Single Scattering of 2-Bev/c Muons in Nuclear Emulsions*

C. Y. Kim, S. Kaneko,† Y. B. Kim, G. E. Masek, and R. W. Williams
Department of Physics, University of Washington, Seattle, Washington
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The single scattering of high-energy muons from emulsion nuclei was measured using a monoenergetic beam of muons produced at the Berkeley bevatron. The median momentum of muons was 2.00 ± 0.03 Bev/c with a spread of no more than $\pm3.5\%$. Nuclear emulsion stacks were exposed to this muon beam behind the targets used for a counter experiment. The results of the counter experiment were reported earlier. The muon tracks recorded in nuclear emulsions were followed by a special fast-scanning technique, and a total of 682 single scattering events were found from 743 meters of track following. For the muon beam accepted in the emulsion scanning, the pion contamination was measured to be $1.3\pm0.2\%$. These pions contribute to the integral muon scattering data by about 3% for scattering angles greater than 1°. The observed scattering distribution which extends up to 3° scattering angle, or momentum transfer of about 100 Mev/c, is in good agreement with the electromagnetic theory predictions and complements the counter experiment which covered momentum transfers of 70–400 Mev/c.

I. INTRODUCTION

THE nuclear scattering of 2-Bev/c muons produced at the Berkeley bevatron was measured by means of scintillation counter hodoscopes and nuclear emulsions. In the counter experiment, the results of which were reported in an earlier paper,[1] the cross sections of muon scattering in carbon and lead were measured up to elastic momentum transfers of 400 Mev/c. There it was concluded that the observed cross sections are in good agreement with the electromagnetic theory predictions, and do not support the "anomalous" scattering reported by some cosmic-ray muon experiments.[2] The apparatus used in the counter experiment was not very sensitive to small angle scattering corresponding to momentum transfers of less than about 100 Mev/c. This region of low momentum transfers has been covered separately by investigating the nuclear emulsion stacks exposed to the same muon beam. Here again, the conclusion reached by the counter experiment is substantiated, and no evidence for the "anomalous" scattering is found. The details of this emulsion experiment are reported in the present paper.

In the emulsion experiment, muon scattering events are identified from sudden deflections of the beam tracks. To detect such events efficiently, the beam tracks were followed individually by a special fast-scanning technique described in Sec. II. In this method, large-angle scattering events can be detected as efficiently as small-angle events, but the expected cross section diminishes very rapidly for large-angle scattering. For this reason, the scattering events, obtained in this experiment from 743 meters of track following, are mostly of small angles, corresponding to momentum transfers of less than about 100 Mev/c. In emulsions, the target nuclei cannot be identified for individual scattering events; therefore, the observed scattering distribution is compared to the theoretical predictions averaged over all emulsion nuclei. On the other hand, the emulsion experiment is characteristic in that each event is identified visually at the point of scattering, and hence, no correction is required for multiple scattering. This advantage is particularly significant in the region of low momentum transfers.[3]

II. EXPERIMENTAL

Exposure

Stacks of G-5 stripped nuclear emulsions (2 in.$\times$4 in. $\times 600\,\mu$) were exposed to the same muon beam used in the counter experiment. This muon beam was produced at the Berkeley bevatron from 3.5-Bev/c negative pions decaying in flight. The characteristics of the beams were described in the earlier paper on the counter experiment.[1] The median momentum of the muon beam was 2.00 ± 0.03 Bev/c with a spread of no more than $\pm3.5\%$. The locations of emulsion stacks in relation to the counter experiment apparatus are shown in Fig. 1. The muon beam entering the stack between the iron absorbers is degraded in momentum to about 1.5 Bev/c, with the pion contamination reduced by a factor of about 10. This reserve stack, however, was not used in the analysis since the pion contamination in the 2-Bev/c plates was found sufficiently low. At the same locations, another set of emulsion stacks were exposed to 2-Bev/c negative pions. These plates were used in the analysis to compare the interactions of muons and pions at the same momentum. The particle fluxes in emulsions were $8.1\times10^3/\text{cm}^2$ for the 2-Bev/c muon plates and $17.4\times10^3/\text{cm}^2$ for the 2-Bev/c pion plates. Beam collimation was good in both sets of plates, and 80% of the beam particles were found within 1° of the beam direction.

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† On leave of absence from Osaka City University, Osaka, Japan.
[1] G. E. Masek, L. D. Heggie, Y. B. Kim, and R. W. Williams, Phys. Rev. **122**, 937 (1961).
[2] For a summary, see G. N. Fowler and A. W. Wolfendale, *Progress in Elementary Particle and Cosmic-Ray Physics* (North Holland Publishing Company, Amsterdam, 1958), Vol. 4, p. 123.
[3] In our counter experiment, the lead scattering data were compared with the distribution calculated by Cooper and Rainwater (reference 4) which explicitly includes the effect of multiple scattering. In the carbon scattering data, however, the multiple scattering correction was negligible at high momentum transfers.

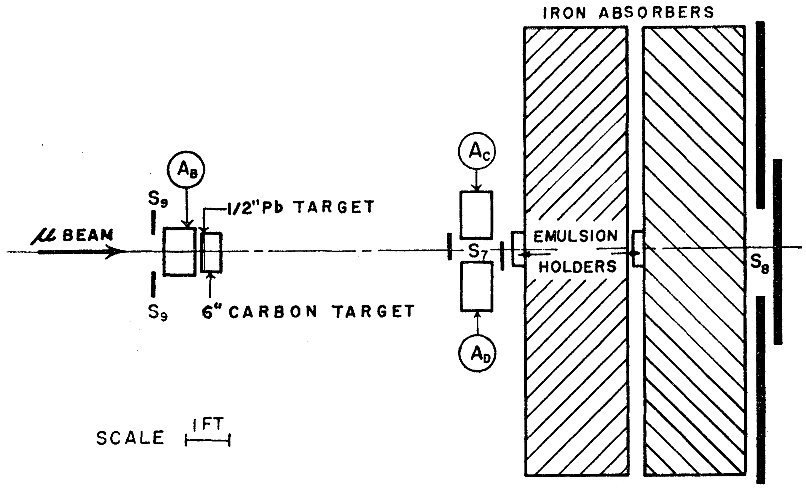

FIG. 1. Experimental arrangement for emulsion exposure. The locations of emulsion stacks are shown in relation to the counter experiment apparatus. S's are scintillation counters and A's are scintillation counter hodoscopes.

Scanning

The processed emulsion plates were scanned for single scattering of muons from emulsion nuclei. Such scattering events appear as sudden deflection of a minimum track and could be detected efficiently only by on-track following. The total length of muon tracks followed in this experiment is 743 meters. The scanning time, however, was greatly economized by the adoption of a fast scanning method described below.

In this method the x axis of a microscope stage is aligned parallel to the beam direction, and the stage is moved rapidly so that grains of a minimum track, kept in focus by the z adjustment, appear as a continuous line. As the y coordinate of the microscope is kept fixed during this rapid stage motion, the line image of an inclined track moves slowly in the field of view along the y axis with a speed $v_y = v_x \alpha_x$, where v_x is the speed of the stage motion and α_x is the projected angle of track inclination with respect to the x axis. Any change in α_x due to scattering can be recognized from the sudden change in v_y. Scattering events with projected angle greater than about 0.5° are readily detected by this method. When the track being followed goes out of the field of view, the stage motion is stopped and another track is selected for scanning. Since only parallel tracks ($\alpha_x \leq 1°$, dip angle$\leq 4°$) are accepted for scanning, the track length followed is given essentially by the distance of the stage motion. Including the time for track changeovers, which averaged about 4 times for one complete stage motion of 7 cm, the scanning speed was 0.5 to 1 m/hr. This is about 5 times faster than the conventional track following.

Each scattering event detected by the fast scanning was re-examined, and the projected scattering angle, $\alpha = \Delta\alpha_x$, was measured with an accuracy of 0.1°. Events with $\alpha < 0.5°$ were not included in the data, since the detection efficiency for these events was very low. Spurious scattering due to local emulsion distortions as well as low-energy electron scattering were eliminated by this re-examination.

In the fast scanning, the track density per field of view was kept at a very low level in order to avoid possible detection biases arising from track confusion. The scanners were specifically cautioned against the loss of a track, which might have resulted from scattering. When the scanner lost a track he was following, he was instructed to reverse the stage motion and relocate the original track. Detection biases arising

TABLE I. The mean free paths of stars and scattering events observed in the 2-Bev/c pion and muon plates. The numerical values given refer to track length in meters/number of events.

Plates and events	Fast track scanning	Slow track scanning	Area scanning
Pion plates[a]			
Stars	44.8/ 95=0.47±0.05	14.1/37=0.38±0.06	
Scattering with $\alpha \geq 1°$	44.8/ 45=1.0 ±0.15	14.1/15=0.94±0.24	
Stars and scattering	44.8/140=0.32±0.03	14.1/52=0.27±0.04	
Stars with $N_h \geq 3$			101/110=0.92±0.09
Muon plates			
Stars with $\alpha_x \leq 1°$	743/ 11=68±20		
Scattering with $\alpha \geq 1°$, $\alpha_x \leq 1°$	743/158=4.7±0.4		
Stars with $N_h \geq 3$			4460/70=64± 8
Stars with $N_h \geq 3$, $\alpha_x \leq 1°$			3570/42=85±13

[a] Pion track lengths shown here were corrected for 9% muon contamination in the pion plates.

TABLE II. The observed and predicted numbers of single scattering for 2-Bev/c muons traversing 743 meters of nuclear emulsions. The numbers refer to the scattering events with the projected scattering angle of α or greater.

α in degrees	0.5	0.7	1.0	1.2	1.5	2.0	2.5	3.0
Observed number	682	421	158	84	37	11	2	1
Predicted from extended nucleus	923	403	152	86	39	11	3.9	1.7
Predicted from point nucleus	1147	585	286	199	127	71	46	32

from other uncontrollable sources were estimated by the following tests.

(i) Using 2-Bev/c pion plates, the fast scanning was compared to a slow scanning in its efficiency of detecting star events and scattering events with $\alpha \geq 1°$. For these events, the detection efficiency of the slow scanning is almost 100%. The observed numbers of events and the corresponding mean free paths are shown in Table I. Including both the star and scattering events, the detection efficiency of the fast scanning relative to the slow scanning is (0.27 ± 0.04) m/(0.32 ± 0.03) m$=85\pm14$%. Although the statistics are rather limited, the detection efficiency for the scattering events alone appears somewhat better.

(ii) A random sample of 100 scattering events detected by the fast scanning was selected for rescanning. Each event was followed under the same criteria as applied to the fast scanning. Only 5 events with $\alpha=0.5°$ were missed in this rescanning. Since 24 events with $\alpha=0.5°$ were contained in the sample, this test indicates that the detection efficiency is about 80% at 0.5° and close to 100% at larger scattering angles. These figures are somewhat optimistic in that those scattering events which are intrinsically difficult to detect would have not been included in the sample.

From these tests, it is felt that the detection efficiency of the fast scanning employed in the present experiment is about 90% for scattering with $\alpha \geq 1°$.

Beam Impurities and Background Events

The pion contamination in the muon beam can be determined by observing interactions produced predominantly by the pions. In the emulsion experiment, this was done by observing star events produced by the beam particles. The star events observed in different ways are listed in Table I. Using these data and other facts, the pion contamination in the muon beam accepted for scanning ($\alpha_x \leq 1°$) are obtained in a number of different ways.

(i) In the area scanning, only those stars which had three or more heavy prongs, or $N_h \geq 3$, were selected. In the muon plates, such stars were observed with a mean free path of 85 ± 13 m for the beam particles with $\alpha_x \leq 1°$. Attributing all of these stars to the pions, for which the mean free path to produce such stars was measured from the area scanning of the pion plates to be 0.92 ± 0.09 m, one obtains the pion contamination in the muon beam as $0.92/85=(1.1\pm0.2)$%.

(ii) In the fast scanning of muon plates, stars were observed with a mean free path of 68 ± 20 m. On the other hand, the fast scanning of the pion plates gave a mean free path of 0.47 ± 0.05 m for the similar stars. The pion contamination obtained from these two numbers is $0.47/68=(0.7\pm0.2)$%.

(iii) In the counter experiment, the pion contamination of the muon beam striking the target was measured to be (3.4 ± 0.2)%. In passing through the target and scintillation counter material in front of the emulsion, about 30% of the pions are filtered out of the beam. In the beam particles with $\alpha_x \leq 1°$ accepted for scanning, the pion intensity is further reduced to about $42/70=60$%, while 80% of the muons remain. The pion contamination deduced in this way gives a value $3.4\%\times0.7\times0.6/0.8=(1.8\pm0.4)\%$.

From the three values obtained above, the pion contamination of the muon beam used in the emulsion experiment is determined to be (1.3 ± 0.2)%. Diffraction and Coulomb scattering of these pions contribute to the muon scattering data. This contribution is calculated to be about 3% for scattering events with $\alpha \geq 1°$. This number can be checked in a different way from the data given in Table I. In the fast scanning of pion tracks, the stars and scattering with $\alpha \geq 1°$ were observed in a ratio 95:45. Since 11 stars were observed in the muon track scanning, $11\times(45/95)=5.2$ pion scattering events would contribute to the 158 muon scattering observed. The pion contribution so deduced is again about 3%.

Electrons in the primary muon beam were effectively filtered out by a 0.25-in. lead degrader left in front of the last bending magnet. Knock-on electrons produced in the counter experiment apparatus may get into the emulsions. These electrons are, however, of low energy (less than 300 Mev), and can be readily distinguished from the 2-Bev muons. Background events due to other possible sources, such as μ-e decays, π-μ decays, and scattering of K^-, p^- are completely negligible in the present experiment.

III. RESULTS AND DISCUSSION

In the course of following 743 meters of muon tracks, a total of 682 single scattering events were found. These events are interpreted as 2-Bev/c muons scattered from emulsion nuclei. The numbers of observed events with the projected scattering angle greater than a given value α are shown in Table II. The data are also shown in Fig. 2 as an integral scattering distribution vs α. The vertical flags indicate statistical errors only. In the abscissa is also shown $q'=2p\sin(\alpha/2)=4\sin(\alpha/2)$ Bev/c. As the space scattering angle θ is always greater

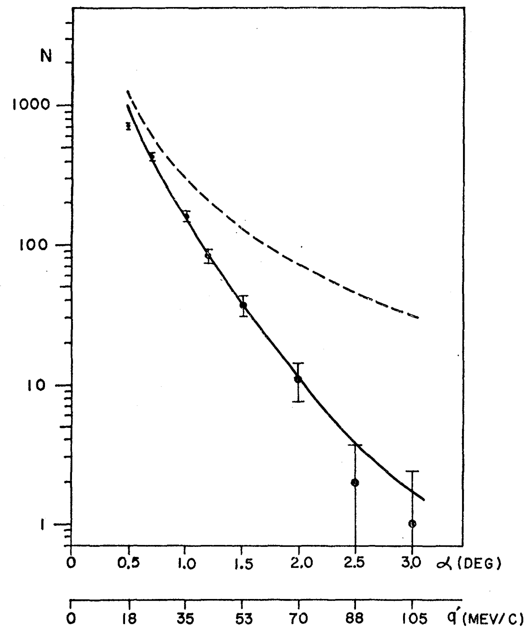

FIG. 2. The experimental and theoretical integral scattering distributions of 2-Bev/c muons in the traversal of 743 meters of nuclear emulsions. α is the projected scattering angle and $N(\alpha)$ is the number of single scattering events with the projected scattering angle of α or greater. q' is the momentum transfer defined in the text. The errors shown on the experimental points are statistical only. The dashed curve is the theoretical predictions based on the point nucleus model, and the solid curve is that based on the extended nucleus model.

than α, the elastic momentum transfer, $q=2p\sin(\theta/2)$, involved in the scattering is larger than q'.

Of the sources of nonstatistical errors discussed in the previous section, only two items need to be cited here. (a) The detection efficiency is low for $\alpha\leq 0.5°$, but it is estimated to be about 90% for $\alpha\geq 1°$. (b) The pion contamination was measured to be $(1.3\pm0.2)\%$. Diffraction and Coulomb scattering of these pions are contained in the observed distribution by about 3% at $\alpha=1°$, and about 20% at $\alpha=2°$. Since these two corrections tend to cancel each other, the net nonstatistical error in the observed scattering distribution is about 10% for $\alpha\geq 1°$.

The observed scattering distribution is now compared with the theoretical predictions based on two different models of nuclear charge distribution: the point nucleus and the extended nucleus.

(A) *Point Nucleus.* For a muon traversing 1 cm of nuclear emulsions, the differential probability of single scattering from emulsion nuclei is written in the form

$$dP(\theta)=\sum_i N_i\sigma_i(\theta)d\Omega, \tag{1}$$

where θ is the scattering angle in space, N_i is the number of atoms per cc, and i refers to the individual elements constituting the emulsions. For a point nucleus, $\sigma(\theta)$ for any element is given in the Born approximation by the Mott formula:

$$\sigma(\theta)=\tfrac{1}{4}Z^2\left(\frac{e^2}{pv}\right)^2\frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}\left[1+\frac{2pv}{Mc^2}\sin^2(\theta/2)\right]^{-1}. \tag{2}$$

Here p and v are, respectively, the momentum and velocity of the incoming muon. The atomic number Z and the mass M refer to the target nucleus in question. The nuclear recoil term involving M is negligible in the present case. After the conversion of space angle θ to the projected angle α, (1) can be expressed in the form

$$dP(\alpha)=G(\alpha)d\alpha, \tag{3}$$

and the integral distribution for single scattering is obtained by

$$P(\geq\alpha)=\int_\alpha^\pi G(\alpha)d\alpha. \tag{4}$$

The numbers of scattering events expected from this calculation are entered in Table II and shown in Fig. 2 by the dashed curve.

The assumption of a point nuclear charge is very unrealistic in the present scattering experiment. For scattering events with $\alpha\geq 1°$, the impact parameters are well within the emulsion nuclear radii, and the calculation based on this assumption will grossly overestimate the scattering probability. This assumption is, nevertheless, considered here since some of cosmic-ray experiments have reported that the muon scattering is anomalously large and closely follows the predictions based on the point nuclear charge. From Fig. 2, it is seen that this is not the case with the present experiment. The experimental curve is widely different from the point nucleus predictions both in the magnitude and shape.

(B) *Extended Nucleus.* For the range of momentum transfers covered in the present experiment, the structure effects of nuclear charge distribution can be adequately taken into account by modifying the basic cross section (2) to

$$\sigma(\theta)\rightarrow\sigma(\theta)F_p^2(q)F_N^2(\theta). \tag{5}$$

The nuclear form factor F_N is not calculable in its precise form, and the approximate expression given by Cooper and Rainwater[4] is used in the present calculation. As for the proton form factor $F_p(q)$, the expression given by Hofstadter *et al.*,[5]

$$F_p=[1+(q^2a^2/12)]^{-2}, \tag{6}$$

is used with $a=0.7$ fermi and $q=2p\sin(\theta/2)$. The numbers of scattering events calculated on the basis of this modified cross section are shown in Fig. 2 by the solid curve.

Provided that the muon scattering results from the

[4] L. N. Cooper and J. Rainwater, Phys. Rev. **97**, 492 (1955).
[5] R. Hofstadter, F. Bumiller, and M. R. Yearian, Revs. Modern Phys. **30**, 482 (1958).

electromagnetic interactions only, the theoretical calculation is expected to give an accuracy of about 10%. Within the limits of this theoretical accuracy and the experimental errors already quoted, the experimental scattering distribution is in good agreement with the theoretical predictions.

The experiments on high-energy muon scattering prior to our own work were performed with cosmic-ray muons. These cosmic-ray experiments were cited in our previous paper,[1] and it was pointed out that they are not in agreement with each other regarding the existence of an anomaly in muon scattering. In the latest cosmic-ray experiment by Fukui *et al.*,[6] muons were identified, and their momenta were estimated by requiring the muons to traverse a thick block of iron and stop and decay in a thin layer of carbon. The scattering of these muons was measured by means of a cloud chamber containing lead plates. These authors found no anomaly in the range of momentum transfers up to about 100 Mev/c and cast serious doubt on the anomaly reported by some of the earlier cosmic-ray experiments. Our own work confirms the results of Fukui *et al.*, while avoiding many of the uncertainties connected with cosmic-ray muon experiments.

In summary, the present emulsion experiment supports the conclusion already reached by the counter experiment. The scattering distributions of high-energy muons observed in our experiments are in good agreement with the expected electromagnetic predictions and give no evidence for an anomaly.

[6] S. Fukui, T. Kitamura, and Y. Watase, Phys. Rev. **113**, 315 (1959).

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Cusp Phenomena in the Region of Two Neighboring Thresholds*

J. SUCHER, G. A. SNOW, AND T. B. DAY
Physics Department, University of Maryland, College Park, Maryland

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Previous discussions of the cusp phenomena at the threshold for a new reaction are extended to the case of two neighboring thresholds. The S matrix is constructed from an $n\times n$ κ matrix in such a way as to ensure that the physical S matrix is unitary when only r of the n channels are open. As a special application, the amplitude for the reaction $\pi^-+p\rightarrow\Lambda^0+K^0$ is studied in the region of the Σ^- and Σ^0 thresholds.

I. INTRODUCTION

RECENTLY, considerable interest has arisen in the behavior of various cross sections near the threshold for a new reaction. In particular, the possibility of determining the relative parity of the hyperons by study of the production amplitude in a two-body reaction has been pointed out by several authors.[1] Experiments to test this possibility and, hopefully, to measure the (Λ^0,Σ^0) relative parity are in progress.[2]

The previous analyses[3] of the cusp phenomena have been made for a region near a single threshold. However, in the case of actual interest being studied[2] there are really two neighboring thresholds separated in energy by the mass difference $(m_{\Sigma^-}+m_{K^+})-(m_{\Sigma^0}+m_{K^0})$.[4]

It is the purpose of this note to generalize the previous treatments to the case of neighboring thresholds. In so doing, we develop a simple and transparent treatment of the requirements of unitarity on the S matrix in the various energy regions. In Sec. II, the general formalism is set up. For illustration, the well-known results for the case of a single threshold are rapidly rederived in Sec. III. Section IV treats the case when there are two neighboring thresholds, and the results are illustrated and applied to the case of the reaction $\pi^-+p\rightarrow\Lambda^0+K^0$ in the neighborhood of the two ΣK thresholds. Coulomb effects in the (Σ^-,K^+) channel are ignored. A brief summary is given in Sec. V.

II. GENERAL FORMALISM[5]

For simplicity, we consider the case of n coupled two-body channels; each channel consists of one particle of spin zero, and one of spin one-half. We are interested in the submatrix of the S matrix with $J=\frac{1}{2}$ and definite parity.

We adopt the normalization such that the differential cross section for the reaction from channel i to channel j,

* This work was supported in part by the Air Force, the Atomic Energy Commission, and by a National Science Foundation grant.

[1] See, for example, R. K. Adair, Phys. Rev. **111**, 632 (1958); A. N. Baz and L. B. Okun, Soviet Phys.-JETP **35**, 526 (1959).

[2] A. M. Schwartz, Bull. Am. Phys. Soc. **5**, 516 (1960); F. S. Crawford, Jr., Bull. Am. Phys. Soc. **5**, 516, (1960).

[3] See, for example, R. G. Newton, Ann. Phys. **4**, 29 (1958), wherein there is a complete list of earlier references; R. G. Newton, Phys. Rev. **114**, 1611 (1959).

[4] W. H. Barkas and A. H. Rosenfeld give 0.6±0.8 Mev. *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 878. F. S. Crawford, Jr., gives 2.2±0.6 Mev, reference 2 above.

[5] The general formalism described herein follows closely the formalism developed by Dalitz and Tuan to treat low-energy K^- p interactions. See R. H. Dalitz and S. F. Tuan, Ann. Phys. **3**, 307 (1960).