purity indium available was used in the liquid scintillator;[11] (2) the spectrum obtained does not have the structure associated with a possible alpha-emitter contaminant such as thorium, nor is it similar to that obtained with gamma rays; (3) the activity has been observed for a period of $1\frac{1}{2}$ yr and has not changed in this time.

ACKNOWLEDGMENTS

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Coherent Scattering of 1.17-Mev and 1.33-Mev Gamma Rays through Small Angles*

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The dependence of the differential cross section for the coherent scattering of 1.17-Mev and 1.33-Mev gamma rays on atomic number was investigated. An empirical procedure, which makes an absolute determination of the cross sections unnecessary, was used to estimate the Compton scattering cross sections. The latter were subtracted from the measured cross sections in order to obtain the relative coherent scattering cross sections, which were found to vary as Z^n. Our average value for n is 3.07 ± 0.18.

The angular distribution of the total (coherent and Compton) scattering cross section was also investigated in the case of copper and lead between 2.43° and 5.79°. Our results are compared with the existing theoretical predictions and with the results of the earlier experiments, wherever the latter are available.

I. INTRODUCTION

NUCLEAR resonance scattering, Delbrück scattering by the nuclear Coulomb field, Thomson scattering by the nuclear charge, and Rayleigh scattering by the bound electrons in an atom represent different processes involving the coherent scattering of energetic gamma rays by atoms. Of these processes, resonance scattering plays an important role only under very special conditions.[1] Since Delbrück scattering[2] does not follow from the classical linear field equations of Maxwell, a great deal of attention has been focused recently on its experimental verification. Experiments[3] on the scattering of bremsstrahlung photons of 87-Mev mean energy through angles of a few milliradians appear to have established the existence of Delbrück scattering. Detailed calculations[4] of Delbrück scattering cross sections are extremely difficult and have not yet been made. Previous experiments,[5–9] though somewhat conflicting among themselves, have shown that Delbrück scattering is so small for large scattering angles at 1.17 and 1.33 Mev as to be almost undetectable. The corresponding situation for small scattering angles is not clear at the moment. Since the rest energy of a nucleus is much greater than the energy of gamma rays used in our experiments, scattering by the nuclear charge as a whole can be evaluated by the classical Thomson expression. For angles less than six degrees and for gamma-ray energies in the neighborhood of 1 Mev, nuclear Thomson scattering cross sections are expected to be smaller than 0.7% of the Rayleigh scattering cross sections. Therefore, scattering experiments in the small-angle region are very useful in the investigation of the Rayleigh scattering process. Similar experiments have been previously carried out at 0.411 Mev,[10] at 0.325 and 0.660 Mev,[11] and at 1.17 and 1.33 Mev.[12]

Calculations of Rayleigh scattering cross sections in the energy region of interest were first carried out by Franz,[13] with the assumption of the Fermi-Thomas

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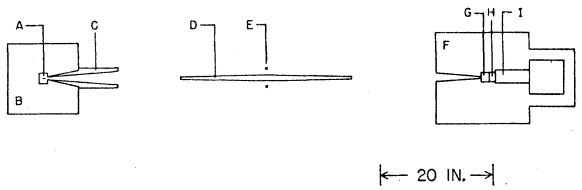

FIG. 1. A diagram of the experimental apparatus to scale. A is the Co^{60} source, B is the lead shielding around the source, C is an extra collimator of brass, D is the direct beam absorber, E is the scatterer for 4.68° measurements, F is the lead shielding around the detector, G is the sodium iodide crystal, H is the light guide, and I is the photomultiplier tube.

distribution for the electrons in an atom. He obtained the following expression for the differential cross section $d\sigma/d\Omega$ of an atom for Rayleigh scattering.

$$\frac{d\sigma}{d\Omega}=8.67\times10^{-33}\left(\frac{E_0}{E}\right)^3 Z^3\frac{1+\cos^2\theta}{2}\frac{1}{\sin^3(\theta/2)}\text{cm}^2/\text{sr}, \quad (1)$$

where E_0 is the rest energy of the electron, E is the energy of the incident gamma ray, and θ is the angle of scattering. Similar nonrelativistic calculations were done by Nelms and Oppenheim[14] with the help of Hartree wave functions for the electrons. The cross sections calculated by them depend in a complicated way upon atomic number of the scatterer, angle of scattering and energy of the gamma ray. Approximate relativistic calculations were performed by Levinger[15] and by Greifinger *et al.*[16] Binding of the electrons in the intermediate virtual states, which was neglected in the previous relativistic calculations, was considered by the Birmingham group in their extensive calculations for the K electrons of mercury at 0.32 E_0, 0.64 E_0, 1.28 E_0, and 2.56 E_0.[17] As far as the dispersive part of the scattering amplitude is concerned, the results of the detailed calculations do not differ much from those of the nonrelativistic calculations. The absorptive part, which is neglected in the nonrelativistic treatments, was calculated to be about one fifth of the dispersive part. Since similar calculations have not been done for small scattering angles and for other elements, it is sometimes necessary to use the simpler theory in an *ad hoc* manner.

We investigated the Z^n variation of the coherent cross section for different momentum transfers and the angular distributions of the total cross sections of copper and lead for 1.17- and 1.33-Mev gamma rays. The details of the experimental method are given in Sec. II. Corrections that have to be applied to the data are described in Sec. III. The experimental results are discussed in Sec. IV.

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II. EXPERIMENTAL DETAILS

The arrangement of the experimental apparatus is shown in Fig. 1. It is very similar to that used in the work of Storruste and Tjom. The cylindrical cobalt-60 source had a strength of about one curie. The aluminum can containing the source absorbed out the beta rays. The diameter and the length of the source were $\frac{1}{8}$ in. and $\frac{5}{8}$ in., respectively. In the final arrangement, the longer dimension of the source was made to coincide with the axis of the direct beam absorber. The cylindrical lead shielding around the source had a diameter of 12 in. and a length of 12 in. The beam was defined by a conical opening in the shield of 4.3° half-angle. The tapered direct beam brass absorber had a length of 30.0 in. Its thickness was 1.000 in. over the central 3 in. and 0.500 in. at the two ends. The gamma rays were detected by means of a cylindrical thallium activated sodium iodide crystal mounted on a Dumont 6292 photomultiplier. Both the diameter and the thickness of the crystal were $1\frac{1}{2}$ in. Pulses from the photomultiplier were amplified and then analyzed by means of one-hundred channel pulse-height analyzer[18] or two single-channel analyzers in parallel. The detector was shielded by about 8 in. of lead in all directions except that of the source. The hole in that direction was a cone of 3.6° half-angle and had a minimum diameter of 0.300 in. at the crystal end. The distance between the center of the source and the center of the 0.300-in. aperture was 76.5 in.

The cross sections of the ring scatterers with the exception of that of carbon were 0.500 in.×0.375 in., the longer dimension being parallel to the axis of the direct beam absorber. The cross sections of the carbon scatterers were 2.000 in.×0.375 in. The choice of the longer dimension of the carbon scatterer was dictated by a compromise between the necessity to provide very nearly the same number of electrons as in the case of other scatterers and the desire to avoid geometrical corrections. The carbon scatterers used by us had between 70 to 80% as many electrons as the other scatterers. The mean diameters of the scattering rings were 1.625 in., 2.375 in., 3.125 in., and 3.875 in., the corresponding angles of scattering being 2.43°, 3.57°, 4.68°, and 5.79°, respectively. By means of aluminum pins, the semicylindrical scattering rings could be accurately positioned with respect to a light Lucite holder mounted rigidly at the center of the direct beam absorber.

In order to carry out reliable measurements, it is necessary to align very carefully the source, the direct beam absorber axis and the axis of the conical opening in front of the detector. The last two were aligned with the help of an optical telescope. Then, counting rates were measured for different source positions in the absence of any ring scatterers. The correct position

[18] Manufactured by the Radiation Instrument Development Laboratory, Chicago, Illinois.

for the source was indicated by a minimum in the counting rate. The hundred-channel analyzer permitted determinations of the correct position to about $\pm\frac{1}{32}$ in.

Our primary aim was to determine Z^n dependence of the coherent scattering cross section for different momentum transfers. Since in the small-angle region, direct experimental separation of the Compton (inelastically) scattered gamma rays from the elastically scattered ones is not possible with present techniques, it is necessary to estimate the Compton scattered intensity at each angle. In the previous experiments, Compton scattering cross sections were calculated and then subtracted from the measured cross sections to obtain the Rayleigh cross sections. This method involves a determination of the absolute values of the cross sections and is consequently subject to error. Therefore, we decided to estimate empirically the relative Compton scattered intensity in each case. Since the coherent scattering cross section decreases rapidly as atomic number decreases and since the atomic number of carbon is only six, the entire scattering cross section $d\sigma/d\Omega_C$ of carbon was taken as a measure of its Compton scattering cross section. By multiplying the previous result by Z_{rel}, where Z_{rel} is equal to $Z/6$, we obtained an empirical measure of the Compton cross section for any other scattering atom. We subtracted the latter from the measured cross section in the case of each scatterer in order to determine the relative coherent scattering cross section, $(d\sigma/d\Omega - Z_{rel}d\sigma/d\Omega_C)_{rel}$.

Typical data obtained at 2.43° are presented in Fig. 2. It shows that the background was quite low in our experiment. The number of scattering atoms was determined in each case in a straightforward manner from an accurate measurement of the mass of the ring. Similar observations were made at 4.68°. During the course of one run at a given angle, a minimum of ten independent counts, each of 5-min duration, were taken for each element. Two or three independent complete runs were made at each scattering angle. The subtraction method, discussed in the previous paragraph, demands an accurate determination of the relative carbon cross section. So counts were taken with carbon scatterers for much longer periods than those used in the case of other scatterers. Corrections, which are described in Sec. III, were applied to the raw data. By dividing the corrected number of counts by the number of scatterers, we determined the relative cross sections of the various elements.

Throughout the course of these measurements, in order to ascertain possible drifts in electronics, a test Co^{60} source of about 1 mC strength was used in a fixed position in front of the conical opening close to the detector. The opening in the lead shielding surrounding the strong Co^{60} source was plugged during the course of these checks. Corrections for small drifts in pulse height were applied in the manner to be described in Sec. III.

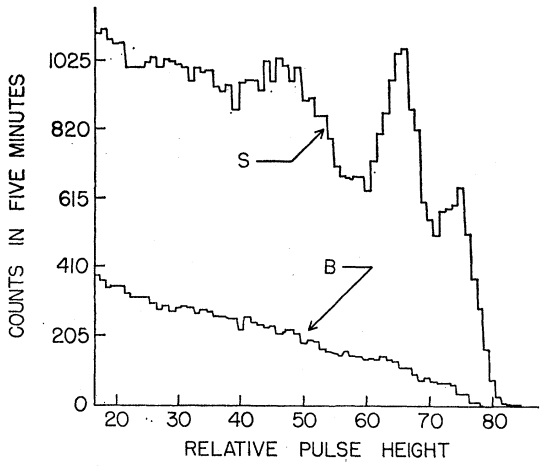

FIG. 2. Typical pulse-height spectra obtained in short test runs. B is a pulse-height spectrum of background. S is a pulse-height spectrum obtained with the lead scatterer for a scattering angle of 2.43°. The peaks in S around channels 65 and 73 represent the 1.17- and 1.33-Mev photopeaks, respectively.

III. CORRECTIONS

In order to correct for slight drifts in pulse height, a lead scatterer was used at regular intervals for normalization purposes and counts were taken. During a given run, one of these readings was taken as the standard lead count. A reading for any other element was multiplied by the ratio of the standard lead count to the average of the two lead counts adjacent to the particular reading under consideration. The success of the normalization procedure followed from the fact that the lead counts could be taken to 1% statistical precision in a period of 10–15 min. The normalization ratio was in the range 0.98 to 1.02 for the 1.17-Mev gamma-ray counts and in the range 0.84 to 1.04 for the 1.33-Mev gamma-ray counts. In several cases normalization was not necessary.

Slight shifts in pulse height caused by changes in pulse rate were estimated by interpolation between data taken in the direct beam for widely different pulse rates. These shifts have a negligible effect on the determinations of n at 1.17 Mev and increase the n value for 1.33 Mev by a possible maximum of 3%. No corrections for this effect were applied to the data.

The observed counts had to be corrected for attenuation within the scatterers. Theoretical absorption coefficients, obtained by interpolation from the tables of Davisson and Evans,[19] were used for calculating the attenuations. As a check, the absorption coefficients were also determined in the case of carbon, copper, zirconium, and lead with the help of rings of different thicknesses parallel to the direct beam absorber axis. Agreement between the experimental and the theoretical values was found to be about 10%.

Scattering of gamma rays from the molecules of air in the path between the source and the scatterer

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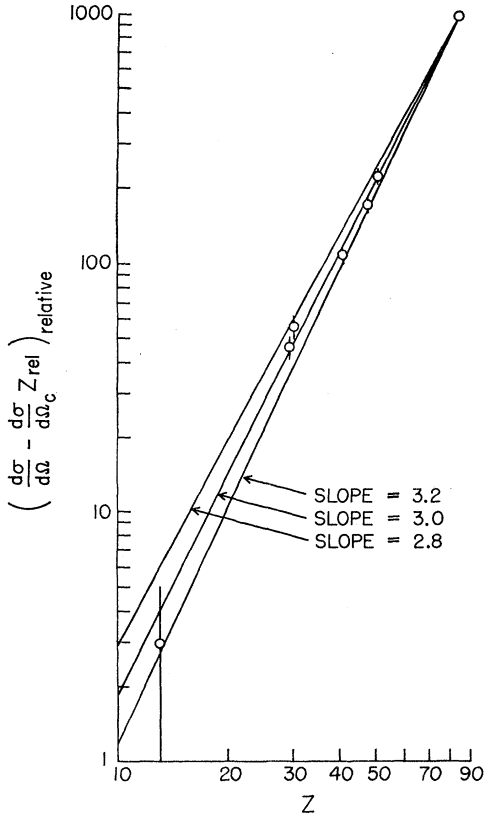

FIG. 3. Variation of the coherent scattering cross section $(d\sigma/d\Omega - Z_{rel}d\sigma/d\Omega_C)_{rel}$ with atomic number Z. Angle of scattering $=2.43°$. Energy of the gamma ray $=1.17$ Mev. The middle solid line in each of Figs. 3 to 6 represents the best visual fit to the data. The other two represent extreme fits.

contributed to the background. Attenuation of such gamma rays within the ring scatterer changes the background. Therefore, the background is different in the two cases of with and without scatterer. A correction of $+6\%$ to lead scattering counts at the smallest angle has been estimated. The percentage correction decreases rapidly with increasing scattering angle and increases slowly with decreasing atomic number. The resultant correction for the index n turns out to be about $+1\%$ at 2.43° and negligible at 4.68°.

The contribution of bremsstrahlung from electrons generated within the scatterer has been estimated to be negligible in the case of the pulse-height channels employed.

Since the angular distribution of the differential cross section exhibits a sharp rise at very small angles and since the rate of rise increases with increasing atomic number Z, the values of the index n are subject to a -12% correction at 2.43° and a negligible correction at 4.68°.

In the determination of relative coherent scattering cross sections, the Compton scattering cross section was estimated empirically. The estimation depended upon the exact proportionality of the Compton scattering cross section to the atomic number Z. From the theoretical standpoint, such a proportionality holds only if the electrons responsible for Compton scattering are assumed to be entirely free. Since the momentum transfer to an electron in the scattering of a photon through an angle θ is proportional to $\sin(\theta/2)$, the last assumption does not really hold in the case of scattering through small angles particularly from heavy elements, whose inner electrons have large binding energies of the order of 70 kev. Various attempts[20–22] have been made to include the effects of binding on the Compton scattering cross sections. The earliest treatment is valid only for gamma-ray energies less than the self-energy of the electron. The last calculation was made for a few elements and only for angles greater than about 60°. In the case of all the three calculations, the results cannot be expressed in a closed form applicable to all elements. So no correction on this account has been applied to the values of n.

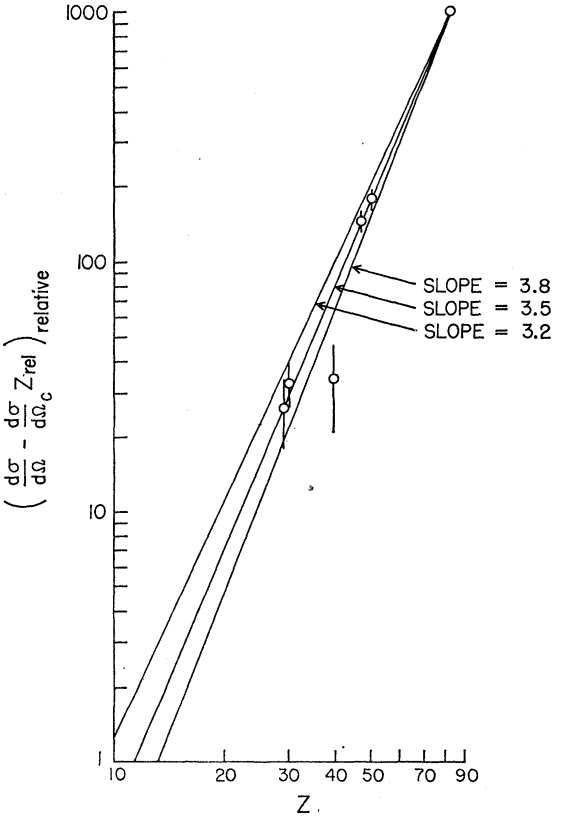

FIG. 4. Variation of the coherent scattering cross section $(d\sigma/d\Omega - Z_{rel}d\sigma/d\Omega_C)_{rel}$ with atomic number Z. Angle of scattering $=2.43°$. Energy of the gamma ray $=1.33$ Mev.

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IV. RESULTS AND DISCUSSION

Typical sets of data at 2.43° and 4.68° scattering angles are presented in Figs. 3 to 6. As pointed out in Sec. II, $(d\sigma/d\Omega - Z_{\text{rel}} d\sigma/d\Omega_C)_{\text{rel}}$ represents an experimental measurement of the coherent part of the differential scattering cross section. Since the coherent scattering cross section decreases rapidly as the scattering angle increases, the data at 4.68° are not as precise as those at 2.43°. In Table I, values of the index n and the associated corrections are listed for 1.17- and 1.33-Mev gamma rays at 2.43° and 4.68° scattering angles. Even though the calculations of Nelms and Oppenheim predict an increase of n with the momentum change in the scattering, the data do not reveal any such dependence within the limits of experimental accuracy. Therefore, an average of all the corrected values of n was taken. It was found to be 3.07. The standard deviation of the average n was computed as ± 0.10. If, instead, the error of each individual measurement of n is taken as the difference between the values of n determined from the extreme and best fits to the data, the error of the average n turns out to be ± 0.18. So we obtain agreement with the value 3 predicted by Franz for Rayleigh scattering. The discussion of the next paragraph suggests that the agreement is likely to be fortuitous. The calculations of Nelms and Oppenheim give a Rayleigh scattering cross section that varies approximately as, although not exactly as, $Z^{2.8}$ for the momentum transfers involved in our experiment. Our average value for n is also in agreement with that for 0.411 Mev at 77° scattering angle.[5] But it is much lower than the value 4 for 0.66 and 1.12 Mev[23] at 105°. The latter might be attributable to the very large momentum transfers involved in that experiment and to an overestimate of the Compton scattered intensity.

TABLE I. Values of the index n for different momentum transfers. The coherent part of the differential scattering cross section is found to vary as Z^n, where Z is the atomic number of the scatterer. The total correction, as discussed in Sec. III, for air scattering and for rapid angular variation of the cross section is listed in the fourth column.

Energy (Mev)	Angle	Average value of n	Correction	Corrected value of n
1.17	2.43°	3.03	−11%	2.70
1.17	4.68°	3.55	+ 1%	3.59
1.33	2.43°	3.28	−11%	2.92
1.33	4.68°	3.04	+ 1%	3.07

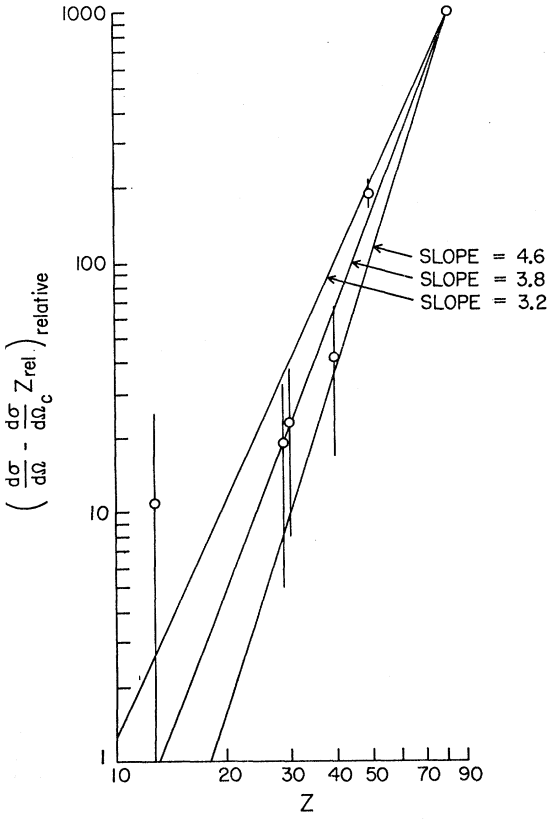

FIG. 5. Variation of the coherent scattering cross section $(d\sigma/d\Omega - Z_{\text{rel}} d\sigma/d\Omega_C)_{\text{rel}}$ with atomic number Z. Angle of scattering $= 4.68°$. Energy of the gamma ray $= 1.17$ Mev.

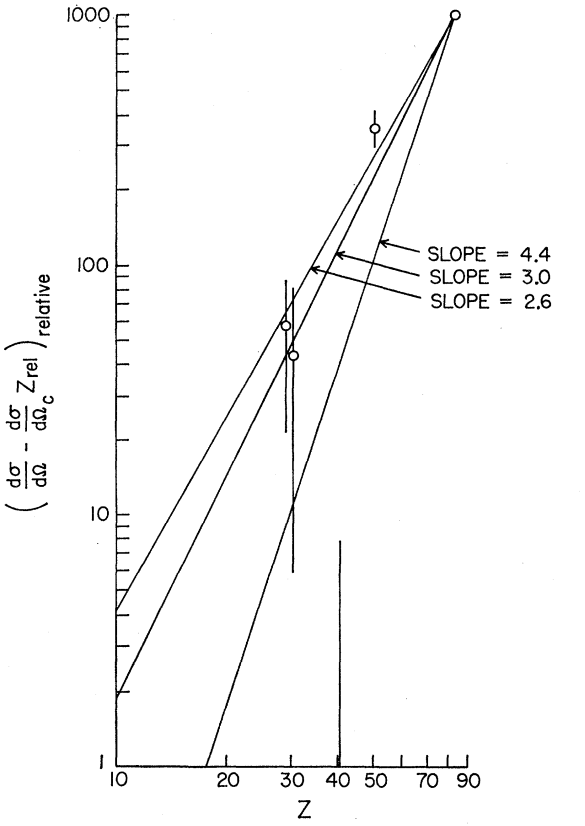

FIG. 6. Variation of the coherent scattering cross section $(d\sigma/d\Omega - Z_{\text{rel}} d\sigma/d\Omega_C)_{\text{rel}}$ with atomic number Z. Angle of scattering $= 4.68°$. Energy of the gamma ray $= 1.33$ Mev.

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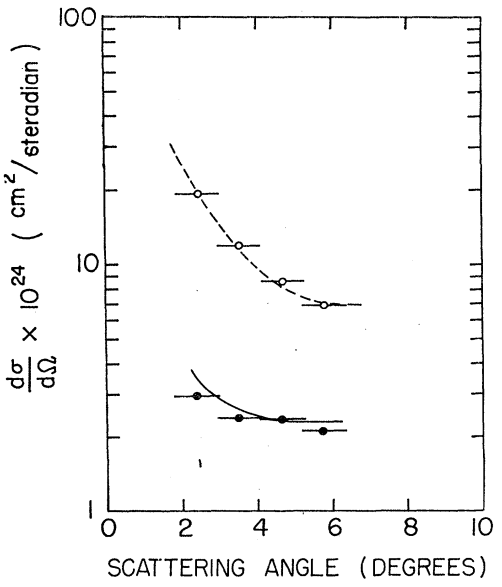

FIG. 7. Angular distributions of the total scattering cross sections of lead and copper for 1.17-Mev gamma rays. The top curve for lead is obtained by a smooth interpolation between the data of Storruste and Tjom. The lower curve for copper is obtained on the basis of nonrelativistic theoretical calculations of the type of Nelms and Oppenheim.

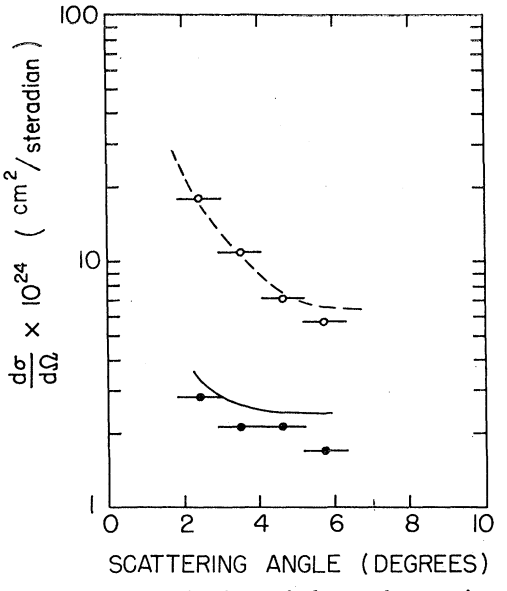

FIG. 8. Angular distributions of the total scattering cross sections of lead and copper for 1.33-Mev gamma rays. The top curve for lead is obtained by a smooth interpolation between the data of Storruste and Tjom. The lower curve for copper is obtained on the basis of nonrelativistic theoretical calculations of the type of Nelms and Oppenheim.

The augular distributions of the total (coherent and Compton) cross sections are shown in Figs. 7 and 8. Absolute values of the cross sections were not determined in our experiments. So all our data were normalized to the value of the differential cross section of lead for 1.17-Mev gamma rays at 2.43°, obtained by a smooth interpolation between the data of Storruste and Tjom. The distributions for lead are in excellent agreement with the earlier ones. In general, calculations based on the Franz formula for Rayleigh scattering give much too large total cross sections. However, the experimental cross sections in the case of copper are seen not to deviate much from the values computed on the basis of calculations of the type of Nelms and Oppenheim. But in the case of lead, the latter calculations give a cross section at 2.43°, which is about 50% higher than the corresponding experimental value. The scattering from L-shell electrons,[24] Delbrück scattering and relativistic corrections to the form factor amplitudes are some of the likely sources of this discrepancy. Since these effects have not yet been calculated adequately, it is diffcult to give an unambiguous interpretation of the results. In particular, definite conclusions cannot be reached regarding the presence of Delbrück scattering at these energies.

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It is a pleasure to acknowledge the kind assistance of Professor H. L. Schulz of the Electron Accelerator Laboratory at Yale University in arranging a loan of the Co^{60} source, and the help of W. Sochaki and W. Miglus in the construction of apparatus. This work was facilitated by the fact that Professor F. I. Boley kindly loaned to us many electronic instruments.

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