$K^- - D$ Absorption and a $\pi - \Sigma$ Resonance

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The branching ratios for the $K^- + d \rightarrow \pi + Y + N$ reactions are compared with those for the $K^- + p \rightarrow \pi + Y$ reactions. Certain of these deuteron branching ratios are shown to be independent of hyperon-nucleon final state interaction and are inconsistent with the proton branching ratios. The most likely explanation of the discrepancy is the presence of an isospin zero $\pi - \Sigma$ resonance a few Mev below the $K^- + p$ threshold. This resonance is consistent with the $(b-)$ set of $K^- - p$ scattering lengths determined by Dalitz and Tuan.

DALITZ and Tuan have analyzed K^--proton scattering processes in a scattering length approximation and have found four sets of scattering lengths which fit the data.[1] Two of these solutions (the "minus" solutions) have negative real parts for the scattering lengths. Dalitz and Tuan point out that these "minus" solutions could lead to a resonance in pion-hyperon scattering at an energy slightly below the K^--p threshold.[2] One suggested way of observing this resonance was to look at the energy spectra of the particles resulting from K^--deuteron interactions at rest, since the resulting Y-π center-of-mass energy will be a few Mev below the K^--p threshold because of nucleon recoil and the deuteron binding energy. However, Karplus and Rodberg have shown that strong final-state hyperon-nucleon scattering will greatly alter the energy spectra, particularly that of the nucleon.[3]

The purpose of this paper is to show that, in the absence of such a resonance, one can derive approximate sum rules which are independent of the hyperon-nucleon force. These sum rules relate the branching ratios for K^- absorption in deuterium to those in hydrogen, and do not fit the experimentally observed branching ratios.[4] The $(b-)$ solution of Dalitz and Tuan, which has an isotopic spin zero resonance, easily explains the observed branching ratios.

In deriving the approximate sum rules we wish to consider eigenstates of the nucleon-hyperon scattering. If charge independence is valid, the amplitudes for the production of the seven π-Y-N charge states by K^- absorption in deuterium may be written as linear combinations of three independent amplitudes, T_f, which are labeled by the isospin of the hyperon and the total isospin of any pair of particles. In hydrogen there is no nucleon in the final state so the natural labels are those referring to the π-Y isospin. In deuterium the Y-N isospin is the convenient label since this quantity is conserved by the final state interaction. The connections between the various amplitudes are given in Table I.

TABLE I. The production amplitudes for the seven π-Y-N charge states in terms of the three independent amplitudes, T_f, which refer to pure isospin states of πY (Col. 2) or of YN (Col. 3). The data given at the Kiev Conference (see reference 4) are listed in Cols. 4 and 5. The neutron data (in parentheses) were obtained by charge independence from the proton data listed. The $\pi^-\Lambda^0 p$ events have been subdivided into those in which the Λ was produced directly and those in which a Σ was produced which was charge-exchange scattered to become a Λ.

Charge state	Production amplitudes labeled by πY isospin	YN isospin	Events observed Deuterium	Hydrogen
$\pi^+\Sigma^- n$	T_0+T_1	$T_{\frac{3}{2}}$	353 ± 19	45 ± 1
$\pi^-\Sigma^+ n$	T_0-T_1	$(\frac{1}{3}T_{\frac{3}{2}}+\frac{2}{3}T_{\frac{1}{2}})$	354 ± 30	21 ± 1
$\pi^0\Sigma^0 n$	$-T_0$	$-(\frac{2}{3}T_{\frac{3}{2}}+\frac{1}{3}T_{\frac{1}{2}})$	470 ± 22	27 ± 2.5
$\pi^0\Lambda^0 n$	$-T_\Lambda$	$-T_\Lambda$		7 ± 1.5
$\pi^0\Sigma^- p$	$\sqrt{2}T_1$	$\sqrt{2}(\frac{1}{3}T_{\frac{3}{2}}-\frac{1}{3}T_{\frac{1}{2}})$	53 ± 7	(12 ± 5)
$\pi^-\Sigma^0 p$	$-\sqrt{2}T_1$	$-\sqrt{2}(\frac{1}{3}T_{\frac{3}{2}}-\frac{1}{3}T_{\frac{1}{2}})$	59 ± 9	(12 ± 5)
$\pi^-\Lambda^0 p$	$\sqrt{2}T_\Lambda$	$\sqrt{2}T_\Lambda$	357 ± 21	(14 ± 3)
			161 direct	
			196 indirect	

Consider first the state $\pi^+ + \Sigma^- + N$ which is a pure state of Y-N isospin $\frac{3}{2}$. Using the final state interaction formalism of Karplus and Rodberg,[3] the matrix element for this state is given by $T_{\frac{3}{2}} = \langle \chi^{(-)} | M_{\frac{3}{2}} | \phi_i \rangle$, where ϕ_i is the wave function describing the deuteron and the K^- meson bound in a Coulomb orbit, $M_{\frac{3}{2}}$ is the exact transition amplitude for the reaction $K^- + p \rightarrow \pi^+ + \Sigma^-$, and $\chi^{(-)}$ is the product of the ingoing wave solution, $g^{(-)}(\mathbf{r}_\Sigma - \mathbf{r}_N)$, of the hyperon-nucleon scattering problem and plane waves describing the motion of the pion and the nucleon-hyperon center of mass. Because of the large size of the Coulomb orbit, the K^- meson wave function may be assumed constant if, as is now believed,

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[1] R. H. Dalitz and S. F. Tuan, Ann. phys. **8**, 100 (1959); **10**, 307 (1960).

[2] R. H. Dalitz and S. F. Tuan, Phys. Rev. Letters **2**, 425 (1959).

[3] Robert Karplus and Leonard S. Rodberg, Phys. Rev. **115**, 1058 (1959); see also, T. Kotani and M. Ross, Nuovo cimento **14**, 1282 (1959).

[4] Luis Alvarez, Ninth Annual Conference on High-Energy Physics, at Kiev, 1959 (unpublished); also, Lawrence Radiation Lab. Rept. UCRL-9354, 1960 (unpublished).

the capture occurs from S states. The initial state wave function, ϕ_i, is then just the deuteron wave function, $\phi_d(\mathbf{r}_p-\mathbf{r}_n)$. If there are no resonances $M_{\frac{3}{2}}$ will not be very energy dependent and can be replaced by a constant or, in configuration space, by a delta function times a constant. This constant should be the amplitude observed in K^- absorption by hydrogen. With these approximations the amplitude $T_{\frac{3}{2}}$ becomes

$$T_{\frac{3}{2}}=M_{\frac{3}{2}}\int g^{(-)}(\mathbf{r})^* e^{-i\mathbf{q}\cdot\mathbf{r}}\phi_d(\mathbf{r})d^3r, \qquad (1)$$

where $\mathbf{q}=\mathbf{p}M_N/(M_N+M_\Sigma)$, $\mathbf{p}$ is the pion momentum, and M_N and M_Σ are the masses of the nucleon and hyperon, respectively. The total transtition rate will then be given by

$$W_{\frac{3}{2}}=A\int |T_{\frac{3}{2}}|^2\rho_f, \qquad (2)$$

where A is a constant which is independent of the final state and ρ_f is the density of final states. Let $\mathbf{k}$ be the relative momentum of the nucleon and hyperon. The momenta $\mathbf{k}$ and $\mathbf{p}$ are related through the total energy equation,

$$E=M_Y+(p^2+M_\pi{}^2)^{\frac{1}{2}}+\frac{p^2}{2(M_N+M_Y)}+\frac{k^2}{2}\frac{M_N+M_Y}{M_NM_Y}. \qquad (3)$$

After evaluating $T_{\frac{3}{2}}$ for typical functions $g^{(-)}(\mathbf{r})$ one sees that when $1/k$ becomes smaller than the radius of the deuteron, the value of $T_{\frac{3}{2}}$ becomes quite small, or more precisely $T_{\frac{3}{2}}\approx 0$ for $k>150$ Mev/c. The pion momentum is a slowly varying function of k for $k<150$ Mev/c and has essentially the same value as in the free proton reaction. Therefore, ρ_f becomes $\rho_\Sigma d^3k$, where ρ_Σ is approximately the two-particle density of final states which determines the transition rate in hydrogen. Then, again because $T_{\frac{3}{2}}$ falls off rapidly with increasing k, the integral over momenta in the formula for $W_{\frac{3}{2}}$ [Eq. (2)] can be extended to infinity. The only dependence of the integrand on k is in the wave functions $g^{(-)}(\mathbf{k},\mathbf{r})$. These functions form a complete set and so by closure we have

$$W_{\frac{3}{2}}=A\rho_\Sigma|M_{\frac{3}{2}}|^2 \times\int\int\int d^3rd^3r'd^3k[\phi_d(\mathbf{r})^*e^{i\mathbf{q}\cdot\mathbf{r}}g^{(-)}(\mathbf{k},\mathbf{r}) \times g^{(-)}(\mathbf{k},\mathbf{r}')^*e^{-i\mathbf{q}\cdot\mathbf{r}'}\phi_d(\mathbf{r}')]=A\rho_\Sigma|M_{\frac{3}{2}}|^2, \qquad (4)$$

which says that the total rate is not changed by the final-state interactions. Numerical evaluation for typical functions $g^{(-)}(\mathbf{k},\mathbf{r})$ shows that the approximations used in applying closure should be valid to within 30%.

For the Y-N isospin $\frac{1}{2}$ case, the function $g^{(-)}(\mathbf{r})$ has two components to take into account the $\Sigma+N\leftrightarrow\Lambda+N'$ transitions. The primary amplitude M also has two components to describe primary production of a Σ or a Λ (denoted by $M_{\frac{1}{2}}$ and M_Λ, respectively). However, the criterion that the hyperon-nucleon momentum k should be small in order to give a large value for the integral in Eq. (1) and the large $\Sigma-\Lambda$ mass difference mean that $M_{\frac{1}{2}}$ and M_Λ will contribute to different regions of the pion momentum spectrum. M_Λ contributes when $p\approx 250$ Mev/c, $M_{\frac{1}{2}}$ contributes when $p\approx 180$ Mev/c, and there is not very much overlap between the two contributions. The pion spectrum associated with $\pi^-+\Lambda+p$ events shows these two peaks quite clearly.[3,4] The pion spectrum associated with Σ particles is forbidden by energy conservation from going beyond 180 Mev/c, so that M_Λ will not contribute to Σ production. This separation of the events coming from primary Σ and Λ production allows one to make use of the sum rules derived by applying closure to the complete set of two component functions $g^{(-)}(\mathbf{k},\mathbf{r})$. The results, in analogy to the $I=\frac{3}{2}$ case, are

$$W_\Lambda=A\rho_\Lambda|M_\Lambda|^2, \qquad (5)$$

$$W_{\frac{1}{2}}=W_{\Sigma\frac{1}{2}}+W_{\Lambda\frac{1}{2}}=A\rho_\Sigma|M_{\frac{1}{2}}|^2, \qquad (6)$$

where the constant, A, is the same as in Eq. (4), $W_{\Sigma\frac{1}{2}}$ refers to the process $K^-+d\rightarrow\pi+\Sigma+N\rightarrow\pi+\Sigma'+N'$ with $\Sigma+N$ in an isospin $\frac{1}{2}$ state, and $W_{\Lambda\frac{1}{2}}$ refers to $K^-+d\rightarrow\pi+\Sigma+N\rightarrow\pi+\Lambda+N'$ events (also called indirect Λ events).

These three sum rules can be combined in various ways. For example, consider the ratio

$$\pi^+:\pi^-:\pi^0=W_{\frac{3}{2}}:(\tfrac{1}{3}W_{\frac{3}{2}}+\tfrac{2}{3}W_{\frac{1}{2}}+2W_\Lambda):(\tfrac{2}{3}W_{\frac{3}{2}}+\tfrac{1}{3}W_{\frac{1}{2}}+W_\Lambda), \qquad (7)$$

which should be independent of final interactions and should, therefore, be predictable from the hydrogen branching ratios. The predicted ratio 45:47:46 is quite different from the observed ratio 353:764:529. [It should be noted that $\pi^0=\frac{1}{2}(\pi^++\pi^-)$ by charge independence alone.] By choosing appropriate combinations of charge states one can also determine the ratio $W_{\frac{3}{2}}:W_\Lambda:W_{\frac{1}{2}}$ which is predicted to be 45:7:27 and is observed to be 353:80:736 which indicates that the predicted $W_{\frac{1}{2}}$ is too small by a factor of about 3.

We have estimated the effects of multiple scattering in the final state in an effort to understand this discrepancy and we find effects of only a few percent. There is also the possibility that the K^--$\bar{K}^0$ mass difference may make our charge independence argument for determining the K^--n amplitudes invalid. This effect was estimated by extrapolating the K^--p amplitudes to high momenta where mass difference effects should be negligible, finding the K^--n amplitude for this high momentum, and extrapolating back to threshold. For this extrapolation we used the formalism of Dalitz and Tuan[1] and their values for the scattering lengths. No effects larger than 20% result. Multiple scattering in the initial state may appreciably alter the total transition rate; however, the effect of such multiple scattering on the branching ratios has been estimated and is less

than 30%. The pion-nucleon energy is well below the 3-3 resonance so that pion-nucleon scattering is negligible. The validity of the impulse approximation itself is shown by the lack of non-mesonic events and by the good agreement of the pion spectrum with the prediction of the impulse model. The conclusion is that the observed ratios cannot be explained by any of these mechanisms.

However, our assumption of constant M in obtaining Eq. (1) is not justified if there is a nearby resonance as is the case for the "minus" solutions of Dalitz and Tuan. Since final state interactions will probably still have little effect on the total rates $W_{\frac{3}{2}}$, $W_{\frac{1}{2}}$, and W_Λ, let us consider only the spectator approximation which neglects final state scattering. If all interactions are described by potentials (this is not essential), the exact transition amplitude T_f is given by

$$T_f = \langle \phi_f | V - V(H-E-i\epsilon)^{-1} V | \phi_i \rangle, \tag{8}$$

where ϕ_f and ϕ_i are plane waves describing the $\pi+Y+N$ and $K+d$ states, respectively, H is the total Hamiltonian, and V is the interaction part of H. For the spectator approximation, baryon interactions with baryons are neglected, and the deuteron is replaced by a distribution of neutron-proton plane wave states. Since one of the nucleons does not interact with any of the other particles, H and E separate into two parts. One part is the kinetic energy of the noninteracting nucleon and of the center of mass motion of the meson-baryon system. This part cancels out in the H-E operator leaving $H'-E'-i\epsilon$ where H' and E' refer to the internal motion of the meson-baryon system. That is,

$$T_f = M_f \int e^{-i(\mathbf{q}+\mathbf{k})\cdot\mathbf{r}} \phi_d(\mathbf{r}) d^3r, \tag{9}$$

where

$$M_f = \langle \phi_{\pi Y} | V - V(H'-E'-i\epsilon)^{-1} V | \phi_{KN} \rangle \tag{10}$$

is the transition amplitude for the reaction $K^- + N \rightarrow \pi + Y$. Since the relative $K^- - N$ momentum is small and the interaction is expected to be of short range, nearly all of the energy dependence of M is given by E', the final $\pi - Y$ relative energy. In deuterium the energy E' must be below the $K^- - N$ threshold by more than the deuteron binding energy because of the recoil energy of the noninteracting nucleon.

Above threshold we have

$$M(E') = M'[1 - ik'(a+ib)]^{-1}, \tag{11}$$

for each πY isospin state, in the "constant K matrix" approximation of Dalitz and Tuan, neglecting the $K^- - \bar{K}^0$ mass difference. The constants a, b, and M' are determined by Dalitz and Tuan from $K^- + p$ reactions and k' is the relative $K^- - N$ momentum corresponding to energy E'. Below threshold we analytically continue M with the prescription $k' \rightarrow i\kappa$, $\kappa > 0$. If a is negative and large in absolute value, a resonance

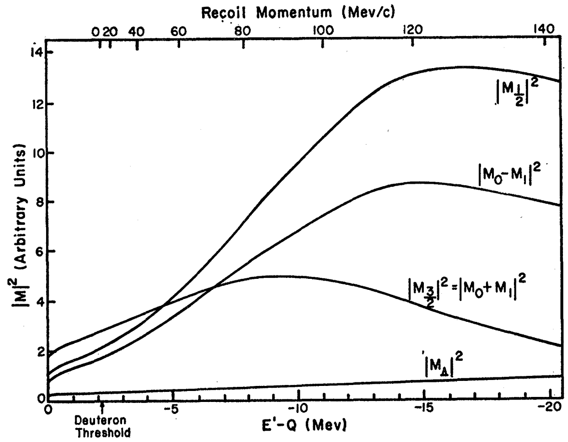

FIG. 1. The amplitudes for the $\bar{K}+N \rightarrow \pi+Y$ reactions are plotted for the $(b-)$ solution of Dalitz and Tuan as a function of πY center-of-mass energy, E', below the physical threshold, Q. The various amplitudes, M, are defined in Eq. (9) and Table I. The upper scale shows the momentum of the recoil nucleon in $K^- + d$ events which have relative πY energy E'.

in $M(E')$ will occur for $\kappa \approx -1/a$. For the $(b-)$ solution of Dalitz and Tuan this resonance would occur in the isospin zero state at an energy only about 10 Mev below threshold. Figure 1 shows the variation of $M_{\frac{3}{2}}$, $M_{\frac{1}{2}}$, and M_Λ as a function of energy below threshold for the $(b-)$ solution. (The $K^- - \bar{K}^0$ mass difference is included, and the sign of the phase difference between M_0' and M_1' is chosen to fit the observed variations in the $\Sigma^- + \pi^+ / \Sigma^+ + \pi^-$ ratio for in-flight $K^- + p$ events; see reference 1.) Note that the rapid change of the phase of M_0 causes a drastic change in the $|M_{\frac{3}{2}}| : |M_{\frac{1}{2}}| : |M_\Lambda|$ ratio and that this change is in the direction required by the deuteron data.

The above method of using the impulse approximation and extrapolating the $\bar{K}+N \rightarrow \pi+Y$ amplitude cannot be strictly correct, as may be seen by examining the implications of unitarity and analyticity near threshold. The energy derivative of the amplitude for production from free nucleons is infinitive at the $\bar{K}+N$ threshold, as may be seen by expanding Eq. (11) in powers of k'. Such a cusp results from the requirements of unitarity and analyticity and, in the region above $\bar{K}+N$ threshold, may be considered as arising from transitions through real $\bar{K}+N$ states. Similarly the amplitude for production from deuterons, calculated as a function of energy, must contain a cusp at the $K^- + d$ threshold resulting from real $K^- + d$ intermediate states. Hence the neglect of the neutron-proton interaction in the intermediate states omits an effect which must be present in the exact expression. Furthermore, if the $K^- + d$ energy were a few Mev above threshold, the calculated distribution of events in $\pi + Y$ energy would have a cusp at the $\bar{K}+N$ threshold [i.e., at $k'=0$ in Eq. (11)]; this cusp results from real $\bar{K}+p+n$ intermediate states and would be reduced if one took into account the tendency of the proton and neutron to stick together. However, the large value of

$|M_{\frac{1}{2}}|:|M_{\frac{3}{2}}|$ computed with the impulse model does not result from the proximity to the singularity at $k'=0$ in Eq. (11) but rather from proximity to the pole at $k'=-i(a+ib)^{-1}$. We adopt the philosophy that the existence and position of this pole and the associated $\pi+Y$ resonance are not sensitive to the details of the extrapolation procedure.

The $(b-)$ solution, extrapolated in the above way, also predicts that for K^- absorption in heavier nuclei the $\Sigma^-+\pi^+/\Sigma^++\pi^-$ ratio given by $|M_0+M_1|^2/|M_0-M_1|^2$, which is 2.2 in hydrogen, should be less than one for energies which are more than about 7 Mev below threshold (see Fig. 1). In heavy nuclei, where final state interactions should be symmetric in Σ^- and Σ^+ except for coulomb effects, the data of the K^--collaboration group[5] show $\Sigma^-+\pi^+/\Sigma^++\pi^-=0.45\pm0.10$. In helium[6] the ratio $\Sigma^-+\pi^++H^3/\Sigma^++\pi^-+H^3$ is 48/79. In the deuteron where final state interactions are not symmetric in that only the Σ^+ can undergo charge exchange scattering, the ratio comes back up to 1.

If the $(a-)$ solution of Dalitz and Tuan were correct, the resonance would be expected to occur in the isospin 1 $\pi-Y$ states. In this case, however, the ratio $|M_{\frac{1}{2}}|:|M_{\frac{3}{2}}|$ can be substantially increased for deuterium only if the resonance energy is appreciably closer to the K^--p threshold than is predicted by the Dalitz and Tuan analysis. The small number of $\pi^0+\Sigma^-+p$ and $\pi^-+\Sigma^0+p$ events also makes an $I=0$ resonance appear more probable than an $I=1$ resonance, although these numbers are not independent of final state interactions. The behavior of the $W_\Lambda:W_{\frac{3}{2}}$ ratio should give an important clue as to the isospin of the resonance. If the $I=1$ amplitude is resonant, W_Λ should be increased while $W_{\frac{3}{2}}$, which has destructive interference between isospins 0 and 1 near the resonance, would not increase very much, as one approached the resonant energy. If isospin zero is resonant, this destructive interference keeps the $|M_\Lambda|:|M_{\frac{3}{2}}|$ ratio roughly independent of energy. The fact that the $W_\Lambda:W_{\frac{3}{2}}$ ratio seems to be about the same in deuterium as in hydrogen is evidence against an $I=1$ resonance. On the other hand, the analysis by the helium bubble chamber group of $K^-+\mathrm{He}^4\rightarrow\pi+Y+\mathrm{H}^3$ and $K^-+\mathrm{He}^4\rightarrow\pi+Y+\mathrm{He}^3$ events indicates that the percentage of directly produced Λ's is much higher than in hydrogen[6], so the present evidence on the $|M_\Lambda|:|M_{\frac{3}{2}}|$ ratio is inconclusive.

One unfortunate feature of a resonance this close to the threshold is that the pion spectrum will be almost the same as predicted by an impulse approximation without the resonance. The nucleon and hyperon spectra will be distorted by the final state interaction, so that it is probably not possible to see the resonance in any of the momentum spectra.

It is concluded from this analysis of the K^--p and K^--d absorption data that there is likely to be a $J=\frac{1}{2}$ resonance (S wave if the K^- is pseudoscalar) in $\pi-Y$ scattering at an energy corresponding to a "bound" state of $\bar{K}+N$. The "binding energy" of this state is less than 20 Mev and the isotopic spin is probably zero.

Note. O. Dahl, N. Horwitz, D. Miller, J. Murray, and P. White [Phys. Rev. Letters **6**, 142 (1961)] have reclassified the $\pi^-\Lambda p$ events into three groups, (a) direct production, (b) internal conversion of Σ's or indirect production, and (c) events associated with the $\pi-\Lambda$ resonance 50 Mev below the K^--p threshold. This resonance may have a simple interpretation in terms of the Dalitz-Tuan (a^-) solution. However, the reclassification of $\pi^-\Lambda p$ events does not greatly alter the experimental $W_{\frac{3}{2}}:W_{\frac{1}{2}}$ ratio, which, as shown above, requires a $\pi-\Sigma$ resonance within about 20 Mev of the K^--p threshold. [For example, if one associates the Y^* production events with the direct $\Lambda+\pi$ production, the reclassification of Dahl, *et al.* leads to a $W_{\frac{3}{2}}:W_\Lambda:W_{\frac{1}{2}}$ ratio of 353:114:635, where the numbers have been normalized to facilitate easy comparison with the ratio following Eq. (7).] Since the zero-range Dalitz-Tuan analysis would predict that $I=1$ $\pi-\Lambda$ and $\pi-\Sigma$ resonances should occur at almost the same energy, it is not possible to explain both the Y^* (at -50 Mev) and the resonance suggested in this paper with the zero-range parameters of Dalitz and Tuan.

[5] K^--collaboration, Nuovo cimento **14**, 315 (1959).

[6] Martin Block and Jack Leitner, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), and private communication.