

# Intrinsic Excited States in $\text{Hf}^{178}$ Populated by the Allowed Decay of 9.3-min $\text{Ta}^{178}$

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The levels in  $\text{Hf}^{178}$  populated by the allowed positron and electron capture decay of 9.3-min  $\text{Ta}^{178}$  have been investigated with the Copenhagen six-gap spectrometer. Conversion-electron spectrum and beta-gamma coincidence measurements have established the spin and parities of levels in  $\text{Hf}^{178}$  with energies (in keV) 93(2+), 307(4+), 1197(0+), 1277((2+)), 1430((1+)), 1440(0+), and 1483(2+), and possibly a level at 1550. (Double parentheses indicate that the spin is not definitely established.) Intrinsic configuration assignments for the excited states are discussed in terms of recent theoretical developments. The half-life of the 93-keV level has been measured and found to be  $(1.25 \pm 0.08) \times 10^{-9}$  sec. The  $\beta^+/K$  capture ratio measured for the allowed decay to ground is within experimental error of theory. Reduced  $E0/E2$  transition probability ratios for the transitions depopulating the 1197- and 1440-keV 0+ levels have been calculated from the observed  $K$ -conversion line intensities of the  $E0$  and  $E2$  transitions depopulating the levels.

## I. INTRODUCTION

THE large beta-decay<sup>1,2</sup> rate ( $\log ft = 4.6$ ) of the 9.3-min  $\text{Ta}^{178}$  isomer to the ground-state band of  $\text{Hf}^{178}$  is unique among the beta-decay rates presently known for ground-state to ground-state decays of deformed odd-odd nuclei. Recently it has been found possible to assign a unique configuration to this isomer which clearly explains the decay rate.<sup>3</sup> The unambiguous configuration assignment for this isomer makes it an excellent subject for further study because, if  $\text{Ta}^{178}$  decay populates excited states in  $\text{Hf}^{178}$ , it may be possible through a study of the decay rates to these states to assign definite configurations to them. Assignments of intrinsic configurations to excited levels in deformed even-even nuclei have been made on only a limited number of cases to date<sup>3</sup> and, consequently, much remains to be done in order to achieve a clearer understanding of the excited intrinsic levels and the particle coupling in these nuclei. It therefore seemed worthwhile to study the decay of this isomer in more detail than has been done earlier.

## II. PREVIOUS EXPERIMENTAL INVESTIGATIONS

Wilkinson<sup>4</sup> first characterized the 9.3-min  $\text{Ta}^{178}$  isomer, reporting that it decayed by positron emission ( $\beta^+_{\text{max}} = 1.06$  MeV) and electron capture. He also reported electrons of 0.08 MeV and a gamma ray of 1.5 MeV. Bisi, Terrani, and Zappa<sup>5</sup> later reported that there was no positron branch (upper limit 1%) and in addition that there were no gamma rays at 1.5 MeV (upper limit 1.5%). Felber<sup>1</sup> obtained results essentially in agreement with Wilkinson's, and in disagreement with those of Bisi *et al.*, and reported a value  $\beta^+_{\text{max}} = 880$

keV. He determined that at least two excited states of  $\text{Hf}^{178}$  were populated, one at  $93.17 \pm 0.09$  keV, the other at either 1257 or 1350 keV. From the energy of the  $\beta^+$  group he concluded that the  $\text{Ta}^{178}$  decay to the  $\text{Hf}^{178}$  ground state had a  $\log ft = 4.7$ , on which basis he assigned spin 1, positive parity, to the isomer. He also reported a  $\beta^+/K$  capture ratio of 0.02. Carver and Turchinets<sup>2</sup> produced 9.3-min  $\text{Ta}^{178}$  in a study of  $(\gamma, xn)$  reactions on  $\text{Ta}^{181}$ , and definitely established that there was a  $\beta^+$  group belonging to  $\text{Ta}^{178}$  by measuring the decay rate of the annihilation radiation. They measured a beta spectrum with an end-point energy of  $1.65 \pm 0.05$  MeV, which they assigned as the  $\text{Ta}^{178}$  positron group, in disagreement with the results of Felber and Wilkinson. They also reported 4 gamma rays, at 93, 1160, 1390, and 1480 keV, the latter three of which were assigned as depopulating a 2+ level at 1480 keV. They also assigned the isomer as 1+ on the basis of the values of  $\log ft$ .

In addition to these studies of the decay of the 9.3-min  $\text{Ta}^{178}$ , studies of the decay of the 2.1-hr  $\text{Ta}^{178}$  isomer<sup>2,6</sup> have established levels in  $\text{Hf}^{178}$  at 1480(8, 9-), 1148.5(8-), 1059.7(8+), 632.7(6+), 306.87(4+), and 93.17(2+) keV. The 2+ level has also been identified in Coulomb excitation experiments<sup>7-9</sup> on  $\text{Hf}^{178}$ , the highest resolution study<sup>9</sup> yielding  $93.14 \pm 0.02$  for the energy of the level.

## III. EXPERIMENTAL

### A. Apparatus, Calibration, and Techniques

Gamma-ray spectrometers consisting of 3-in. diam by 3-in. long NaI(Tl) scintillation crystals attached

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<sup>1</sup> F. F. Felber, Jr. (Master's thesis) University of California Radiation Laboratory Report UCRL-3618, January, 1956.

<sup>2</sup> J. H. Carver and W. Turchinets, Proc. Phys. Soc. (London) **71**, 618 (1958).

<sup>3</sup> C. J. Gallagher, Nuclear Phys. **16**, 215 (1960).

<sup>4</sup> G. Wilkinson, Phys. Rev. **80**, 495 (1950).

<sup>5</sup> A. Bisi, S. Terrani, and L. Zappa, Nuovo cimento **3**, 661 (1956).

<sup>6</sup> F. F. Felber, Jr., F. S. Stephens, and F. Asaro, J. Inorg. Nuclear Chem. **7**, 153 (1958).

<sup>7</sup> C. McClelland, H. Mark, and C. Goodman, Phys. Rev. **97**, 1191 (1955); P. H. Stelson and F. K. McGowan, *ibid.* **99**, 112 (1955); N. P. Heydenburg and G. M. Temmer, *ibid.* **100**, 150 (1955).

<sup>8</sup> T. Huus, J. H. Bjerregaard, and B. Elbek, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **30**, No. 17 (1956).

<sup>9</sup> E. L. Chupp, J. W. M. DuMond, F. J. Gordon, R. C. Jopson, and H. Mark, Phys. Rev. **112**, 518 (1958).

to EMI 9531 phototubes, with 100<sup>-10</sup> and 256<sup>-11</sup> channel pulse-height analyzers were used to measure the gamma-ray spectrum. Gamma-gamma coincidence measurements were made with a Bell-type fast-slow triple coincidence circuit with a 10<sup>-7</sup>-sec resolving time. The gamma-ray energy spectrum was calibrated using conventional radioactive energy standards, such as  $\text{Co}^{60}$  and  $\text{Na}^{22}$ .

The electron spectrum was measured with the Copenhagen six-gap "orange" spectrometer.<sup>12,13</sup> The electron detector in the spectrometer is a cylindrical anthracene crystal of 3 cm diam. The effective length of the crystal can be varied as a function of the resolution required, because the electrons strike the crystal perpendicular to its axis of highest symmetry. Beta-gamma coincidence measurements were also made with this instrument, using a coincidence circuit identical to that used in the gamma-gamma coincidence measurements. The circuit resolving time was  $2\tau = 1.4 \times 10^{-7}$  sec. The absolute energy calibration of the orange spectrometer is known as a function of exciting current to  $\approx 1\%$  assuming the spectrometer magnet has been demagnetized. The relative energies of close-lying transitions can be determined with considerably higher resolution, however.

Gamma-ray intensity measurements were made in the well-calibrated geometry of the beta-gamma coincidence setup. The efficiency curve for the low-energy region ( $\leq 100$  kev) was checked by comparison between calculated and true coincidence sum peaks.

Pulse-height analyses of the energies of the positrons incident on the electron detector were made during the measurement of the positron spectrum to ensure that all positrons had been accounted for correctly. As the positron spectrum is measured with the field opposite to that used in the measurement of the electron spectrum, degraded electrons from the high-energy transitions did not influence our measurement.

The half-life of the 93-kev level was measured with an improved version of the Bell-type fast coincidence circuit due to Bell.<sup>14</sup>

## B. Source Preparation and Chemical Procedure

The 9.3-min half-life of  $\text{Ta}^{178}$  is too short to have allowed us to carry out the necessary experiments with reasonable statistics using directly-produced  $\text{Ta}^{178}$ . Fortunately  $\text{Ta}^{178}$  is populated by readily-produced 21-day  $\text{W}^{178}$ , and we therefore decided to use an equilibrium  $\text{W}^{178}$ - $\text{Ta}^{178}$  mixture. Study of the equilibrium system is simplified by the fact that the electron capture

decay of  $\text{W}^{178}$  populates only the 9.3-min  $1+$  level of  $\text{Ta}^{178}$ . A source of  $\text{W}^{178}$  in equilibrium with 9.3-min  $\text{Ta}^{178}$  will thus display a pure  $\text{Ta}^{178}$  spectrum when proper corrections have been made for Ta x rays and Auger electrons belonging to the  $\text{W}^{178}$  spectrum.

Tantalum metal powder (0.45 g) of 99.9% purity (from L. Light and Company, England) was irradiated in the synchro-cyclotron of the Gustaf Werner Institute for Nuclear Chemistry, Uppsala, Sweden, with 50-Mev protons for 5 hr. This bombarding condition produced  $\text{W}^{178}$  by a  $\text{Ta}^{181}(p,4n)\text{W}^{178}$  reaction.

The target was dissolved in a mixture of concentrated hydrofluoric acid to which a small amount of concentrated nitric acid had been added. Following evaporation to dryness and redissolving in 10M HF, the bulk of the tantalum was removed from the solution by repeated extractions with di-isopropylketone, which had been equilibrated with 10M HF. After evaporation of the aqueous phase to near dryness, the wolfram activity was taken up in a few drops of water and absorbed on top of a Dowex 1 $\times$ 10 anion exchange column contained in a polyethylene tube with dimensions 1 $\times$ 10 mm. Elution of carrier-free wolfram was performed with a solution 1M in HCl and 0.3M in HF. Sources for the  $\beta$  spectrometer were made by collecting the drops containing the activity on a VYNS foil, 50  $\mu\text{g}/\text{cm}^2$  thick. The total source activity was approximately 10  $\mu\text{C}$ .

The purity of the chemical procedure was checked by observing the decay of the separated tantalum and wolfram fractions. The radioactive decay of the tantalum fraction was followed using a windowless flow-type proportional counter operated on the  $\beta$  plateau. Four components, with half-lives 9.2 min,  $\sim 9$  hr, 57 hr, and 200 days, were detected. We identify these activities as  $\text{Ta}^{178}$  (9.3 min),  $\text{Ta}^{176}$ , and/or  $\text{Ta}^{180}$  (8 hr),  $\text{Ta}^{177}$  (53 hr), and  $\text{Ta}^{179}$  ( $\sim 600$  days), where the previously reported half-lives are given in parentheses.<sup>15</sup> Activity measurements on a sample of the wolfram eluate were carried out with the flow counter as well as with a scintillation counter set on the K x rays. Growth of a 9-min half-life going over into a 21-day activity clearly indicates that the activity is  $\text{W}^{178}$ .<sup>1</sup>

## C. Experimental Measurements and Results

We have measured the energies and intensities of 26 electron lines in the electron spectrometer. These are interpreted to establish the energies of 15 transitions following the decay of  $\text{Ta}^{178}$ . The energies of the transitions are reported in the second column of Table I. The intensities of the electron lines, normalized so that the intensity of the composite L line of the 93-kev transition is 100, are given in columns four and five. These intensities are estimated to have errors of  $\pm 20\%$ .

<sup>15</sup> D. Strominger, J. M. Hollander, and G. T. Seaborg, *Revs. Modern Phys.* **30**, 585 (1958).

<sup>10</sup> Type RT 5965, manufactured by Philips, Balham, England.  
<sup>11</sup> Model 20611, manufactured by Radiation Counter Laboratories, Inc., Skokie, Illinois.

<sup>12</sup> O. Kofoed-Hansen, J. Lindhard, and O. B. Nielsen, *Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd.* **25**, No. 16 (1950).  
O. B. Nielsen and O. Kofoed-Hansen, *ibid.* **29**, No. 6 (1955).

<sup>13</sup> K. M. Bisgaard (to be published).

<sup>14</sup> R. E. Bell and M. H. Jørgensen, *Can. J. Phys.* **38**, 652 (1960).

TABLE I. Transitions in  $\text{Hf}^{178}$  following 9.3-min  $\text{Ta}^{178}$  decay.

Assignment of transition	Energy of transition (keV)	Photon intensity	Conversion electron intensity		Normalized conversion coefficient		Multipolarity	
			K	L	$\alpha_K$	$\alpha_L$	Exp. <sup>a</sup>	Assignment in decay scheme
BA	93 $\pm$ 2	100	11	100	>0.65 <sup>b</sup>	(2.70)	E2	E2
	203 $\pm$ 4	2.8	0.022	0.060	>0.018	$\geq 0.058^c$	E2	E2
CB	214 $\pm$ 4		0.089		$\geq 0.089$			
EC	970 $\pm$ 10	0.7 <sup>d</sup>	0.0012		$\leq 0.004$		E2	E2
DB	1105 $\pm$ 11	11	0.0085		0.0026		E2	E2
HC	1176 $\pm$ 12	3.8	0.0087	0.0015	$\geq 0.0065$	0.0010	M2, +	E0+E2
EB	1180 $\pm$ 12							
DA	1197 $\pm$ 12							
IC	1260 $\pm$ 25							
FB	1335 $\pm$ 13	46	0.012	<0.0047 <sup>e</sup>	$\geq 0.00073$		E1, +	M1
GB	1345 $\pm$ 13		0.011		$\geq 0.00065$		E1, +	E2
HB	1390 $\pm$ 14		0.018 <sup>h</sup>	0.004	$\geq 0.0011$	0.0024	E1(+M2), +	E2+(+E0)
FA	1430 $\pm$ 14		0.013	0.004 <sup>i</sup>	$\geq 0.0038$	0.00073	M1, E1+M2, +	M1
GA	1440 $\pm$ 14	9	0.058	0.011	$\geq 0.0175$	0.0033	M4, +	E0
IB	1460 $\pm$ 15		0.0034		$\geq 0.0010$		E2, +	E2
HA	1483 $\pm$ 15		<0.004 <sup>i</sup>		0.0013		E2, +	E2
	511	10						
	K x rays	1270	26 <sup>j</sup>		0.037 <sup>k</sup>			

<sup>a</sup> The + sign in this column indicates that the transition can have a higher multipole order than that indicated.

<sup>b</sup> The normalized value is low because of absorption in the source. A direct measurement yields  $\alpha_K(93) = 1.03 \pm 0.15$ .

<sup>c</sup>  $K/L(\text{exp}) = K/L(\text{theor } E2) = 1.8$ .

<sup>d</sup> Calculation of the intensity of these photons from the number of 970-keV photons coincident with the K line of the 214-keV transition yields a value of 2.1, where we have assumed that the observed 214-keV photon peak results from a single transition. The value of 0.7 is calculated from a resolution of the singles spectrum. The discrepancy probably indicates that the 214-keV photon peak is complex.

<sup>e</sup> Calculated from intensity observed in coincidence with K 214. Value may be high. (See reference d).

<sup>f</sup> K 1250, L 1190 superimposed. L 1190/(M 1190+N 1190+K 1250)=2.

<sup>g</sup> L 1335, K 1390 only partially resolved; hence the upper limit on the intensity of the L 1335.

<sup>h</sup> L 1345, K 1390 superimposed.

<sup>i</sup> L 1430, K 1483 superimposed.

<sup>j</sup> Sum of unresolved KLL and KLM groups.

<sup>k</sup> This Auger coefficient is lower than the accepted values, indicating probably that there has been some source absorption.

for all but the weakest lines, for which the limits are larger. A portion of the electron spectrum in the high energy region is shown in Fig. 1.

The complex gamma-ray spectrum was only partially resolved. As a consequence, intensities of the partially resolved groups of gamma rays are given in column 3 of Table I. The spectrum observed is shown in Fig. 2.

Normalized conversion coefficients are reported in columns six and seven, and are, as a consequence of the unresolved gamma-ray spectrum, lower limits on the actual conversion coefficient. The conversion coefficients reported have been calculated assuming that the total intensity of the corresponding gamma-ray peak results from the transition in question alone.

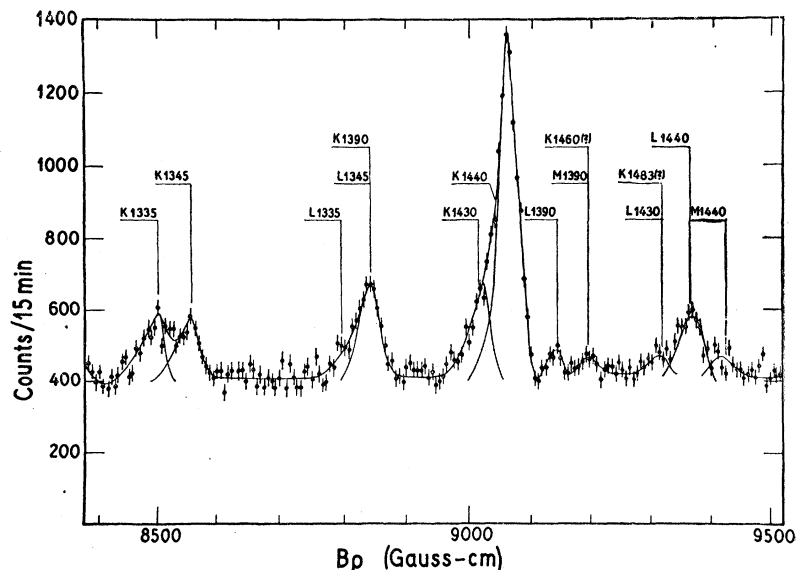


FIG. 1. The conversion electron spectrum observed at 0.4% momentum resolution between 1260 and 1460 keV.

TABLE II. Comparison of experimental and theoretical conversion coefficients. The theoretical results are those of Sliv and Band.

Energy of transition (kev)	Theoretical $K$ conversion coefficient					Experimental $K$ conversion coefficient	Assignment in decay scheme
	$\alpha_1$	$\alpha_2$	$\beta_1$	$\beta_2$	$\beta_3$		
93	0.35	1.10	4.2	33	930	1.03	$E2$
203						$\geq 0.018$	
214	0.042	0.14	0.40	1.8	76	$\geq 0.089$	$E2$
970	0.0015	0.0035	0.0075	0.018	0.125	$\leq 0.004$	$E2$
1105	0.0012	0.0027	0.0054	0.013	0.080	0.0026	$E2$
1176	0.0011	0.0024	0.0046	0.011	0.065	$\geq 0.0065$	$E2$
1180	0.0011	0.0024	0.0046	0.011	0.065		$E0 + E2$
1197	0.0010	0.0023	0.0044	0.011	0.061	$\geq 0.017$	$E0$
1260	0.00092	0.0022	0.0040	0.0098	0.053		$E2$
1335	0.00082	0.0019	0.0034	0.0082	0.042	$\geq 0.00073$	$M1$
1345	0.00082	0.0019	0.0034	0.0082	0.047	$\geq 0.00065$	$E2$
1390	0.00076	0.0018	0.0031	0.0074	0.037	$\geq 0.0011$	$E2 (+E0)$
1430	0.00073	0.0017	0.0029	0.0067	0.033	$\geq 0.0038$	$M1$
1440	0.00073	0.0017	0.0029	0.0067	0.033	$\geq 0.0175$	$E0$
1460	0.00017	0.0017	0.0028	0.0065	0.032	$\geq 0.0010$	$E2$
1483	0.00069	0.0016	0.0027	0.0063	0.030	0.0013	$E2$

The normalization used is that  $\alpha_L(93) = \alpha_L(\text{theor})$ , where  $\alpha_L(\text{theor})$  is the sum of the  $L_I$ ,  $L_{II}$ , and  $L_{III}$  conversion coefficients for a pure  $E2$  transition, as calculated by Sliv and Band.<sup>16</sup> This normalization has been checked by measuring the  $K$ -conversion coefficient of the 93-kev transition directly by a measurement of the ratio of  $K$  x rays to 93-kev photons coincident with positrons.  $\alpha_K(93)$  determined is  $1.03 \pm 0.15$ , in excellent agreement with the theoretical  $\alpha_K = 1.05 \pm 0.05$  for a pure  $E2$  transition. The value  $\alpha_K(93) = 0.65 \pm 0.50$  determined by normalization of electron and photon intensities is lower than the directly measured value, but this disagreement is not serious, as it is obvious that some of the 28-kev  $K$ -conversion electrons will be absorbed in the rather thick source. Similarly, the  $K$ -Auger electrons might be expected to suffer some absorption, and the value of the  $K$ -Auger coefficient observed, 0.037, is indeed smaller than the value of 0.045 deduced from empirical curves.<sup>17</sup> However, the percentage absorption is less in the latter case, as expected.

A comparison between these experimental limits on the conversion coefficients and the theoretical values of Sliv and Band for  $E1$ ,  $E2$ ,  $M1$ ,  $M2$ , and  $M5$  transitions is made in Table II. The comparison definitely restricts the possible multipolarity assignments for most of the transitions. In some cases a definite value has been determined. In two cases, the 1197- and 1440-kev transitions, the conversion coefficient is consistent only with the interpretation that these transitions are either  $E0$  or have a high magnetic multipole. In column 8 of Table I is given the lowest multipole order assignable to each transition on the

basis of the observed conversion coefficient. The interpretation assigned in the decay scheme is shown in column 9.

The beta-gamma coincidence measurements were gated on the internal conversion electron. The gamma rays coincident with the conversion electrons of the low-energy transitions are shown in Table III. The energy and intensity of the 1260-kev transition were established on the basis of these measurements alone. Coincidence relationships between the conversion lines of the high-energy transitions and the 55- and 93-kev photons are shown qualitatively in Table IV. These results were decisive in the assignment of transitions in the decay scheme.

The gamma-gamma coincidence measurements were used only as a gross check on the beta-gamma coincidence results. As the results of the two sets of

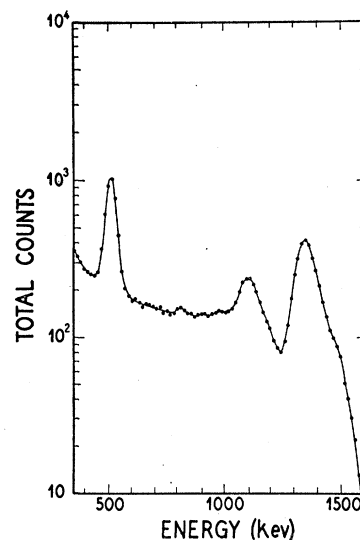


Fig. 2. The  $\text{W}^{178}\text{-Ta}^{178}$  equilibrium-mixture gamma-ray spectrum taken with a 3 in.  $\times$  3 in.  $\text{NaI}(\text{Tl})$  crystal.

<sup>16</sup> L. A. Sliv and I. M. Band, Leningrad Physico-Technical Institute Reports, 1956 and 1958 [translation: Reports ICC 57 K 1, and ICC 58 L 1, issued by Physics Department, University of Illinois, Urbana, Illinois (unpublished)].

<sup>17</sup> *Nuclear Spectroscopy Tables*, prepared by A. H. Wapstra, G. J. Nijgh, and R. V. Van Lieshout (Interscience Publishing Company, Inc., New York, 1959).

TABLE III. Gamma rays coincident with low-energy conversion electron lines.

Conversion-line gate	Energy of coincident radiations (kev) <sup>a</sup>
<i>L</i> 93	55 ( <i>K</i> x rays), 93, 214(0.62), 511(0.43 <sup>b</sup> ), $\approx$ 970(0.45), $\approx$ 1100(2.0), $\approx$ 1180(0.76), $\approx$ 1350(7.5)
<i>K</i> 203	$\approx$ 1000(22), $\approx$ 1270(27), $\approx$ 1340(65)
<i>K</i> 214	$\approx$ 970(60), $\approx$ 1180(20), $\approx$ 1250(30)

<sup>a</sup> Values in parentheses indicate percentage in coincidence with the conversion line.<sup>b</sup> A relative intensity deduced from this number would be incorrect because all positrons do not annihilate at the source.

measurements were consistent, we do not report the results of the less sensitive gamma-gamma measurements here.

Bertelsen, Borggreen, and Nathan<sup>18</sup> have measured the angular correlation between the 93- and 1350-kev photon peak. As the 1350-kev peak is known to be complex from the internal conversion data, they divided the observed peak into upper and low energy parts and determined the correlation coefficient of the two parts. Their analysis indicates that the lower part of the peak definitely contains a 0-2-0 component and that there is also a  $n$ -2-0 component, where  $n=1, 2$ , or 3. The correlation observed for the upper part is consistent with a 2-2-0 cascade. They did not attempt to determine the angular correlation of the 1200-93 kev cascade because of the large uncertainty introduced into the angular correlation by the underlying Compton peak of the more intense 1350-kev peak.

The positron spectrum was measured from 200 kev to above its end point. A Fermi-Kurie analysis of the spectrum showed the presence of two groups, in agreement with the positron-photon coincidence measurements. The ratio of the intensity of the group populating the 93-kev level to that populating the ground state is 0.7. The limits of error on this ratio are difficult to evaluate because the resolved portion of the upper group is only 90 kev long, and because source scattering introduces appreciable uncertainties in the low-energy

region. The positron end point energies are  $890 \pm 10$  and  $800 \pm 15$ , the energy difference being equal within experimental error to the energy of the 93-kev first excited state.

The positron to *K*-capture ratio for the ground state band was determined. The *K* x ray intensity was corrected for the presence of the  $W^{178}$  parent by determining the ratio of *K* x rays to 93-kev photons in a  $W^{178}$ - $Ta^{178}$  equilibrium mixture, and comparing it to the same ratio determined from a sample of  $Ta^{178}$  prepared by a DIPK extraction from the  $W^{178}$  parent. The corrected *K* x ray intensity was then further corrected for the contribution to it from *K* conversion of the 93-kev transition, *K* capture to the excited states, and the *K*-Auger effect. In measuring the intensity of the 511-kev annihilation peak we took care to provide enough low-*Z* material around the source so that all positrons annihilated in the immediate vicinity of the source. The annihilation peak intensity was halved to account for the two quanta per annihilation event. The ratio of the 511 intensity to the corrected *K* x ray intensity then yields directly the  $\beta^+/K$  capture ratio to the ground state band,  $\beta^+/K$  capture =  $0.9 \pm 0.6\%$ .

The half-life of the 93-kev level was measured by observing coincidences between 93-kev photons and positrons and high energy gamma rays. The measured half-life is  $(1.25 \pm 0.08) \times 10^{-9}$  sec.

TABLE IV. Coincidences between high-energy conversion electron lines and *K* x rays and 93-kev photon.

Conversion-line gate	<i>K</i> x rays	93
<i>L</i> 93	x	...
<i>K</i> 214	x	x
<i>K</i> 1105	x	x
<i>K</i> 1176	x	x
<i>K</i> 1180	x	x
<i>K</i> 1197	x	...
<i>K</i> 1335	x	x
<i>K</i> 1345	x	x
<i>K</i> 1390	x	x
<i>K</i> 1435	x	...
<i>K</i> 1445	x	...
$\beta^+$	x	x <sup>a</sup>

<sup>a</sup> The ratio  $I_{K \text{ x ray}}/I_{\gamma 93}$  determines the *K*-conversion coefficient of the 93-kev transition.

<sup>18</sup> U. Bertelsen, J. Borggreen, and O. Nathan (private communication, July, 1960). We are indebted to them for permission to quote these results in advance of their publication.

#### IV. INTERPRETATION OF RESULTS

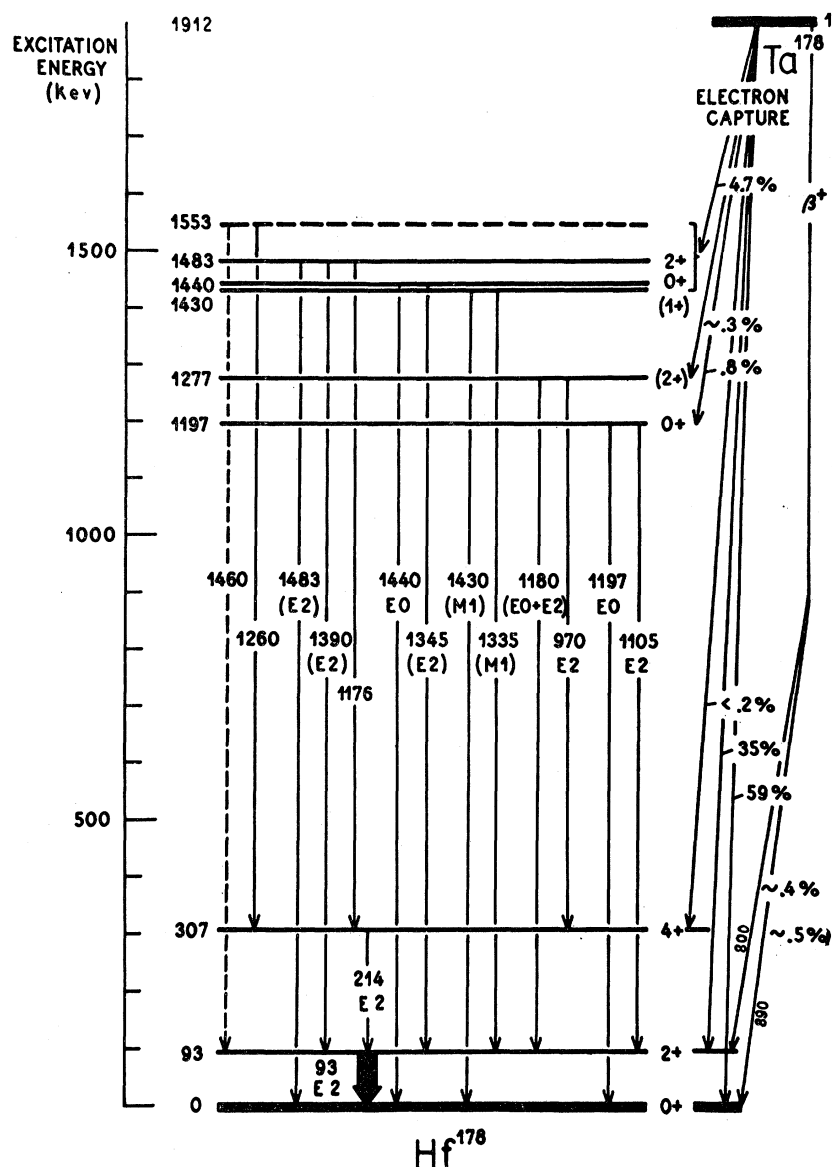
##### A. Decay Scheme

We are able to propose a consistent interpretation of the data obtained in the present study which we believe establishes seven excited levels in  $Hf^{178}$ , and possibly an eighth. This decay scheme is shown in Fig. 3. We discuss first the experimental measurements which lead to the level energies, spin, and parity assignments shown in the figure, and secondly we compare these results to those reported previously.

##### Present Results

The 2+ level at 93 kev is established by the coincidence observed between positrons and 93-kev photons, and the observed energy difference of  $\approx 90$  kev between the positron groups. In addition, the photon and conversion electron intensities of this transition are too intense to assign it elsewhere in the

FIG. 3. A partial decay scheme for the 9.3-min  $\text{Ta}^{178}$  isomer.



decay scheme. The  $2+$  character of the state is established by the measured  $K$ -conversion coefficient of the 93-keV transition, and the predominant  $L$ -subshell conversion.

The  $K$ -,  $L$ -, and  $M$ -shell conversion lines of the 214-keV transition have been observed. The photon is coincident with the  $L$  93-conversion electrons, and the  $K$ - and  $L$ -lines of the transition are 100% coincident with the sum of the intensities of 3 high-energy transitions. The measured  $K$ -conversion coefficient and  $K/L$  ratio for the transition are equal within experimental error to the theoretical values for a pure  $E2$  transition. It is therefore assigned to depopulate a level at 307 keV, which is probably the  $4+$  rotational state of the ground state band.

The assignment of the  $0+$  level at 1197 keV is based

on the probable  $E0$  multipolarity of the 1197-keV transition. The  $K$ -conversion coefficient of this transition is comparable only with the theoretical conversion coefficients of a high magnetic multipole or of an  $E0$ . When this is considered together with the  $E2$  multipolarity of the 1105-keV transition to the  $2+$  level at 93 keV, only a  $0+$  assignment for the level is consistent with all of the data.

The 1277-keV level is established by the observed coincidences between the  $L$  line of the 93-keV transition and the 1180-keV photons. The 214-970 coincidences support the assignment. The  $E2$  multipolarity of the 970-keV transition establishes positive parity and a spin of 2, 3, or 4 for the level. Because the level is populated directly (and strongly) by the beta decay of a  $1+$  state, only  $2+$  is probable. No direct evidence

for the 1277-keV transition has been obtained, although we have looked for the  $K$ -conversion line of the transition. We can state that the  $K$ -conversion line is weaker than that of the 970-keV transition. The large conversion coefficient of the 1180-keV transition is of interest, as it probably indicates an appreciable  $E0$  admixture in the transition. Evidence that  $E0$  admixtures occur in transitions between states with  $I \neq 0$ , but with  $K_i = K_f = 0$ , has recently been obtained in both the rare earth<sup>19</sup> and heavy element<sup>20</sup> regions. Thus the large conversion coefficient of this transition favors the assignment of this state as the  $2+$  rotational state based on the 1197-keV  $0+$  level.

The 1440-keV level is established as  $0+$  because the large conversion coefficient of the 1440-keV transition is consistent only with an  $M4$  or higher magnetic multipole or an  $E0$ . Only an  $E0$  assignment is consistent with all the data. The 1345-keV transition assigned to depopulate this state is coincident with the 93-keV photon. The limit on the conversion coefficient of the 1345-keV transition is consistent with  $E2$  multipolarity assignment, but does not exclude other multipoles except pure  $E1$ .

The 1430-keV level is established by the energies of the 1430 and 1335-keV transitions, the latter being coincident with the 93-keV photon while the former is not. The spin of the state is limited to 0 or 1, positive parity, or 2 or 3, negative parity, by the limit on the conversion coefficient of the 1430-keV transition and the fact that the state is directly populated by the beta decay of a  $1+$  state. The anisotropy of the angular correlation between the  $\approx 1350$ -keV peak and the 93-keV photon indicates that the transition is not pure  $0-2-0$ .<sup>18</sup> This result effectively rules out a  $0+$  assignment for the 1430-keV level, because the 1440-keV level is  $0+$ , and if the peak consisted of two  $0-2-0$  transitions the correlation observed would probably show a greater anisotropy than is observed. An assignment of  $2-$  or  $3-$  is not probable because of the low  $\log ft$  for the direct beta population of the state (see Table V). It must be admitted, of course, that the  $\log ft$  for beta decay to this state is calculated assuming that both the 1335- and 1430-keV transitions are pure  $M1$ . However, if the state were  $2-$  or  $3-$ , only  $E1$  or  $M2$  multipolarities would be reasonable;  $E1$  assignments would lead to an even lower  $\log ft$  than reported, whereas  $M2$  assignments would increase the  $\log ft$  reported by at most 0.6. The conclusion that the transition rate is very large would therefore not be altered, making it unlikely that the  $EC$  transition is first forbidden. The 1430-keV level is therefore most probably  $1+$ .

The 1483-keV level is established as  $2+$  because of the observed decay from this state to the  $0+$ ,  $2+$ , and

$4+$  levels of the ground state band, the limits that can be set on the conversion coefficient of the 3 transitions, and the fact that it is strongly populated by the decay of a  $1+$  state. The  $K$  line of the  $2+$  to  $4+$  transition ( $\approx 1180$  keV) is not resolved from that of the more intense  $2+$  to  $0+$  1180-keV transition; the argument for its presence is that a photon of 1180 keV is coincident with  $\approx 50\%$  of the  $K$ -conversion electrons of the 214-keV transition, whereas the total intensity of the 214-keV transition is less than the total intensity of the 1180-keV peak.

The level at 1550 keV is the least certain assignment in the decay scheme, the tentative assignment being based on the coincidences observed between a transition of 1260 keV and the fact that there is a weak electron line which can be assigned as the  $K$  line of a 1460-keV transition, which could be the transition to the 93-keV state from a 1550-keV level. We have no direct evidence for the 1550-keV transition to ground.

The level spectrum above 1 MeV thus far discussed is based solely on the high energy transitions. We might also expect to see a number of low energy transitions, corresponding to transitions between the observed states above 1 MeV, and perhaps also to and from other states which are only weakly populated by beta decay and hence not assigned in our decay scheme. Evidence for such increased complexity exists, in that the region of the electron spectrum between the  $M$  lines of the 93-keV transition and the  $L$  lines of the 214-keV transition shows a great deal of unresolved structure, the intensity of the structure being greatest between roughly the  $K$  and  $L$  lines of the 214-keV transition. In this region we have resolved only one line, at 138 keV. Coincidences gated on this line show gamma-ray peaks at 1000 and 1350 keV, and the latter is obviously complex. In view of the similarity in the energy of the 1350-keV peak to the intense 1350-keV peak in the singles spectrum, this datum cannot be interpreted unambiguously, and hence we have not assigned it in the decay scheme. In general, because of the extremely weak intensity and complexity of this portion of the spectrum, much greater source strengths and higher resolution than are currently available to us will be needed to clarify it. A higher resolution study of this region with more intense sources would undoubtedly yield much additional information about the levels in  $\text{Hf}^{178}$ .

## B. Comparative Lifetimes

Comparative lifetimes for the electron capture and positron branchings to the levels in  $\text{Hf}^{178}$  have been calculated using Moszkowski's nomograph<sup>21</sup> and are shown in Table V. The percentage of primary branchings to the levels at 1400 keV have been calculated assuming the multipolarities assigned in Table I. The values of  $\log ft$  for these states are therefore dependent

<sup>19</sup> O. Nathan and S. Hultberg, Nuclear Phys. **10**, 118 (1958).

<sup>20</sup> C. J. Gallagher, Jr., and T. D. Thomas, Nuclear Phys. **14**, 1 (1959/60).

<sup>21</sup> S. A. Moszkowski, Phys. Rev. **82**, 25 (1951); reproduced also in references 15 and 17.

TABLE V. Values of  $\log ft$  for the electron capture branching to the excited levels.

Energy of state (kev)	Percentage primary EC branching	$\log ft$	Percentage primary $\beta^+$ branching	$\log ft$
0	$59 \pm 12$	4.6	0.5	4.8
93	$35 \pm 7$	4.8	$\leq 0.4^a$	$\geq 4.8$
307	$< 0.02$	$> 7.9$		
1197	$0.8 \pm 0.2$	5.1		
1277	$0.2 \pm 0.4$	5.3 to 5.7		
1430	$1.4 \pm 0.6^b$	4.6		
1440	$1.2 \pm 0.3^b$	4.7		
1483	$1.7 \pm 0.7^b$	4.4		
1550	$0.4 \pm 0.1^b$	4.8		

<sup>a</sup> Source scattering in the thick source produced appreciable distortion of the positron spectrum, making the analysis of the intensity of the inner group uncertain.

<sup>b</sup> These percentage branchings are dependent on the interpretation given the data. The sum of all of the observed photon intensities is independent of the interpretation, however, and therefore indicates very large transition probabilities to the high lying states.

on the decay scheme. A discussion of absolute transition rates is given below.

### C. Comparison with Previous Results

The energy and  $2+$  assignment of the 93-kev level are as previously reported by Felber,<sup>1</sup> Carver and Turchinets,<sup>2</sup> and from the Coulomb excitation measurements.<sup>7-9</sup> Similarly, our assignment of the  $4+$  level at 307 kev is as established by the cascade decay of  $\text{Hf}^{178}$  as reported by Felber *et al.*,<sup>16</sup> and by Carver and Turchinets. Our results for the high-energy region indicate a much greater complexity than previously suspected, and as a consequence differ considerably from the previous results which now appear to have been incorrect. Our results support the positron end point of 880 kev reported by Felber, and we therefore suggest that the source of the 1.65 Mev  $\beta$  activity reported by Carver and Turchinets was incorrectly identified. We agree with the previous workers that the low values of  $\log ft$  and predominant decay to the  $0+$  and  $2+$  levels of  $\text{Hf}^{178}$  are best interpreted if  $\text{Ta}^{178}$  has a  $1+$  configuration. The  $\beta^+/K$  capture ratio,  $0.9 \pm 0.6\%$ , determined in the present study is somewhat smaller than the  $2\%$  reported by Felber, although the difference is within experimental error.

## V. DISCUSSION AND CONCLUSIONS

### A. $\beta^+/K$ Capture Ratio

The  $\Delta I=1$  transitions and large decay rate observed in the decay of  $\text{Ta}^{178}$  to the ground state band of  $\text{Hf}^{178}$  ensure that only allowed Gamow-Teller ( $\beta$ ) matrix elements will be important in both the electron capture and positron branches. The decay of this isotope should therefore provide the best case yet observed in the high- $Z$  region for testing the theoretical  $\beta^+/K$  capture ratio. The experimental value of  $0.9\% \pm 0.6\%$  is lower than, but within experimental error of, the theoretical ratio  $\approx 1.4\%$  for a nucleus with  $Z=73$  and two posi-

TABLE VI. Positron and electron capture branching ratios to the  $0+$  and  $2+$  states in  $\text{Hf}^{178}$ .

Decay mode	Energy of rotational state (kev)		$ft(1+1 \rightarrow 2+0)/ft(1+1 \rightarrow 0+0)$	
	$0+$	$2+$	Experiment	Theory
$\beta^+$	0	93	$\approx 1$	2
$K$ capture	0	93	$1.7 \pm 0.8$	2

tron groups which have end point energies of 800 and 890 kev and roughly equal intensity. The theoretical value was interpolated from the plot of Zweifel's<sup>22</sup> results prepared by Wapstra, Nijgh, and Van Lieshout.<sup>17</sup>

### B. $E0$ Matrix Elements

Absolute  $E0$  matrix elements have been shown by Church and Weneser<sup>23</sup> and Reiner<sup>24</sup> to be sensitive to details of nuclear structure, and therefore their calculations assuming various nuclear models should provide a test of the models. Because we do not know the half-lives of the  $0+$  levels we are not able to report absolute  $E0$  matrix elements for the  $0+$  to  $0+$  transitions we have observed, but we can calculate the  $E0/E2$  reduced transition probability ratios for both the 1197- and 1440-kev  $0+$  levels from the relative  $K$ -conversion line intensities of the transitions depopulating them. In some cases this ratio may be a more significant test of the nuclear models than the absolute matrix elements.<sup>24,25</sup>

Rather than report the total  $E0/E2$  transition probability ratio  $\mu$  as proposed by Reiner, we report the dimensionless  $E0/E2$  reduced transition probability  $X = (\rho^2 e^2 R_0^4) / [B(E2)]$ .<sup>25</sup>  $\rho^2 = T(E0)/\Omega_K$  is the  $E0$  reduced transition probability and equals the total  $E0$  transition probability divided by the factor  $\Omega_K$ , which contains all contributions from electronic terms. Values of  $\Omega_K$  as a function of the energy of the monopole transition and  $Z$  are available from the figures of Church and Weneser.  $\Omega_K$  is to some extent dependent on the choice of the nuclear charge distribution, as shown clearly by Reiner. The nuclear-theory-dependent part of the  $E2$  transition probability is  $B(E2) = T(E2)/C$ , where  $C = (4\pi/75)(1/\hbar)(\omega/c)^5$ . The product  $e^2 R_0^4$  is introduced to make  $B(E2)$  a dimensionless quantity to compare with  $\rho^2$ .  $e$  is the electron charge and  $R_0 = 1.20$ ,  $A^{1/3}$  fermis is the nuclear radius.

The observed ratios of  $K$ -conversion line intensities are ( $K$  1197/ $K$  1105) =  $2.8 \pm 0.6$  and ( $K$  1440/ $K$  1345) =  $5.3 \pm 1.1$ , where the error in the ratio has been estimated as  $20\%$  because the lines are close-lying in energy. If we assume that the 1105- and 1345-kev

<sup>22</sup> P. F. Zweifel, Phys. Rev. **107**, 329 (1957).

<sup>23</sup> E. L. Church and J. Weneser, Phys. Rev. **103**, 1035 (1956).

<sup>24</sup> A. Reiner, thesis, University of Amsterdam, 1958 (unpublished).

<sup>25</sup> J. O. Rasmussen, Nuclear Phys. **19**, 85 (1960).



transitions are pure  $E2$ , with theoretical  $E2$   $K$ -conversion coefficients  $\alpha_K(1105)=0.0026$  and  $\alpha_K(1345)=0.0019$ , we calculate total  $E0/E2$  transition probability ratios of 0.0076 and 0.010 for the 1197- and 1440-keV levels, respectively. The values of  $X$  we obtain are then, respectively,  $0.18 \pm 0.04$  and  $0.53 \pm 0.16$ . We have not attempted to determine the  $E0/E2$  ratio for the  $2+$  to  $2+$  1190-keV transition (and possibly for the 1390-keV transition) because we do not know the photon intensity sufficiently well.

A theoretical treatment of the problem of  $E0$  matrix elements and  $E0/E2$  reduced transition probability ratios which will, for example, enable the experimentalist to assign a definite intrinsic configuration to an excited  $0+$  state, or to choose conclusively among various nuclear models, does not exist at present, in spite of the progress that has been made in understanding the problem.<sup>23-25</sup> We therefore shall not attempt a detailed comparison of our results with theory. Instead, we shall only mention two points which are raised by this and other experimental work, namely that (1) the values of  $X$  for the  $\beta$ -vibrational  $0+$  in the strongly deformed nuclei  $\text{Hf}^{178}$  (0.18) and  $\text{Pu}^{238}$  (0.14)<sup>26</sup> are essentially constant, whereas  $X$  for the only-just deformed spheroidal nucleus  $\text{Sm}^{152}$  is 0.016,<sup>25,27</sup> and (2)  $X$  for the 1440-keV intrinsic  $0+$  level in  $\text{Hf}^{178}$  is 3 times the  $X$  of the 1197-keV  $\beta$ -vibrational state.

### C. $\text{Ta}^{178}$ $1+$ State

The  $1+$  assignment of  $\text{Ta}^{178}$  is based on its large decay rate to the  $0+$  and  $2+$  levels of the  $\text{Hf}^{178}$  ground state band. The  $1+$  state has been interpreted<sup>3</sup> as the spin triplet state of the 73rd proton and 105th neutron,  $514\uparrow$  and  $514\downarrow$ , respectively (in the  $Nn_z\Lambda\Sigma$  notation of the asymptotic-limit Nilsson wave functions<sup>28</sup>). The assignment of the  $9/2$  state as the 73rd proton state is not completely consistent with the observed  $7/2+$  ground state of  $\text{Ta}^{181}$ , but, because the  $9/2$  level in  $\text{Ta}^{181}$  is only 6 keV above the  $7/2+$  ground state,<sup>29</sup> it seems possible that the level might appear in  $\text{Ta}^{178}$  as an isomeric state. The observed  $\log ft=4.6$  for the beta branching to the ground state is then understood as the allowed unhindered decay of the  $514\uparrow$  proton to the  $514\downarrow$  neutron state.<sup>3</sup>

A check on the  $K$ -quantum number of  $\text{Ta}^{178}$  (and hence on the asymptotic-limit configuration of the  $\text{Ta}^{178}$  state) should be provided by the  $\beta$ -branching ratio to the  $0+$  and  $2+$  rotational levels of the  $\text{Hf}^{178}$  ground state band,<sup>30</sup> the electron capture and positron branchings providing independent checks on the

theoretical ratio. It should be noted here that this case should be considered an important test of the asymptotic limit configuration assignments. This is because the branching ratio predicted by the strong-coupling model is dependent only on geometrical considerations, assuming only one matrix element is important, which criterion we should obtain here, because the large transition rate and spin change  $\Delta I=1$  for the decay to both the  $0+$  and  $2+$  states require that only allowed Gamow-Teller matrix elements are effective in both branches. The experimental ratios are  $1.7 \pm 0.8$  and  $\approx 1$  for the  $K$  capture and  $\beta^+$  branchings, respectively. The  $K$ -capture ratio is somewhat lower than the theoretical ratio of 2, but within experimental error of it. The  $\beta^+$ -branching ratio indicates a larger population of the  $2+$  state than theoretically expected. However, if source- and back-scattering degraded some positrons which were then detected, the net result would be an increase in the intensity of the lower energy group. We conclude therefore that the experimental  $\beta^+$ -branching ratio we report is probably low, owing to the rather thick source used in the experiment, and the spectrum should be investigated using thinner, more intense sources, before a definite conclusion is reached on this point.

### D. $\text{Hf}^{178}$ Levels

The level at 1197 keV is assigned as the base state of a  $K=0$  band. As discussed above, the  $2+$  level at 1277 is assigned as the first rotational state based on it. The energy of the state, 78 keV above the  $0+$  state, indicates that this band has a larger moment of inertia than the ground state band. On the basis of the systematics of energy levels in this region we tentatively call this state a  $\beta$ -vibrational state. It should be noted that the decay rate to this state is smaller than the decay rate to the ground state band.

It should also be noted that we have not observed any transition which we can assign to establish the  $\gamma$ -vibrational state in  $\text{Hf}^{178}$ , indicating that the decay to this state is retarded relative even to the decay to the  $\beta$ -vibrational state.

We propose that the 1430- and 1440-keV levels represent intrinsic excitations. On the basis of the probable  $2+$  assignment of the 1483-keV level we tentatively assign it as the first rotational state based on the 1430-keV level. There is nothing obviously inconsistent with the assignment of the 1550-keV level as a  $2+$  rotational state band on the 1440-keV level, although such an assignment is at best speculative. It should be noted here that, because there are at least three states with a fourth expected, (the  $\gamma$ -vibrational state) with  $2+$  character in an energy interval of only  $\approx 300$  keV in this nucleus, these states will probably not have a clearly defined  $K$ -quantum number.

Assuming that the fast decay rate to the ground state is associated with the transition from the proton

<sup>26</sup> Unpublished results of F. Asaro, F. S. Stephens, and I. Perlman, quoted in reference 25.

<sup>27</sup> I. Marklund, O. Nathan, and O. B. Nielsen [Nuclear Phys. **15**, 199 (1960)] first pointed out this anomaly.

<sup>28</sup> S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 16 (1955).

<sup>29</sup> A. H. Muir and F. Boehm, Bull. Am. Phys. Soc. **4**, 367 (1959).

<sup>30</sup> G. Alaga, K. Alder, A. Bohr, and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 9 (1955).

state  $514\uparrow$  to the neutron state  $514\downarrow$ , we conclude that it indicates a large probability of finding the  $514\downarrow$  neutron pair in the excited state also. If we assume a two-particle configuration (as is done, for example, in reference 3) and if the  $514\downarrow$  neutron state is on the Fermi surface (as deduced from the large decay rate to ground), it is qualitatively difficult to understand why the decay rate to the excited state is unhindered relative to the decay rate to the ground state. A simple two-particle picture, therefore, does not seem to describe the state very well. However, Mottelson has suggested<sup>31</sup> on the basis of calculations of even-even excited state spectra based on pairing correlation forces<sup>32</sup> that two-quasi-particle  $0+$  states should exist which have intrinsic structure closely related to that of the ground state. It is expected,<sup>33</sup> furthermore, that the transition rates to these states should approximately equal the rate to the ground state. Although the physical concepts underlying pairing-correlation calculations have been tested to some extent already,<sup>34,35</sup> there is at present very little experimental evidence that supports the calculational result that excited spectra of deformed even-even nuclei should be relatively simple, at least with respect to the number of two-quasi-particle intrinsic states at excitation energies up to about 2 Mev. It therefore seems important that the present results provide evidence supporting these ideas.

<sup>31</sup> B. R. Mottelson, lecture notes on a Nuclear Physics course given at the Institute (spring, 1960).

<sup>32</sup> S. T. Belyaev, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **31**, No. 11 (1959).

<sup>33</sup> Private communication from B. R. Mottelson.

<sup>34</sup> L. Kisslinger and R. A. Sorensen, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **32**, No. 9 (1960).

<sup>35</sup> J. Griffin and M. Rich, Phys. Rev. **118**, 850 (1960). O. Prior and S. G. Nilsson (to appear in Kgl. Danske Videnskab. Selskab).

The state at 1430 kev assigned as  $1+$  is not easily understood on the basis of the expected two-quasi-particle spectrum although both a low-lying proton state ( $404\downarrow-402\uparrow$ ) and a low-lying neutron state ( $514\downarrow-512\uparrow$ ) with a  $1+$  configuration are expected. The reason is that the decay to this state is as fast as that to the ground state, indicating clearly that the intrinsic structure is similar to the intrinsic structure of the  $\text{Ta}^{178}$  ground state. A possible explanation of the state is that it is the first rotational state of the intrinsic  $0+$  state as 1440 kev; however, if this state is correctly assigned above we would not expect odd spin rotational states in its rotational band, as the wave function of the state has the same symmetry properties as that of the ground state.

The large percentage branching to the two other states is explained qualitatively if the states are rotational states. Quantitative checks on the  $K$ -quantum number assignments can be made in principle from the beta-gamma-ray branching ratios to and from these states (see above). However, because we have not resolved the complex gamma-ray spectrum, our estimates of these ratios are not quantitatively meaningful. A direct gamma-ray intensity measurement is needed and would be very important in checking our assignments.

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