

Hyperon Decay in the Nonleptonic Mode

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An interaction Hamiltonian of the weak interaction between baryons and pions is described in the three-dimensional charge space. In order to determine the types of interaction, a conservation law of extended chirality is speculated; the branching ratios and the absolute lifetimes of the hyperon decays in the nonleptonic modes are calculated and the results are compared with the experiments.

ACCORDING to the experiments of Cool *et al.*,¹ it has been found that the up-down asymmetry of the hyperon decay in the nonleptonic mode cannot be described by the $V-A$ interaction of the Yukawa type. Attempts have been made by several authors²⁻⁶ to understand the strange behavior of the hyperon decay in terms of a linear combination of the $V+A$ interaction and the $V-A$ interaction. In this article, a variant theory of the weak interaction is proposed in which a new conservation law could explain the experiments quite well.

In the current formulation of the isospin I , an invariant Hamiltonian in the usual isospin space cannot be given for the weak interaction since this obeys the selection rule $\Delta I = \frac{1}{2}$. An alternative formulation in the three-dimensional isospin space proposed by d'Espagnat, Prentki, and Salam⁷ and also by Takeda⁸ leads us to a satisfactory answer, although the same formulation cannot be applied to the strong interactions. Let us follow the M -space formulation⁷ and arrange the baryons and pions into the triplet and the singlet systems:

$$\begin{aligned} p_1 &= (\Xi^- + p)/\sqrt{2}, & p_2 &= (\Xi^- - p)/\sqrt{2}i, \\ p_3 &= (\Xi^0 - n)/\sqrt{2}, & p_0 &= (\Xi^0 + n)/\sqrt{2}, \\ q_1 &= (\Sigma^- + \Sigma^+)/\sqrt{2}, & q_2 &= (\Sigma^- - \Sigma^+)/\sqrt{2}i, \\ q_3 &= \Sigma^0, & q_0 &= \Lambda, \\ \pi_1 &= (\pi^- + \pi^+)/\sqrt{2}, & \pi_2 &= (\pi^- - \pi^+)/\sqrt{2}i, \\ \pi_3 &= \pi^0, & \pi_0 &= \pi^{0'}. \end{aligned} \quad (1)$$

The existence of $\pi^{0'}$ is, of course, quite obscure, but we retain it for the sake of symmetry.

Next, we write the Hamiltonian of the weak interaction between baryons and pions as an invariant in

the three-dimensional isospin space:

$$H = a(\bar{\mathbf{p}} \times \boldsymbol{\pi}) \cdot \mathbf{q} + b\bar{p}_0\boldsymbol{\pi} \cdot \mathbf{q} + c\bar{\mathbf{p}}\pi_0 \cdot \mathbf{q} + d\bar{\mathbf{p}} \cdot \boldsymbol{\pi}q_0 + e\bar{p}_0\pi_0q_0 + \text{H.c.} \quad (2)$$

Here $\mathbf{p} = (p_1, p_2, p_3)$, and similarly for $\boldsymbol{\pi}$ and \mathbf{q} . The letters a, b, c, d , and e are the coupling constants.

Now, suppose the following transformation S of charge symmetry:

$$S: \begin{aligned} p_2 &\rightarrow -p_2, & q_2 &\rightarrow -q_2, & \pi_2 &\rightarrow -\pi_2, \\ (p &\leftrightarrow \Xi^-, & \Sigma^+ &\leftrightarrow \Sigma^-, & \pi^+ &\leftrightarrow \pi^-). \end{aligned} \quad (3)$$

The transformation S will change the sign of the electric charge of charged particles without transforming them into their antiparticle states. It will be easily seen that the term with the coefficient a in (2) changes sign under the transformation S , while the other terms remain invariant under S .

For the universal Fermi interaction, Sudarshan and Marshak⁹ succeeded in deriving the $V-A$ type by requiring invariance under the chirality transformation.¹⁰ It is clear, however, that we cannot extend this principle in a straightforward manner to the weak interaction of the Yukawa type for baryons. In fact, if we require the invariance of the Hamiltonian under the chirality transformation X we get exclusively the $V-A$ interaction. Invariance under X means that

$$H = \bar{\psi}_A \psi_B \phi$$

must be invariant under the two independent transformations:

$$\psi_A \rightarrow \gamma_5 \psi_A, \quad \psi_B \rightarrow \psi_B, \quad \phi \rightarrow \phi,$$

and

$$\psi_B \rightarrow \gamma_5 \psi_B, \quad \psi_A \rightarrow \psi_A, \quad \phi \rightarrow \phi. \quad (4)$$

As was mentioned previously, the $V-A$ interaction alone is at variance with the experiments.

In this respect, it is tempting to us to modify the chirality transformation for the case of baryon inter-

¹ R. L. Cool, J. W. Cronin, B. Cork, L. Kerth, and W. A. Wentzel, *Phys. Rev.* **114**, 912 (1958).

² K. Nakagawa, *Nuclear Phys.* **10**, 20 (1959).

³ B. d'Espagnat and J. Prentki, *Phys. Rev.* **114**, 1366 (1959).

⁴ R. S. Sawyer, *Phys. Rev.* **112**, 2135 (1958).

⁵ S. A. Bludman, *Phys. Rev.* **115**, 468 (1959).

⁶ F. Gürsey, *Nuovo cimento*, **16**, 230 (1960).

⁷ B. d'Espagnat, J. Prentki, and A. Salam, *Nuclear Phys.* **5**, 447 (1958).

⁸ G. Takeda, *Progr. Theoret. Phys. (Kyoto)* **19**, 361 (1958).

⁹ E. C. G. Sudarshan and R. E. Marshak, *Phys. Rev.* **109**, 1860 (1958).

¹⁰ S. Watanabe, *Phys. Rev.* **106**, 1306 (1957); T. Tanikawa and S. Watanabe, *ibid.* **110**, 289 (1958).

actions. We require that the interaction Hamiltonian must be invariant under the combined transformation of S and X .¹¹

The terms $\bar{\psi}_A \gamma_\mu (1 + \gamma_5) \psi_B \partial_\mu \phi$ and $\bar{\psi}_A \gamma_\mu (1 - \gamma_5) \psi_B \partial_\mu \phi$ are respectively invariant and pseudoinvariant under the chirality transformation (4). Therefore, it is obvious that if we take $\Omega_a = \gamma_\mu (1 - \gamma_5)$, $\Omega_b = \Omega_c = \Omega_d = \Omega_e = \gamma_\mu (1 + \gamma_5)$, then we are able to prove the SX invariance in the Hamiltonian (2). Here $\Omega_a, \Omega_b, \Omega_c, \Omega_d$, and Ω_e are the Dirac operators to be inserted in the bilinear forms of the two baryon fields in the terms of (2) with the coefficients a, b, c, d , and e , respectively. The final expression of (2) thus obtained is a linear combination of the $V+A$ interaction and the $V-A$ interaction.

The observed value¹ of the asymmetry parameter α in the mode $\Sigma^+ \rightarrow n + \pi^+$ is very small. It is not at variance with (2) if we assume

$$\pm a/i = b. \quad (5)$$

Then we get

$$\begin{aligned} H = & (a/i) \{ \bar{p} \gamma_\mu (1 - \gamma_5) \Sigma^+ \partial_\mu \pi^0 - \bar{p} \gamma_\mu (1 - \gamma_5) \Sigma^0 \partial_\mu \pi^+ \} \\ & + (a/i) (\epsilon/2) \{ \bar{n} \gamma_\mu (1 + \gamma_5) \Sigma^0 \partial_\mu \pi^0 + \bar{\Xi}^0 \gamma_\mu (1 + \gamma_5) \Sigma^0 \partial_\mu \pi^0 \} \\ & + \sqrt{2} (a/i) \{ \bar{n} \Omega_1 \Sigma^+ \partial_\mu \pi^- + \bar{n} \Omega_2 \Sigma^- \partial_\mu \pi^+ \} \\ & + (a/i) \{ \bar{\Xi}^- \gamma_\mu (1 - \gamma_5) \Sigma^0 \partial_\mu \pi^- - \bar{\Xi}^- \gamma_\mu (1 - \gamma_5) \Sigma^- \partial_\mu \pi^0 \} \\ & + \sqrt{2} (a/i) \{ \bar{\Xi}^0 \Omega_1 \Sigma^- \partial_\mu \pi^+ + \bar{\Xi}^0 \Omega_2 \Sigma^+ \partial_\mu \pi^- \} \\ & + c \{ \bar{p} \gamma_\mu (1 + \gamma_5) \Sigma^+ + \bar{\Xi}^- \gamma_\mu (1 + \gamma_5) \Sigma^- \\ & + (1/\sqrt{2}) (\bar{\Xi}^0 - \bar{n}) \gamma_\mu (1 + \gamma_5) \Sigma^0 \} \partial_\mu \pi^0 \\ & + d \{ \bar{p} \gamma_\mu (1 + \gamma_5) \Lambda \partial_\mu \pi^+ - (1/\sqrt{2}) \bar{n} \gamma_\mu (1 + \gamma_5) \Lambda \partial_\mu \pi^0 \\ & + \bar{\Xi}^- \gamma_\mu (1 + \gamma_5) \Lambda \partial_\mu \pi^- + (1/\sqrt{2}) \bar{\Xi}^0 \gamma_\mu (1 + \gamma_5) \Lambda \partial_\mu \pi^0 \} \\ & + (e/\sqrt{2}) (\bar{\Xi}^0 + \bar{n}) \gamma_\mu (1 + \gamma_5) \Lambda \partial_\mu \pi^0 + \text{H.c.} \end{aligned} \quad (6)$$

For Ω_1, Ω_2 , and ϵ , we have two possibilities.

Case (I):

$$\Omega_1 = \gamma_\mu \gamma_5, \quad \Omega_2 = \gamma_\mu, \quad \epsilon = 1, \quad \text{for } +a/i = b. \quad (7a)$$

Case (II):

$$\Omega_1 = \gamma_\mu, \quad \Omega_2 = \gamma_\mu \gamma_5, \quad \epsilon = -1, \quad \text{for } -a/i = b. \quad (7b)$$

Since the existence of π^0 is quite doubtful, we omit it tentatively in the later discussions. The main results of this theory which can be directly compared with the experiments are summarized as follows:

1. The consequences derived from the selection rule $\Delta I = \frac{1}{2}$ are reproduced in our Hamiltonian (6). For instance, the branching ratio in the Λ decay and Ξ decay will come out from (6) to be

$$\sigma(\Lambda \rightarrow p + \pi^-) / \sigma(\Lambda \rightarrow n + \pi^0) = 1.9, \quad (8a)$$

$$\sigma(\Xi^- \rightarrow \pi^- + \Lambda) / \sigma(\Xi^0 \rightarrow \pi^0 + \Lambda) = 2.3. \quad (8b)$$

Equation (8a) is in good agreement with the experiments.^{12,13}

2. We predict two types of parity-nonconserving interactions:

$$V-A: \quad \Xi^- \rightarrow \pi^- + \Lambda, \quad \Xi^0 \rightarrow \pi^0 + \Lambda,$$

$$\Lambda \rightarrow \pi^- + p, \quad \Lambda \rightarrow \pi^0 + n,$$

$$V+A: \quad \Sigma^+ \rightarrow \pi^0 + p.$$

An experimental check on this problem may come, for example, from the studies on the sign of the asymmetry factors α of these modes of the polarized hyperon decay.

3. The asymmetry factors α inferred from (6) are as follows¹⁴:

Decay modes	α
$\Lambda \rightarrow p + \pi^-, \quad n + \pi^0$	0.91
$\Sigma^- \rightarrow n + \pi^-, \quad \Sigma^+ \rightarrow n + \pi^+$	0
$\Sigma^+ \rightarrow p + \pi^0$	-1.00
$\Xi^- \rightarrow \Lambda + \pi^-, \quad \Xi^0 \rightarrow \Lambda + \pi^0$	0.97

With regard to the Λ decay and the Σ decay, these values are not at variance with the observed values.¹ The signs of the α 's have not been established, and are subject to further experimental studies.

4. It is interesting to note that the calculated values of the branching ratios in the various modes of Σ decay are quite diverse for the case of (7a) and (7b).

Case (I). Equation (7a) gives

$$\begin{aligned} \sigma(\Sigma^+ \rightarrow n + \pi^+) : \sigma(\Sigma^+ \rightarrow p + \pi^0) : \sigma(\Sigma^- \rightarrow n + \pi^-) \\ = 0.8 : 1 : 1.2, \quad (9a) \\ \tau_{\Sigma^-} / \tau_{\Sigma^+} = 1.4. \end{aligned}$$

Here τ_{Σ^-} and τ_{Σ^+} are the mean lifetimes of Σ^- and Σ^+ with regard to the pion decay (without π^0).

Case (II). Equation (7b) gives

$$\begin{aligned} \sigma(\Sigma^+ \rightarrow n + \pi^+) : \sigma(\Sigma^+ \rightarrow p + \pi^0) : \sigma(\Sigma^- \rightarrow n + \pi^-) \\ = 1.2 : 1 : 0.9, \quad (9b) \\ \tau_{\Sigma^-} / \tau_{\Sigma^+} = 2.5. \end{aligned}$$

The results given by (9a) and (9b) are both in poor agreement with the experiments,¹⁵ which have reported 1:1:1 for the branching ratio. It is instructive to reflect that any theory which leads to parity non-conservation for $\Sigma^+ \rightarrow \pi^0 + p$ and to parity conservation for $\Sigma^- \rightarrow \pi^- + n$ and $\Sigma^+ \rightarrow \pi^+ + n$ should have the same results as ours. The 1:1:1 ratio could only be deduced if we neglect the differences in the coupling types and also the diversity in masses for each member of the same multiplet.

5. Now we shall discuss the relationship among the lifetimes for the pion decays of Λ, Ξ , and Σ . By using the observed values for the mean lifetime of Λ ,

$$\tau_{\Lambda \text{ exp}} = (2.505 \pm 0.086) \times 10^{-15} \text{ sec},$$

¹¹ S. Nakamura, J. Phys. Soc. Japan, **15**, 1543 (1960).

¹² M. Gell-Mann and A. Rosenfeld, Ann. Rev. Nuclear Sci. **7**, 407 (1957).

¹³ S. Okubo, R. E. Marshak, and E. C. G. Sudarshan, Phys. Rev. **113**, 944 (1959).

¹⁴ H. Umezawa, M. Konuma, and K. Nakagawa, Nuclear Phys. **7**, 169 (1958).

¹⁵ L. W. Alvarez, Proceedings of the Ninth Conference on High-Energy Physics, Kiev, 1959 (unpublished).

TABLE I. Mean lifetimes (in units of 10^{-10} sec) for Σ and Λ decays as functions of the $\pi^{0'}$ mass.

	$m_{\pi^{0'}}$ (Mev)	$d^2/4\pi$ ($10^{-15}\mu^{-2}$)	τ_{Σ^-}		τ_{Σ^+}		$\tau_{\Sigma^-}/\tau_{\Sigma^+}$		τ_{Ξ^-}	τ_{Ξ^0}	$\tau_{\Xi^0}/\tau_{\Xi^-}$
			(I)	(II)	(I)	(II)	(I)	(II)			
No $\pi^{0'}$		4.72	0.7	1.0	0.48	0.40	1.4	2.5	1.9	4.3	2.3
	135	3.51	0.9	1.3	0.4	0.3	2.2	3.9	2.5	2.9	1.2
	120	3.29	1.0	1.4	0.4	0.4	2.5	3.9	2.6	2.8	1.1
	105	3.13	1.0	1.5	0.4	0.4	2.4	3.8	2.5	1.8	0.7
Experiments ^a			$1.61_{-0.09}^{+0.1}$		$0.81_{-0.05}^{+0.06}$				$1.28_{-0.30}^{+0.38}$	~ 1.5	

^a See reference 19.

we get from (6)

$$d^2/4\pi = (4.72 \pm 0.2) \times 10^{15} \mu^{-2}, \quad (10)$$

where μ = pion mass = 140 Mev.

The lifetime of the Ξ particles can be calculated from (6). By inserting (10), we get

$$\begin{aligned} \tau_{\Xi^-} &= 1.9 \times 10^{-10} \text{ sec}, \\ \tau_{\Xi^0} &= 4.3 \times 10^{-10} \text{ sec}. \end{aligned} \quad (11)$$

Further, if we assume

$$|a| = |d|,$$

we can infer from (6) the mean lifetime for the Σ , which is found to be different in cases (I) and (II).

Case (I):

$$\tau_{\Sigma^-} = 0.7 \times 10^{-10} \text{ sec}, \quad (12a)$$

Case (II):

$$\tau_{\Sigma^-} = 1.0 \times 10^{-10} \text{ sec}. \quad (12b)$$

The present results are shorter than the experimental values,

$$\tau_{\Sigma^-} \text{ exp} = 1.59 \times 10^{-10} \text{ sec},$$

by a factor of ~ 0.6 . It must be mentioned that the doublet scheme,¹⁶ in which $Y = (1/\sqrt{2})(-\Sigma^0 + \Lambda)$ and $Z = (1/\sqrt{2})(\Sigma^0 + \Lambda)$ are used instead of Σ^0 and Λ , will have the same relationship as ours ($|a| = |b| = |d|$), if we apply the doublet scheme to the weak interaction. In this respect, we are led to consider two alternative possibilities.

(i) It is not surprising to find that the effective coupling constants for the Σ decay and those for the Λ decay are not equal.¹⁷ In other words, the values of $|a|$, $|b|$, and $|d|$ which we have estimated using the experimental lifetimes are the effective coupling constants, including secondary effects such as the corrections due to the strong interactions for the particles involved in these processes. As far as the "bare" coupling constants are concerned,¹⁸ we cannot tell anything definite about their universalities from the present analysis.

(ii) The contribution from $\pi^{0'}$ will bridge over discrepancies. It is premature to have excluded the

effects from $\pi^{0'}$ only because it has not so far been observed. Let us consider what would be involved with $\pi^{0'}$. Consider the possible modes of the Σ^+ decay and the Λ decay which are accompanied by the emission of $\pi^{0'}$:

$$\Sigma^+ \rightarrow \pi^{0'} + p, \quad (13)$$

$$\Lambda \rightarrow \pi^{0'} + n. \quad (14)$$

If the $\pi^{0'}$ mass is different from the π^0 mass (see Table I), it seems likely that $\pi^{0'}$ might have escaped observation in the investigations of the normal mode of Λ decay. The well-established value of 2:1 for the charged/neutral branching ratio would not, therefore, be upset even if we take into account the contribution from the channel (14) to the Λ decay. In this case the coupling constants estimated from the observed lifetime of the Λ decay would be revised. It is clear that the result depends on the mass of $\pi^{0'}$ as is shown in Table I.

In estimating the lifetime of the Ξ particles, we must now add the decay processes involved with the emission of $\pi^{0'}$:

$$\Xi^- \rightarrow \pi^{0'} + \Sigma^-, \quad (15)$$

$$\Xi^0 \rightarrow \pi^{0'} + \Lambda, \quad \pi^{0'} + \Sigma^0. \quad (16)$$

If we take into account the contributions from (15) and (16), the values of τ_{Ξ^-} and τ_{Ξ^0} will be modified:

$$\begin{aligned} \tau_{\Xi^-} &= 2.5 \times 10^{-10} \text{ sec}, \quad m_{\pi^{0'}} = 135 \text{ Mev} \\ &= 2.5 \times 10^{-10} \text{ sec}, \quad m_{\pi^{0'}} = 105 \text{ Mev}. \end{aligned} \quad (17)$$

$$\begin{aligned} \tau_{\Xi^0} &= 2.9 \times 10^{-10} \text{ sec}, \quad m_{\pi^{0'}} = 135 \text{ Mev} \\ &= 1.8 \times 10^{-10} \text{ sec}, \quad m_{\pi^{0'}} = 105 \text{ Mev}. \end{aligned} \quad (18)$$

These results are not at variance with the observed ones^{15,19} so far reported.

The experiments on the absolute lifetimes and the relative probabilities for the pion decay of Σ and Ξ particles seem to us to be not accurate enough to rule out all possible cases given in Table I, or to permit a selection among them.

¹⁶ M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).¹⁷ O. Hara, Nuovo cimento **14**, 114 (1959). The author assumed two coupling constants, g_{Σ} and $g_{\Lambda} = \sqrt{2}g_{\Sigma}$.¹⁸ S. Barshay, Phys. Rev. Letters **4**, 618 (1960).¹⁹ W. H. Barkas and A. H. Rosenfeld, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 877.

6. Lastly, we shall discuss the weak interaction of leptons. If we assume that the neutrino belongs to the singlet state in the M space, we have good reason to expect exclusively the $V-A$ interaction for the decay process involving the emission of neutrinos, since then the interaction terms which are S pseudo-invariant and

X pseudo-invariant will be lost. We could thus extend the principle of SX invariance to the universal $V-A$ theory^{9,20,21} for the weak interaction of leptons.

²⁰ R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

²¹ R. H. Dalitz, Revs. Modern Phys. **31**, 823 (1959).

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Direct Nucleon-Nucleon Collisions inside the Nucleus According to the Impulse Approximation*

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Direct nucleon-nucleon collisions play an important role in high-energy nuclear reactions. The importance of such collisions at lower energies is not clear. To aid in the interpretation of nuclear reactions, we have analyzed the collisions between an incident nucleon and nucleons in a Fermi gas by means of the impulse approximation. The treatment given here is based on information from nucleon-nucleon scattering experiments. Collisions inside a nucleus are considered to be the same as those in the unbound state at the same center-of-mass energy, except for the effect of the Pauli exclusion principle. The effective elastic

and inelastic cross section, $\langle\sigma\rangle$, between like and unlike nucleons is computed for incident energies from 10 Mev to 6 Bev at several values of the Fermi energy. The properties of the struck nucleons in allowed collisions are also calculated. This information may prove useful in interpreting some recoil experiments. Analytical expressions for $\langle\sigma\rangle$ and quantities related to the struck nucleon are given for elastic collisions in which the scattering is isotropic and the free-particle cross sections are either constant or vary inversely as the bombarding energy.

THE processes that occur during a nuclear reaction are not well known. One usually assumes that the collision between a high-energy particle and a nucleon inside a nucleus is essentially the same as in the unbound state at the same center-of-mass energy, except for the effect of the Pauli exclusion principle. In line with this "impulse approximation," information obtained from p - p , n - p , and π - p scattering experiments has been used in the analysis of nuclear reactions induced by high-energy particles.¹ This type of analysis has been successful in interpreting the available information on reactions induced by particles with energies of 1 Bev or less. However, serious discrepancies appear at higher energies.²

An analysis of nuclear reactions by means of the impulse approximation would be facilitated if the values of the effective collision cross sections $\langle\sigma\rangle$ of neutrons and protons inside nuclear matter were known at all energies. This information would be especially helpful for a study of the simpler reactions in which one or at most a few nucleons are emitted. The distance the incident particle penetrates the nucleus and the probability of escape of the collision products could be directly calculated from the values of $\langle\sigma\rangle$. The experimentally determined values of the cross sections, angu-

lar distributions, and kinetic energies of the products of the simpler nuclear reactions could then be compared with the results of a "one-step" calculation. In the calculation of Metropolis *et al.*, these reactions are treated along with all the other reactions that occur.¹

The values of $\langle\sigma\rangle$ have been previously estimated over a restricted range of incident energies, usually limited to cases in which the scattering is isotropic in the center-of-mass system.³⁻⁹ Because of their usefulness, we present analytical expressions for $\langle\sigma\rangle$ and for various quantities related to the momentum of the struck nucleons in allowed collisions. Here we consider only isotropic elastic scattering in which the free-particle cross section remains constant or varies inversely with energy.

The analytical solution of the general case is very complex. For this reason we have computed $\langle\sigma\rangle$ and the properties of the struck nucleons, by means of the Monte Carlo method, for incident energies from 10 Mev to 6 Bev at several values of the Fermi energy.¹⁰

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⁸ R. M. Sternheimer, Phys. Rev. **106**, 1027 (1957).

⁹ J. R. Fulco, Phys. Rev. **114**, 374 (1959).

¹⁰ T. P. Clements and L. Winsberg, University of California Radiation Lab. Rept. UCRL-8982 (1960) (unpublished). This report contains a more complete description of the results than can be presented here.

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² D. R. Nethaway and L. Winsberg, Phys. Rev. **119**, 1375 (1960). Further references are given there.