J is the total angular momentum with $J_z=M$. In the absence of any interaction between the core and the odd

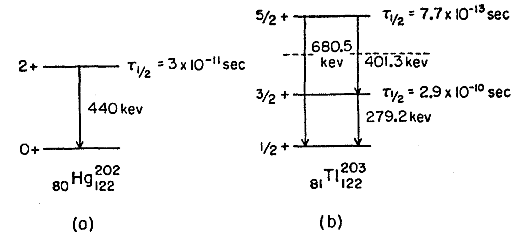

FIG. 1. Energy levels of Hg^{202} and Tl^{203}. Broken line shows position of the center of mass of the $\frac{5}{2}^+$ and $\frac{3}{2}^+$ levels.

particle, all the states characterized by the same pair of values J_c and j, but different values of J (and M), are degenerate. We shall refer to these levels as a "core multiplet." A particle-core interaction will split the degeneracy within a core-multiplet, leaving, in general, only the obvious M degeneracy.

It is tempting to identify the core-state J_c in an odd-even nucleus with the corresponding core-state J_c in the neighboring even-even nucleus. This, however, can be done only with some reservation. The ground state $J_c=0$ of an even-even nucleus, when expanded in terms of the single-particle states of the self-consistent average field, will generally include nucleons also in the state j. When we add a nucleon to the j orbit, the Pauli principle makes it less available for the particles of the core. Consequently, the core state $J_c=0$ in an odd-even nucleus will generally stand for something different from the core state $J_c=0$ of a neighboring even-even nucleus. If, however, the core states represent a thorough mixture of the self-consistent single-particle states, it can be expected that the addition of a single particle to the j orbit will not be very crucial. We shall assume that this is the case and ignore the antisymmetrization of the wave function $|J_c j,JM\rangle$ with respect to the exchange of the odd particle and the particles of the core. The validity of this approximation is not too clear to the author at this stage.

The interaction between the odd particle and the core is obviously a scalar, and we shall take it to be a product of two tensors of degree k: $T^{(k)}(c)$ operating on the core degrees of freedom and $T^{(k)}(p)$ operating on the degrees of freedom of the particle. Of course, the general interaction is a sum of such products taken over all values of k. Since we leave k quite general it is more transparent to consider only one value of k at a time.

The shift in energy $\Delta E(J)$ of the state $|J_c j,J\rangle$ from its unperturbed zeroth-order energy, is given[6] to first order by

$$\begin{aligned}\Delta E_k(J) &= \langle J_c jJ|\mathbf{T}^{(k)}(c)\cdot\mathbf{T}^{(k)}(p)|J_c j\rangle \\ &= (-1)^{J_c+j+J}\langle J_c\|T^{(k)}(c)\|J_c\rangle \\ &\qquad\times\langle j\|T^{(k)}(p)\|j\rangle\begin{Bmatrix} J_c & j & J \\ j & J_c & k\end{Bmatrix}, \quad (1)\end{aligned}$$

[5] Experimental data are taken from Nuclear Data Sheets, unless otherwise stated.

[6] For various relations involving the Racah coefficients and tensor algebra see N. Rotenberg, R. Bivins, N. Metropolis, and J. K. Wooten, Jr., *The 3-j and 6-j Symbols* (The Technology Press, M.I.T., 1959).

where

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ l_1 & l_2 & l_3 \end{Bmatrix} = (-1)^{j_1+j_2+l_1+l_2} W(j_1 j_2 l_2 l_1; j_3 l_3)$$

is a Racah coefficient. We note that the J dependence of $\Delta E(J)$ comes only through universal functions. It then follows from the orthogonality relations of Racah coefficients[6] that for any interaction:

$$\sum (2J+1)\Delta E_k(J) = 0 \quad \text{if} \quad k \neq 0. \tag{2}$$

Thus, inasmuch as the interaction between the core and the odd particle does not involve monopoles ($k=0$), the "center of gravity" of each multiplet coincides with its "unperturbed" position.

For $k=0$, $\Delta E_0(J)$ is independent of J, so that such an interaction produces no splitting in the multiplet. Its only effect, in first order, is to shift the whole multiplet as such, by an energy

$$\Delta E_0 = \frac{\langle J_c \| T^{(0)}(c) \| J_c \rangle}{(2J_c+1)^{\frac{1}{2}}} \cdot \frac{\langle j \| T^{(0)}(p) \| j \rangle}{(2j+1)^{\frac{1}{2}}}. \tag{3}$$

The expression $\langle J_c \| T^{(0)}(c) \| J_c \rangle / (2J_c+1)^{\frac{1}{2}}$ will generally depend on the value of J_c. There are, however, two important cases for which it is independent of J_c. If the core state consists of particles in one orbit only (i.e., the core state is $|j_c{}^n J_c\rangle$), and if $T^{(0)}(c)$ is a sum of single-particle operators, then it is easily verified that $\langle J_c \| T^{(0)}(c) \| J_c \rangle \approx (2J_c+1)^{\frac{1}{2}}$. Similarly, if we consider a sequence of values of J_c arising from a collective motion superimposed on the same intrinsic structure, and if $T^{(0)}(c)$ depends on the intrinsic structure only, then again $\langle J_c \| T^{(0)}(c) \| J_c \rangle \approx (2J_c+1)^{\frac{1}{2}}$. In these cases the separation between the centers-of-mass of multiplets constructed on different core states J_c with the same single-particle state j, should be equal to the separation between the unperturbed multiplets. If, further, the core states in the odd-even nucleus are identical with those of the neighboring even-even nucleus, the multiplets' center-of-mass separation ought to be identical with the separation between the corresponding states in the even-even nucleus. This is the "center-of-gravity" theorem of Lawson and Uretzki.[4] It is seen that in general there is no reason to expect this theorem to hold very precisely. First, for realistic cases, the value of $\langle J_c \| T^{(0)}(c) \| J_c \rangle / (2J_c+1)^{\frac{1}{2}}$ will probably depend, at least to some extent, on J_c. Second, even if the core-particle interaction contains no monopole-monopole part, the core excitation in the odd-even nucleus will generally require different energy than the corresponding excitation in a neighboring even-even nucleus. We can, however, expect the center-of-mass theorem to hold qualitatively, as will be shown when we discuss the empirical data.

The situation is particularly simple when the odd particle is in a state with $j=\frac{1}{2}$. Since J_c generally assumes the values 0, 2, 4, etc., with $j=\frac{1}{2}$ each value of J determines a unique value of J_c. The multiplet structure can then be expected to be very "clean." Furthermore, for $j=\frac{1}{2}$ only $k=0$ and $k=1$ can contribute to (1). We have already seen that $k=0$ does not give rise to any splitting, so that the whole multiplet splitting must be due to the term with $k=1$ in the particle-core interaction. The existence of such a splitting and its size are thus a direct indication and a measure of the dipole-dipole interaction between an odd particle and the even-even core. Furthermore, if the odd particle is in an $s_{\frac{1}{2}}$ orbit, then the only (velocity-independent) operator $T^{(k)}(p)$ for which $\langle s_{\frac{1}{2}} \| T^{(k)}(p) \| s_{\frac{1}{2}} \rangle$ does not vanish is $f(r_p)\boldsymbol{\sigma}_p$. Therefore, the multiplet splitting in this case probably measures the interaction of the *spin* of the odd particle with the core.

Apart from the considerations on energies, the proposed mode of excitation also affects very strongly the electromagnetic transition probabilities between the different states. Let us now consider these in some detail. It will be convenient to introduce the following notation:

$\Omega_\kappa{}^{(k)}$ is the κth component of the tensor operator of degree k whose expectation value (more precisely that of $\Omega_0{}^{(k)}$) gives the conventional definition of the static multipole moment of order k.

$$\begin{aligned} &\text{For } k=1, \quad \boldsymbol{\Omega}^{(1)} = \textstyle\sum_i g_{l,i}\mathbf{l}_i + g_{s,i}\mathbf{s}_i. \\ &\text{For } k=2, \quad \Omega_\kappa{}^{(2)} = (16\pi/5)^{\frac{1}{2}} \textstyle\sum e_i r_i{}^2 Y_{2\kappa}(\theta_i, \varphi_i). \end{aligned} \tag{4}$$

(g_l and g_s are g factors; e_i is the charge in units of the electronic charge.)

The rate of emission of radiation of the corresponding multipolarity is then given in terms of the same operators $\boldsymbol{\Omega}^{(k)}$ by

$$T_{i\to f} = E^{2k+1}\alpha_k[1/(2J_i+1)]|\langle f \| \boldsymbol{\Omega}^{(k)} \| i \rangle|^2. \tag{5}$$

Here i and f stand for initial and final states, respectively, J_i is the total angular momentum of the initial state, E is the energy of the transition in question, and α_k is a numerical constant. If E is measured in Mev, the numerical values of α_k turn out to be

$$\begin{aligned} &\text{For } M1 \text{ radiation}, \quad \alpha_1 = 4.2\times 10^{12} \text{ sec}^{-1} \text{ Mev}^{-3}; \\ &\text{For } E2 \text{ radiation}, \quad \alpha_2 = 3.87\times 10^{12} \text{ sec}^{-1} \text{ Mev}^{-5}. \end{aligned} \tag{6}$$

We now separate each multipole operator $\boldsymbol{\Omega}^{(k)}$ into a part due to the core and a part due to the odd particle

$$\boldsymbol{\Omega}^{(k)} = \boldsymbol{\Omega}_c{}^{(k)} + \boldsymbol{\Omega}_p{}^{(k)}.$$

We then obtain for the general matrix element of interest to us

$$\begin{aligned} &\langle J_c' j, J_f \| \boldsymbol{\Omega}_c{}^{(k)} + \boldsymbol{\Omega}_p{}^{(k)} \| J_c j, J_i \rangle \\ &\quad = (-1)^{J_c'+j+J_f}[(2J_f+1)(2J_i+1)]^{\frac{1}{2}} \\ &\quad \times \left[\langle J_c' \| \Omega_c{}^{(k)} \| J_c \rangle \begin{Bmatrix} J_c' & J_f & j \\ J_i & J_c & k \end{Bmatrix} \right. \\ &\quad \left. + (-1)^{J_i - J_f} \langle j \| \Omega_p{}^{(k)} \| j \rangle \begin{Bmatrix} j & J_f & J_c \\ J_i & j & k \end{Bmatrix} \delta_{J_c J_c'} \right]. \end{aligned} \tag{7}$$

It will be useful to consider in detail some special cases of Eq. (7). Let us first look at magnetic moments and $M1$ radiations. For transitions between states belonging to the same multiplet, we have[7]

$$\frac{T_{i\to f}(M1)}{E^3}=\alpha_1(2J_f+1)\Bigg[\langle J_c\|\mathbf{\Omega}_c{}^{(1)}\|J_c\rangle\begin{Bmatrix} J_c & J_f & j \\ J_i & J_c & 1\end{Bmatrix} +(-1)^{J_i-J_f}\langle j\|\mathbf{\Omega}_p{}^{(1)}\|j\rangle \times\begin{Bmatrix} j & J_f & J_c \\ J_i & j & 1\end{Bmatrix}\Bigg]^2. \quad (8)$$

This relation can be simplified by noting that it is true for $\mathbf{\Omega}_c{}^{(1)}$ and $\mathbf{\Omega}_p{}^{(1)}$ being any vector operators operating on the core and particle coordinates, respectively. We can choose then $\mathbf{\Omega}_c{}^{(1)}=\mathbf{J}_c$ and $\mathbf{\Omega}_p{}^{(1)}=\mathbf{j}$; obviously for $J_f\neq J_i$ the left-hand side of Eq. (7) then vanishes. Noting that $\langle j\|\mathbf{j}\|j\rangle=[j(j+1)(2j+1)]^{\frac{1}{2}}$, we obtain therefore

$$[J_c(J_c+1)(2J_c+1)]^{\frac{1}{2}}\begin{Bmatrix} J_c & J_f & j \\ J_i & J_c & 1\end{Bmatrix} -[j(j+1)(2j+1)]^{\frac{1}{2}}\begin{Bmatrix} j & J_f & J_c \\ J_i & j & 1\end{Bmatrix}=0 \quad \text{for} \quad J_f\neq J_i.$$

Furthermore it can be verified easily that the magnetic moment $\mu(j)$ of a system whose angular momentum is j is given by

$$\mu(j)=jg=\left[\frac{2j}{(2j+1)(2j+2)}\right]^{\frac{1}{2}}\langle j\|\mathbf{\Omega}^{(1)}\|j\rangle.$$

Equation (8) can therefore be written in the form

$$\frac{T_{i\to f}}{E^3}=\alpha_1(2J_f+1)j(j+1)(2j+1) \times\begin{Bmatrix} J_i & J_f & 1 \\ j & j & J_c\end{Bmatrix}^2(g_c-g_p)^2. \quad (9)$$

Thus, like for $M1$ transitions within a rotational band in deformed nuclei, the *ratio* of two $M1$ transitions within one multiplet are independent of the specific values of g_c and g_p. For $g_c=g_p$ all such transition probabilities vanish, since, under this condition, $\mathbf{\Omega}^{(1)}$ becomes proportional to $\mathbf{J}=\mathbf{J}_c+\mathbf{j}$ and cannot connect two different eigenstates of J^2.

To obtain separate values for g_c and g_p we have to know at least one static moment. If the ground state of the odd-even nucleus is assumed to be the state with $J_c=0$, then its g factor is identical with g_p. Alternatively a measured g factor of an excited state can give us another combination of g_c and g_p.

It is evident from Eq. (7) that if the ground state in an odd-even nucleus has $J_c'=0$, then $M1$ radiation from the multiplet constructed on $J_c=2$, say, to the ground state is absolutely forbidden. In checking this prediction one should, however, exercise great care. It is clear that even if the description of the nuclear states as $|J_cj,J\rangle$ is a very good one, it cannot be perfect. For instance we expect the ground state to be $|0j, J=j\rangle$ with at least a little admixture of $|2j, J=j\rangle$ (if $j\geqslant\frac{3}{2}$). Similarly, the state $|2j,J\rangle$ can have small admixtures of the state $|0j',J\rangle$ where $j'=J$, etc. In some cases even a small admixture of 2–3% can lead to an appreciable $M1$ rate for a transition which strictly speaking should have been forbidden. Rather than discuss this question in its full generality, we shall take it up in a concrete case and show there how these admixtures operate.

Finally we want to consider specifically Eq. (7) with $k=2$. Concentrating on $E2$ transitions from the multiplet $|J_c=2, j; J\rangle$ to the ground state $|J_c'=0, j; J_f=j\rangle$, we obtain

$$(T_{i\to f}/E^5)=(\alpha_2/5)|\langle 0\|\Omega_c{}^{(2)}\|2\rangle|^2. \quad (10)$$

Thus the value of $T_{i\to f}/E^5$ should be the same for all $E2$ transitions proceeding from the various members of a multiplet to the ground state. Furthermore this value should be equal to the value of $T_{i\to f}/E^5$ for the $2^+\to 0^+$ transitions in neighboring even-even nuclei, if the core states in odd-even nuclei are identical with the corresponding states in the even-even nuclei.

ANALYSIS OF SOME EXPERIMENTAL DATA

The identification of a group of levels in an odd-even nucleus as belonging to one multiplet is generally quite difficult. Due to the methods through which data on excited states are obtained, one usually discovers levels whose spins are rather close to each other. Thus multiplets with many components may not be known in full and their identification becomes more difficult.

The simplest multiplets are those arising from $j=\frac{1}{2}$. According to the discussion in the previous section, in nuclei whose ground state has $J=j=\frac{1}{2}$, we should find excited states with spins $J=\frac{3}{2}$ and $J=\frac{5}{2}$ and whose parity is identical with that of the ground state. Furthermore, these doublets should be centered roughly around the energy of the first excited state in a neighboring even-even nucleus, and the electromagnetic radiations involving these levels should have the peculiar features indicated in the previous section.

Ignoring the light elements, where isotopic spin may bring in more complications, we have the following regions of the periodic table in which to look for core-excited doublets:

For Z or $N=39$–49, the $p_{\frac{1}{2}}$ level may and does show up as the ground state. Se^{77}, Rh^{103}, Ag^{107}, and Ag^{109} are nuclei in this region which have enough data to be analyzed. Another region is that with $N=63-73$ where the odd neutron is in an $s_{\frac{1}{2}}$ orbit, with Cd^{111} and Cd^{113} being possible cases for detailed study. Then come the Tl isotopes for which the proton is in an $s_{\frac{1}{2}}$ orbit, and

[7] The quantity $T_{i\to f}/E^3$ is proportional to $B(M1)$ (see work cited in reference 3). We prefer to leave it in the slightly more explicit form to avoid confusion of units, decay matrix elements vs excitation matrix elements, etc.

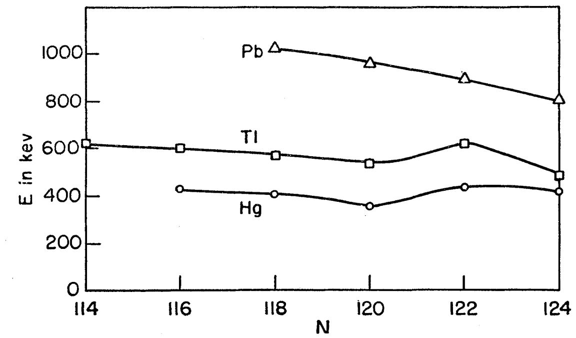

FIG. 2. The 2^+ excitation energies in the even-even isotopes of Hg and Pb and the position of the $\frac{5}{2}-\frac{3}{2}$ center of mass of the odd-A isotopes of Tl, all as a function of the neutron number N.

some odd-A isotopes of Pt, Hg, and possibly Pb for which the neutron is in a $p_{\frac{1}{2}}$ orbit.

Let us consider first the Tl isotopes. All the odd-A isotopes from Tl^{195} to Tl^{205} show the same characteristic spectrum: $\frac{1}{2}^+$ as the ground state with $\frac{3}{2}^+$ and $\frac{5}{2}^+$ the two lowest excited states. Figure 2 shows the center of gravity of the $\frac{3}{2}^+-\frac{5}{2}^+$ doublets in the odd-A Tl isotopes as a function of the neutron number. Shown on the same graph are the 2^+ states in Hg and Pb isotopes with an equal number of neutrons. The Tl line follows closely that of Hg and suggests that the $\frac{3}{2}^+$ and $\frac{5}{2}^+$ excited states could indeed be interpreted as a doublet constructed from the coupling of the $s_{\frac{1}{2}}$ proton to the 2^+ excited state of the core.

For two of the Tl isotopes, Tl^{203} and Tl^{205}, there are also half-lives and branching ratios measurements available. These are indicated in Fig. 3. Using these experimental data we obtain for the reduced $E2$ matrix elements, i.e., $T_{i\to f}(E2)/E^5$ the values shown in Table I.

We see that the reduced matrix element for the two $E2$ transitions in each of the Tl isotopes are equal to each other [compare Eq. (10)] and their value fits well with that obtained for the neighboring even-even isotopes. The data are at best accurate to $\sim 15\%$, so that a more detailed systematic, though apparently there, is not too meaningful.

For the $M1$ transitions the situation is as follows:

The $\frac{3}{2}\to\frac{1}{2}$ $M1$ transition probability is very small. Its reduced matrix elements are $T_{i\to f}/E^3=3\times10^{10}$ sec^{-1} Mev^{-3} in Tl^{203} and 1.2×10^{10} sec^{-1} Mev^{-3} in Tl^{205}. The expected corresponding value for a single-particle transition is[8] 2.8×10^{13} sec^{-1} Mev^{-3}. This highly reduced $M1$ rate has been often associated with the possible Δl forbiddenness of the transition. In fact, if the $\frac{3}{2}^+$ state is interpreted as a $d_{\frac{3}{2}}$ state, and if one takes for the magnetic moment operator the conventional expression $\sum g_{l,i}\mathbf{l}_i+g_{s,i}\mathbf{s}_i$, then the $d_{\frac{3}{2}}\to s_{\frac{1}{2}}$ transition probability vanishes. It was, however, pointed out[9] that there may be corrections to the magnetic moment operator arising from exchange currents and the spin-orbit interaction, and that such corrections would make Δl-forbidden transitions possible. The big reduction of the $\frac{3}{2}\to\frac{1}{2}$ $M1$ matrix element in the Tl isotopes is therefore somewhat surprising on the single-particle picture. With the identification of the $\frac{3}{2}^+$ state as $|2\,\frac{1}{2}\,\frac{3}{2}\rangle$, the situation is different. Now the $M1$ transition is forbidden, not because of the special structure of the $M1$ radiation operator, but simply because this operator is a *vector* (i.e., an irreducible tensor of degree 1), and as such cannot connect the states with $J_c=0$ and $J_c=2$. The tensorial character of the $M1$ radiation operator is, of course, independent of the corrections to it. The existence of a slow $M1$, on this picture, is then indicative of the "impurity" of the state. We can have $|\frac{3}{2}\rangle=A|2\,\frac{1}{2}\,\frac{3}{2}\rangle+(1-A^2)^{\frac{1}{2}}|0\,\frac{3}{2}\,\frac{3}{2}\rangle$, and the small component $|0\,\frac{3}{2}\,\frac{3}{2}\rangle$ could then lead to an $M1$ transition to the ground state via the corrections to the magnetic moment operator.

The situation is different with respect to the $\frac{5}{2}\to\frac{3}{2}$ $M1$ transition. The reduced matrix elements here are $T_{i\to f}/E^3=1.2\times10^{13}$ sec^{-1} Mev^{-3} for Tl^{203} and 5.5×10^{12} for Tl^{205}. The errors in these numbers, especially for Tl^{205}, are rather big since they are derived in an indirect way. One measures the Coulomb excitation cross section for the $\frac{5}{2}$ state and deduces the partial lifetime for the $\frac{5}{2}\to\frac{3}{2}$ transition from the branching ratio for the transitions $\frac{5}{2}\to\frac{1}{2}$ and $\frac{5}{2}\to\frac{3}{2}$. This ratio is very small for Tl^{205}, leading to some uncertainties in the interpretation of the numbers derived from it. Considering only Tl^{203} and using Eq. (9) with the numerical values (6), we obtain

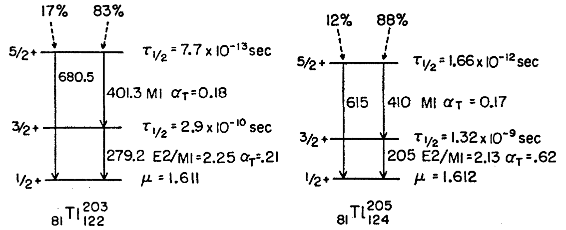

FIG. 3. Energy levels of Tl^{203} and Tl^{205} showing the data used in the calculations. The branching ratios refer to total transitions.

TABLE I. Reduced $E2$ matrix elements for Tl^{203} and neighboring even-even nuclei.

Nucleus	$E2$ transition	$T_{i\to f}/E^5$ in sec^{-1} Mev^{-5}
$_{81}Tl_{122}^{203}$	$\frac{3}{2}\to\frac{1}{2}$, 279 kev $\frac{5}{2}\to\frac{1}{2}$, 680 kev	0.8×10^{12} 0.8×10^{12}
$_{81}Tl_{124}^{205}$	$\frac{3}{2}\to\frac{1}{2}$, 205 kev $\frac{5}{2}\to\frac{1}{2}$, 615 kev	0.6×10^{12} 0.5×10^{12}
$_{82}Pb_{124}^{206}$	$2\to0$, 803 kev	0.3×10^{12}
$_{80}Hg_{124}^{204}$	$2\to0$, 430 kev	0.6×10^{12}
$_{80}Hg_{122}^{202}$	$2\to0$, 440 kev	1.3×10^{12}

[8] S. A. Moszkowski, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955).

[9] See, for instance, R. J. Blin-Stoyle, Revs. Modern Phys. **28**, 75 (1956).

TABLE II. Reduced $E2$ matrix elements for Ag^{107}, Ag^{109}, and neighboring even-even nuclei.

Nucleus	$E2$ transition	$T_{i\to f}/E^5$ in $\text{sec}^{-1}\ \text{Mev}^{-5}$
${}_{47}Ag_{60}{}^{107}$	$\frac{3}{2}^- \to \frac{1}{2}^-$, 324 kev $\frac{5}{2}^- \to \frac{1}{2}^-$, 423 kev	1.4×10^{12} 1.4×10^{12}
${}_{47}Ag_{62}{}^{109}$	$\frac{3}{2}^- \to \frac{1}{2}^-$, 309 kev $\frac{5}{2}^- \to \frac{1}{2}^-$, 416 kev	1.8×10^{12} 1.6×10^{12}
${}_{46}Pd_{60}{}^{106}$	$2^+ \to 0^+$, 513 kev	1.6×10^{12}
${}_{48}Cd_{60}{}^{108}$	$2^+ \to 0^+$, 630 kev	1.3×10^{12}
${}_{46}Pd_{62}{}^{108}$	$2^+ \to 0^+$, 433 kev	1.8×10^{12}
${}_{48}Cd_{62}{}^{110}$	$2^+ \to 0^+$, 656 kev	1.2×10^{12}

$(g_c - g_p) \approx \pm 2.6$. If we take now for g_p the values obtained from the observed ground-state magnetic moment, we get

$$g_c = 0.6 \text{ nm}.$$

It is difficult to estimate the limits of error on this number, but it seems that a value of $g_c = Z/A = 0.4$ is not really excluded by the experimental data on Tl^{203}. The analysis of Tl^{205} along similar lines leads to $g_c = 1.6$, but as was pointed out above the uncertainties in the experimental data are too big to allow any conclusion from this case.

Some of the special features of the coupling we are considering are obscured in the Tl isotopes due to the fact that the single-particle assignments for the levels $\frac{1}{2}^+$, $\frac{3}{2}^+$, and $\frac{5}{2}^+$ would also make the $\frac{3}{2}^+ \to \frac{1}{2}^+$ $M1$ transition a slow one (Δl forbidden) and the $\frac{1}{2}^+ \to \frac{5}{2}^+$ a "normal" $M1$ transition ($d_{\frac{5}{2}} \to d_{\frac{3}{2}}$). The situation is different if we consider nuclei whose ground state is $\frac{1}{2}^-$. As explained in the previous section, they are expected to have an excited "doublet" $\frac{3}{2}^-$ and $\frac{5}{2}^-$. The single-particle interpretation is now $p_{\frac{1}{2}}$ for the ground state, and $p_{\frac{3}{2}}$ and $f_{\frac{5}{2}}$ for the excited states. On this interpretation, therefore, the $\frac{3}{2}^- \to \frac{1}{2}^-$ is a "normal" $M1$ transition, whereas the $\frac{5}{2}^- \to \frac{3}{2}^-$ should be the Δl-forbidden transition. If, however, one adopts the assumption of a "core doublet" the situation is reversed: The $\frac{3}{2}^- \to \frac{1}{2}^-$ is "core forbidden," whereas the $\frac{5}{2}^- \to \frac{3}{2}^-$ $M1$ should proceed with a rate proportional to $(g_c - g_p)^2$.

An example of such a case can probably be found in the Ag isotopes, whose relevant levels are shown in Fig. 4. The doublet center of mass is at 383 kev in ${}_{47}Ag_{60}{}^{107}$ and 373 kev in ${}_{47}Ag_{62}{}^{109}$. This is to be compared with a 2^+ excitation of 513 kev in ${}_{46}Pd_{60}{}^{106}$, 630 kev in ${}_{48}Cd_{60}{}^{108}$, 433 kev in ${}_{46}Pd_{62}{}^{108}$, and 656 kev in ${}_{42}Cd_{62}{}^{110}$. The reduced $E2$ transition probabilities are given in Table II. Again we see that they fall well within the range of values obtained for the neighboring even-even nuclei in agreement with Eq. (10).

The $\frac{3}{2}^- \to \frac{1}{2}^-$ $M1$ transition should be forbidden. In reality we do not expect the states to be very pure, and a slight admixture, say, of $|0p_{\frac{3}{2}}\frac{3}{2}\rangle$ in $|2p_{\frac{1}{2}}\frac{3}{2}\rangle$ will give rise

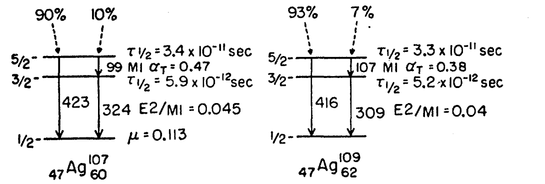

FIG. 4. Energy levels of Ag^{107} and Ag^{109} showing the data used in the calculations. The branching ratios refer to total transitions.

to an $M1$ radiation. Unlike the case of Tl (whose ground state is $\frac{1}{2}^+$), where the admixture could contribute only via the exchange-current correction to the $M1$ radiation operator, here such admixtures can contribute via the main part of the $M1$ operator. Their effect is consequently expected to be considerably bigger. The observed reduced matrix elements for the $\frac{3}{2}^- \to \frac{1}{2}^-$ transitions are $T_{i\to f}/E^3 = 3.3\times10^{12}\ \text{sec}^{-1}\ \text{Mev}^{-3}$ in Ag^{107} and $4.3\times10^{12}\ \text{sec}^{-1}\ \text{Mev}^{-3}$ in Ag^{109}. The single-particle estimates in both cases are $2.8\times10^{13}\ \text{sec}^{-1}\ \text{Mev}^{-3}$. The transitions are thus slowed down by nearly an order of magnitude.

The $\frac{5}{2}^- \to \frac{3}{2}^-$ $M1$ transition is Δl-forbidden on the single-particle assignment for these levels, and allowed if the two states are the components of a "core-doublet." We obtain for the reduced matrix elements: $T_{i\to f}/E^3 = 1.4\times10^{12}\ \text{sec}^{-1}\ \text{Mev}^{-3}$ for Ag^{107} and $0.9\times10^{12}\ \text{sec}^{-1}\ \text{Mev}^{-3}$ for Ag^{109}. Comparing this with the expected single-particle rate[8] for a $\frac{5}{2} \to \frac{3}{2}$ transition, i.e., $T_{i\to f}/E^3 = 3.4\times10^{13}\ \text{sec}^{-1}\ \text{Mev}^{-3}$, these transitions do seem to be even more hindered than the $\frac{3}{2}^- \to \frac{1}{2}^-$ transitions discussed above. However, it turns out that for the Ag isotopes $(g_c - g_p)^2$ is very small so that the slow $\frac{5}{2}^- \to \frac{3}{2}^-$ $M1$ transitions may reflect the smallness of $(g_c - g_p)^2$ rather than a forbiddenness of a $f_{\frac{5}{2}} \to p_{\frac{3}{2}}$ transition. Using indeed Eq. (9) we obtain for Ag^{107} $(g_c - g_p) = \pm 0.91$ and for Ag^{109} $(g_c - g_p) = \pm 0.73$, leading to $g_c = 0.7$ nm and 0.5 nm for Ag^{107} and Ag^{109}, respectively. The similarity between these values of g_c and the one obtained for Tl^{203} ($g_c = 0.6$ nm) further supports the "core-doublet" interpretation. Furthermore we see that the reduced matrix elements of the $\frac{3}{2}^- \to \frac{1}{2}^-$ and the $\frac{5}{2}^- \to \frac{3}{2}^-$ $M1$ radiations are of the same order of magnitude. It is therefore difficult to interpret one as a normal $M1$ and the other as a Δl-forbidden $M1$. On the "core-doublet" interpretation the smallness of the rate of the "normal" transition is due to the smallness of the *static*

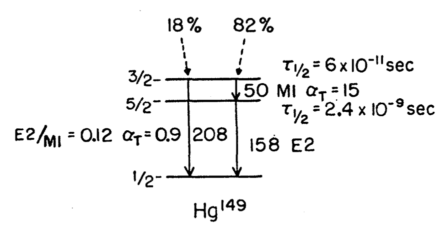

FIG. 5. Energy levels of Hg^{199} showing the data used in the calculations. The branching ratios refer to total transitions. (The 18% and 82% should be interchanged.)

moment of a $p_{\frac{1}{2}}$ proton, the latter being due to a near cancellation of the intrinsic moment by the orbital moment.

A somewhat more convincing example is that of Hg^{199}, the relevant data on which are given[10] in Fig. 5. Here again the $E2$ reduced matrix element for the $\frac{5}{2}^- \rightarrow \frac{1}{2}^-$ and the $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$ transitions are practically identical with each other and equal to that of Hg^{200} and Hg^{202}. For the $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$ $M1$ radiation we obtain $T_{i\rightarrow f}/E^3 = 4\times 10^{11}$ sec^{-1} Mev^{-3} as compared with an expected single-particle value of 2.8×10^{13} sec^{-1} Mev^{-3}, (i.e., a hindrance by a factor of 70). For the $\frac{3}{2}^- \rightarrow \frac{5}{2}^-$ $M1$ radiation, on the other hand, we obtain $T_{i\rightarrow f}/E^3 = 1.0 \times 10^{12}$ sec^{-1} Mev^{-3}. With the observed magnetic moment of the ground state of Hg^{199} this leads to a value of $g_c = 0.4$ nm—in very nice agreement with other values obtained for g_c. This example shows very clearly the superiority of the "core-doublet" interpretation over that of single-particle excitations.[10a]

Other examples can be added, though there are not too many nuclei for which enough reliable information is available. Since, however, we only want to stress the possibility of this mode of excitation in odd-even nuclei, we shall satisfy ourselves with the few examples given above.

DISCUSSION

The idea of coupling an odd nucleon to a core state is, of course, not new. It is the natural thing to do within the framework of Bohr and Mottelson's collective model.[11] Many extensive calculations[12] have been carried out with the aim of relating various quadrupole effects to the surface deformability, or of coupling the vibrational motion to the single-nucleon motion, etc. The purpose of the present note is to draw the attention to the simple consequences of coupling the odd nucleon to the core states. We deliberately avoided any identification of the core state being due to one mechanism of excitation or another. The consequences we have discussed are valid irrespective of the nature of the core excitation. If they are confirmed by further experimental data, we could conclude that, whatever the origin of the core excitation is, the odd nucleon is coupled weakly to it. Furthermore, it seems from an analysis of Au^{197} that the width of the "core-multiplet" is roughly the same for $j=\frac{1}{2}$ and $j=\frac{3}{2}$. If this is confirmed in general, it would indicate that the interaction between the odd particle and the core is predominantly a dipole-dipole interaction, since higher multipoles will not show up in the core-multiplet for $j=\frac{1}{2}$ (and weak-coupling). It is also worthwhile to note that we were able to obtain consistent values for g_c in various nuclei only by using the *observed* g factor for the odd nucleon (derived from the magnetic moment of the ground state). This g factor, as is well known, is significantly different from the Schmidt single-particle value. Thus the simple core-excitation approximation is valid, if at all, only if we assign the odd particle a renormalized effective moment.

ACKNOWLEDGMENTS

I should like to thank Professor L. Grodzins, Professor A. K. Kerman, and Professor V. F. Weisskopf for very helpful discussions.

[10] There is some confusion in the literature in quoting the data on Hg^{199}. We have estimated the 50/208 branching from the intensity of the corresponding conversion lines and the total conversion coefficients as given by P. J. Cressmann and R. G. Wilkinson, Phys. Rev. **109**, 872 (1957).

[10a] *Note added in proof.* R. Bauer, L. Grodzins, and H. Wilson have recently measured the magnetic moment of the $\frac{5}{2}$ state in Hg^{199} obtaining $\mu = +0.85 \pm 0.15$ nm. This leads to $g_c = 0.17 \pm 0.08$ nm. However, L. Grodzins has pointed out some serious discrepancies in available data on relevant branching ratios and conversion coefficients. It is therefore still an open question whether the magnetic moment of the $\frac{5}{2}^-$ state and the rate of the $\frac{3}{2}^- \rightarrow \frac{5}{2}^-$ $M1$ radiation in Hg^{199} are consistent with the model described here.

[11] A. Bohr and B. R. Mottelson, *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press, Inc., New York, 1960), p. 1009.

[12] See, for instance, L. L. Foldy and F. J. Milford, Phys. Rev. **80**, 751 (1950); F. J. Milford, *ibid.* **93**, 1297 (1953); A. K. Kerman, *ibid.* **92**, 1176 (1953); K. W. Ford and C. Levinson, *ibid.* **100**, 1 (1955); R. D. Amado, *ibid.* **108**, 1462 (1957); A. S. Davydov, Nuclear Phys. **16**, 597 (1960); D. Kurath and R. D. Lawson (to be published); N. K. Glendenning, Phys. Rev. (to be published).