

experimental result becomes 0.30. At present, however, the magnitude of this effect is not known accurately enough to correct $|\delta|$ and γ .

ACKNOWLEDGMENTS

The author would like to express his sincere thanks to Professor R. D. Sard under whose guidance this experiment was performed and to Dr. J. M. Fowler

who collaborated with Professor Sard and the author. In addition, I would like to thank Professor H. Primakoff for several stimulating discussions throughout the experiment. Special thanks are due to Dr. E. D. Lambe for his advice and assistance in preparing the computer program. I would also like to express my gratitude to Mr. J. D. Miller for his help in the construction and maintenance of the experimental apparatus.

PHYSICAL REVIEW

VOLUME 122, NUMBER 6

JUNE 15, 1961

Angular Distribution of Shower Particles from 1000-Bev Nucleon Alpha Particles on Emulsion Nuclei

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(Received January 10, 1961; revised manuscript received March 1, 1961)

Twenty-eight interactions of α particles were located in a 22-liter stack of nuclear emulsion by tracing back showers of minimum ionization particles to their origins. The angular distributions of 17 α particles with a dip angle $\leq 20^\circ$ are presented. The inelasticity for these 17 interactions shows large fluctuations for individual events and its mean value is 30%. The angular distributions of these α particles have been transferred into a system in which they are roughly symmetric. The degree of anisotropy of the angular distributions is in disagreement with a hydrodynamical model of nucleon-nucleus collisions. The detailed analysis of the angular distribution of composite stars for events with a high degree of anisotropy of secondaries in the center-of-mass system shows that the shape of the angular distribution is in agreement with the predictions of the "two-fireball" model of multiple meson production, both for nucleon-nucleon and nucleon-nucleus collisions.

1. INTRODUCTION

AT present there are not enough experimental data available to show which one, among the several existing theories of Fermi,¹ Heisenberg,² Landau,³ and the "two-center model,"⁴⁻⁶ can best explain very high energy nucleon-nucleon interactions. The main difficulty is that the energy available in accelerators is not yet high enough (~ 30 Bev) to make any systematic studies. So, for the study of high-energy nuclear interactions, cosmic rays are the only source of high-energy particles. Since the flux of the high-energy particles is much less than of the low-energy particles in cosmic radiation, our present experimental knowledge is based on a small number of events.

In order to observe high-energy interactions, nuclear emulsion is generally used. It has the advantage of being light in weight and can thus be easily flown to very high altitudes. The only disadvantage in using nuclear emulsion is that it consists mainly of heavy elements, and consequently very few ($< 5\%$) of the collisions of the primary particles with the emulsion

nuclei are with free protons. Thus in nuclear interactions in emulsion, we are concerned primarily with nucleon-nucleus collisions, in which several target nucleons may take part. The generally accepted practice for selecting "jets" which are due to nucleon-nucleon interaction is to select those events which have no heavy prongs at all or only those events which have not more than three or four heavy prongs. This is believed to be the best approximation to a collision between a nucleon and a free proton or between a nucleon and only one bound nucleon at the periphery of a heavy nucleus without any visible excitation of the rest of the nucleus. We may point out here that these criteria used for the nucleon-nucleon interaction are not quite safe, because the emission of neutrons alone as a result of some excitation of a nucleus cannot be excluded entirely. As yet there exists no procedure by which it could be ascertained without any doubt whether the collision is a nucleon-nucleon interaction or not.

About one-third of the primary jets produced by cosmic ray particles are initiated by α particles rather than protons. In such collisions it is reasonable to regard the incident α particle as four separate nucleons, each of which, when it interacts, emits mesons from its individual center-of-mass system. On the average, these separate center-of-mass systems will have the same velocity in the laboratory system, and the assumption is made of symmetrical emission of mesons from a com-

¹ E. Fermi, *Progr. Theoret. Phys. (Kyoto)* **5**, 570 (1950); *Phys. Rev.* **81**, 683 (1951).

² W. Heisenberg, *Z. Physik* **126**, 569, (1949); **133**, 65, (1952).

³ S. S. Belenki and L. D. Landau, *Nuovo cimento Suppl.* **3**, 15 (1956).

⁴ P. Ciok, T. Coghien *et al.*, *Nuovo cimento* **8**, 166 (1958).

⁵ G. Cocconi, *Phys. Rev.* **111**, 1699 (1958).

⁶ K. Niu, *Nuovo cimento* **10**, 994, (1958).

mon center-of-mass system. According to the above suggestion we can regard the interactions of the incident α particles as a superposition of the interactions of the individual nucleons. The observed ratio of multiplicities for proton and α particle primaries suggests that on the average, two nucleons^{7,8} of the α particle undergo interaction. The multiplicity of proton-induced jets of median energy 3 Tev (3 Tev=3000 Bev) is about 20 and the multiplicity for primary α -induced jets is about 40. In the present paper we shall consider the angular distribution of shower particles generated in nuclear interactions of high-energy ($\sim 10^{12}$ ev per nucleon) α particles with the nuclei of photographic emulsion. We shall separate these events in different categories according to their N_h (grey and black prongs) and n_s (light prongs) values and shall try to fit the angular distribution with a certain nuclear model. Our object is to see how far we are justified in using the jets produced by α particles for studying the properties of nucleon-nucleon or nucleon-nucleus collisions.

2. EXPERIMENTAL METHOD

A stack of 22 liters of Ilford G-5 emulsion, consisting of 200 pellicles, 60×30 cm, 600μ thick was exposed to cosmic radiation on a Skyhook balloon flight over Texas with the 60-cm side pointing to the vertical direction. The flight remained at an altitude of 116 000 ft for 13 hr. After development, each emulsion was cut into eight pieces of dimensions 15×15 cm.

Every high-energy nuclear event is accompanied by a narrow electromagnetic cascade, arising from the decay of π^0 mesons, which can be found relatively easily. The scanning procedure was therefore designed to detect these cascades and to trace them back to their origin. Each emulsion was scanned along a line about 0.5 cm from the edge, on all sides except the top side of the first section, using a magnification of about $300\times$. More than 120 jets, produced by singly charged particles, neutral primaries, α particles, or heavy nuclei of energy 10^{12} ev per nucleon, were thus located. The angular distribution of 78 nuclear interactions produced by singly charged particles and 6 interactions produced by neutral particles has been discussed separately.⁹ Out of 28 α -particle interactions, we have analyzed 17 events having a dip angle $\leq 20^\circ$. In one event, the incident α particle emerged undeflected, presumably as a He^3 nucleus, where only one neutron interacted.

3. ENERGY DETERMINATION

In the study of high-energy nuclear interactions, one of the most serious problems is that of a reliable determination of the energies of the primary particles. A direct

TABLE I. Details of primary α events.

Group number	Event number	Type	Energy per nucleon in 10^{12} ev	Anisotropy (σ)	
				From Eq. (10)	From Eq. (11)
(a) $N_h \leq 5$	2	5+10	7.20	0.61	1.06
	4	2+78	8.85	0.58	1.07
	5	4+11	1.88	0.47	0.98
	6	3+37	1.10	0.88	0.96
	8	3+27	4.20	0.72	1.03
	9	0+4	8.40	0.20	1.06
	13	0+23	3.24	0.49	1.03
	14	3+34	1.23	0.76	0.96
	17	4+74	0.51	0.59	0.91
	18	3+40	1.04	0.53	0.95
(b) $N_h > 5$	1	8+39	1.22	0.67	0.91
	3	6+26	7.92	0.88	1.01
	7	11+60	11.90	0.66	1.02
	10	15+61	17.30	0.73	1.04
	11	25+163	0.96	0.75	0.89
	12	38+76	0.96	0.64	0.89
	15	29+150	2.60	0.66	0.95

measurement of the energy ($\sim 10^{12}$ ev) of the primary particle from Coulomb scattering of the primary track is practically impossible, and therefore indirect methods based on an assumption regarding the process of emission of the secondary particles have to be employed. The most common method used for finding the energy of the primary particle from the angular distribution of the shower particles under certain assumptions is given by Castagnoli,¹⁰ as

$$\log \gamma_c = -\langle \log \tan \theta_i \rangle, \quad (1)$$

$$E_p = (2\gamma_c^2 - 1)Mc^2, \quad (2)$$

where θ_i is the angle which the i th shower particle makes with the direction of the primary particle in the laboratory system, and γ_c is the energy of the primary particle in the center-of-mass system in units of its rest energy; E_p is the energy of the primary particle per nucleon, Mc^2 the rest energy of a nucleon. The energy values as calculated from Eqs. (1) and (2) do not always give the correct value of the primary energy. A better estimate is obtained if instead of Eq. (2) one uses the following relation¹¹:

$$E_p = N(2\gamma_c^2 - 1)Mc^2, \quad (3)$$

where

$$N = 1.8 \text{ for events with } N_h > 5,$$

or

$$N = 0.75 \text{ for events with } N_h \leq 5.$$

The primary energy of 17 α primary particles was estimated from Eq. (3) and is given in Table I. In order to make use of Eq. (3) for events with $N_h > 5$, we have assumed that the symmetry system for which γ_c was calculated by means of Eq. (1) is the center-of-mass system of the primary nucleon and the tunnel through

⁷ M. V. K. Appa Rao, R. R. Daniel, and K. A. Neelakantan, Proc. Indian Acad. Sci. **43A**, 181 (1956).

⁸ P. L. Jain, E. Lohrmann, and M. W. Teucher, Phys. Rev. **115**, 643, (1959).

⁹ A. Barkow, B. Chanany, D. Haskin, P. Jain, E. Lohrmann, M. Teucher, and M. Schein, Phys. Rev. **122**, 617 (1961).

¹⁰ C. Castagnoli *et al.*, Nuovo cimento **10**, 1539, (1953).

¹¹ E. Lohrmann, M. Teucher, and M. Schein, Phys. Rev. **122**, 672 (1961).

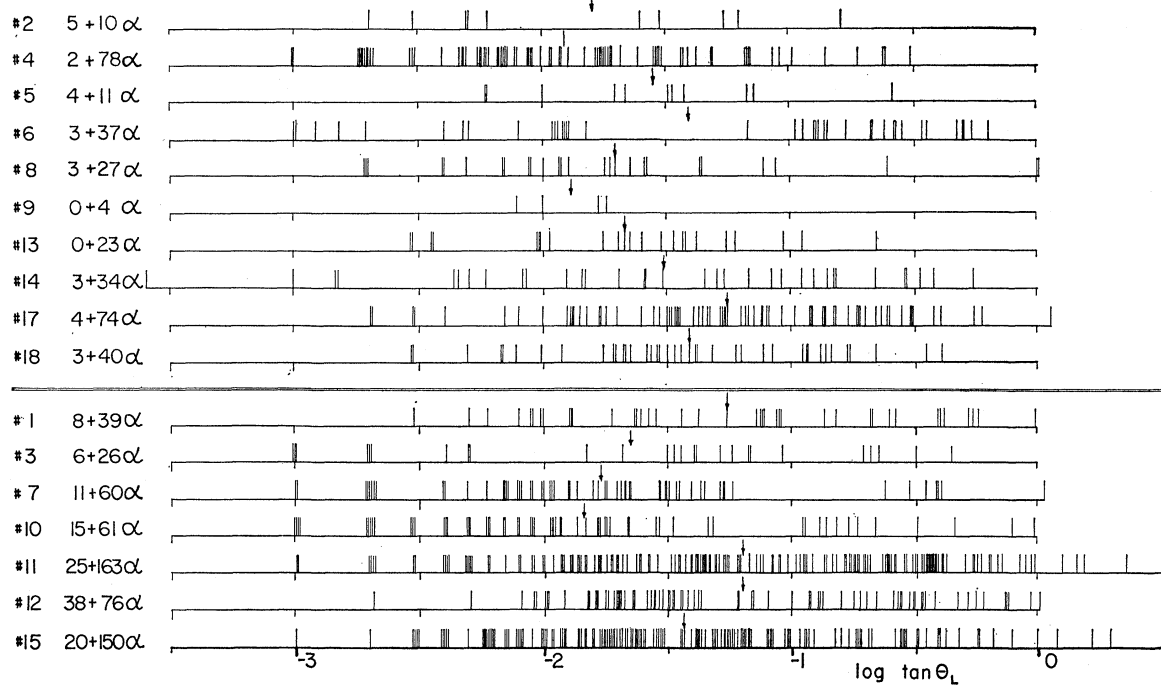


FIG. 1. The angular distribution of shower particles from the primary interactions plotted over $\log_{10} \tan \theta_L$. The center is denoted by an arrowhead.

the struck nucleus. In Sec. 5, we shall examine the symmetry in the angular distribution of the system moving with this γ_c value.

4. INELASTICITY OF THE COLLISION

In the study of high-energy nuclear interactions, one of the important quantities of great interest is the inelasticity parameter η of the collisions. η is defined as the fraction of the total available kinetic energy of the colliding particles, before the collision, which is subsequently used for the production of mesons and other particles. In the laboratory system it is defined by

$$\eta = \sum E_s / E_0, \quad (4)$$

where E_0 is the kinetic energy of the primary particles in the laboratory system and $\sum E_s$ is the sum of the energies, also in the laboratory system, of all the particles created in the collision. A simple and approximate method of determining the energies of the secondary particles is to make use of the principle that their transverse momentum (P_t) is invariant with respect to the angle of emission. The value of the average transverse momentum taken from the several published measurements^{11,12} is equal to 0.32 BeV/c. Thus from the known value of P_t and from the measurements of the angular distribution, one can get the total energy. In a good approximation, it is given by

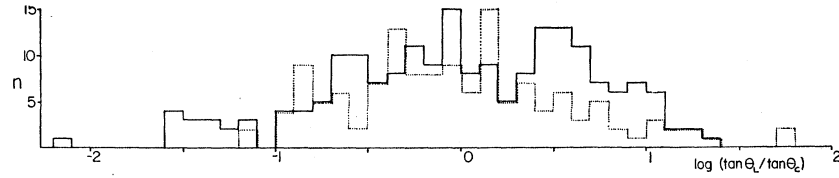
$$\sum E_s = 1.5 \langle P_t \rangle \sum \csc \theta_L. \quad (5)$$

¹² L. F. Hansen and W. B. Fretter, Phys. Rev. 118, 812 (1960).

In the above equation it has been assumed that all the shower particles are π mesons. The influence of nucleons and strange particles, which amount to about 20% among the shower particles, has been neglected. An estimate for the primary energy E_0 per nucleon was obtained from the angular distribution of the shower particles using Eqs. (1) and (3). For these equations one should only include the mesons among the shower particles. It was mentioned in Sec. 1 that on the average two tracks in α interactions continue to move with very small angle θ_L and they most probably correspond to incident nucleons contained in the primary α particle. These two tracks, which generally carry away a large fraction of the primary energy, have been excluded in the determination of the value of η .

As the primary energy E_0 determined from Eqs. (1) and (3) is not very reliable, the inelasticity averaged over all the events, rather than for an individual event, will give a meaningful result. The value of $\langle \eta \rangle$ for all α -particle interactions as calculated from Eqs. (4) and (5), is about 0.30, which is smaller roughly by a factor of 2 than the value of $\langle \eta \rangle$ for proton and neutron interactions⁹ with approximately the same energy per nucleon. Our data show small increase of $\langle \eta \rangle$ with N_h . Apart from the statistical fluctuations, one could explain this if the transverse momentum for events with $N_h > 5$ and $N_h \leq 5$ are slightly different. The only correlation that we have observed between the number of shower particles n_s and the inelasticity parameter η is that with very small n_s , the value of η is also very small.

FIG. 2. Combined angular distribution of shower particles normalized to the same primary energy according to Eqs. (1) and (3). Event numbers 5, 6, 14, 17, and 18 in group (a_1); normalizing energy ~ 1.2 Tev per nucleon (continuous line). Event numbers 2, 4, 8, and 13 in group (a_2); normalizing energy ~ 5.9 Tev per nucleon (dotted line).



5. ANGULAR DISTRIBUTION OF SHOWER PARTICLES

In order to investigate the dependence of the angular distribution on the target nucleus, we have divided our events into two groups, group (a) $N_h \leq 5$ and group (b) $N_h > 5$. In Fig. 1 is shown the angular distribution, in the laboratory system, of the secondary particles produced in events belonging to both groups. These angular distributions are plotted in terms of an x variable, where

$$x = \log_{10} \tan \theta_L. \quad (6)$$

The center of gravity of each event is denoted by an arrowhead which corresponds to a center-of-momentum angle of emission $\theta_c = 90^\circ$.

In order to investigate the dependence of the angular distribution on the primary energy, we have divided the events in groups (a) and (b) further into subgroups (a_1), (a_2), and (b_1), (b_2), respectively, according to their primary energy. Event number 9, because of its low multiplicity as compared to the rest of the events, has not been used in any discussion throughout the paper. In each subgroup, all the events have been normalized to the same primary energy and the combined angular distributions of events belonging to these subgroups (a_1), (a_2), and (b_1), (b_2) are plotted over a y variable, where

$$y = \log_{10}(\gamma_c \tan \theta_L) = \log_{10}(\tan \theta_L / \tan \theta_c). \quad (7)$$

These distributions are shown in Figs. 2 and 3, respectively. The shapes of these distributions are roughly symmetric except at the extremities, the reasons for these asymmetries being given in Sec. 8. We may compare the distributions given in Figs. 2 and 3 by continuous lines which correspond to the same normalized primary energy. It seems that in these high-energy interactions there is a reference system in which the

angular distribution of shower particles produced in nucleon-nucleus and nucleus-nucleus collisions is also symmetric as in the case of nucleon-nucleon interactions. In order to transform the angular distribution of shower particles into this symmetric system, we used the formula¹¹

$$\tan(\theta_s/2) = \gamma_s \tan \theta_L, \quad (8)$$

where

$$\gamma_s = 1.3\gamma_c. \quad (9)$$

This is valid if the velocity of the symmetric (s) system is equal to the velocity of the particles emitted in this system. In Fig. 4 is shown the angular distribution in the s system for all the events with $N_h \leq 5$ and $N_h > 5$ and also for events with $N_h \leq 5$ and having an anisotropy parameter (as explained in Sec. 6) $\sigma > 0.6$. The distribution for events with $N_h \leq 5$ and $\sigma > 0.6$ is more symmetric than the other two distributions; reasons for this symmetry are given in some detail in Sec. 8. Small deviations from symmetry in angular distributions may be due to the fact that the velocities of a few shower particles are not the same as the velocity of the s system. The near symmetry shown for a group of events with $N_h > 5$, where shower particles are presumably produced either in a nucleon-nucleus collision or in a nucleus-nucleus collision, is quite remarkable.

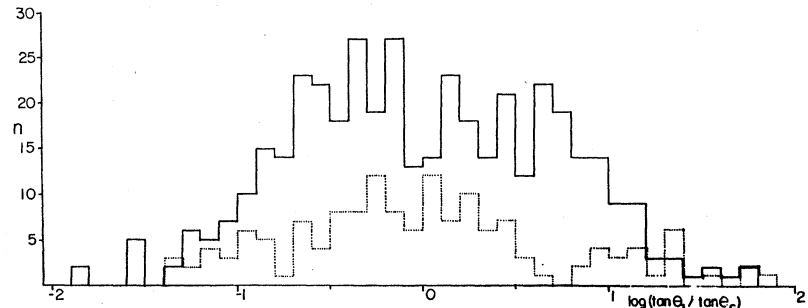
6. SHAPE OF THE ANGULAR DISTRIBUTION

The shape of the angular distribution determines the dispersion which is given by the relation

$$\sigma = \langle (x - \langle x \rangle)^2 \rangle^{1/2}. \quad (10)$$

The dispersion of this distribution is a measure of the degree of anisotropy, and for an isotropic angular distribution σ is equal to 0.39. The differential angular distribution dn/dy vs y , where y is defined in Eq. (7),

FIG. 3. Combined angular distribution of shower particles normalized to the same primary energy according to Eqs. (1) and (3). Event numbers 1, 11, 12, and 15 in group (b_1); normalizing energy ~ 1.4 Tev per nucleon (continuous line). Event numbers 3, 7, and 10 in group (b_2); normalizing energy ~ 12.5 Tev per nucleon (dotted line).



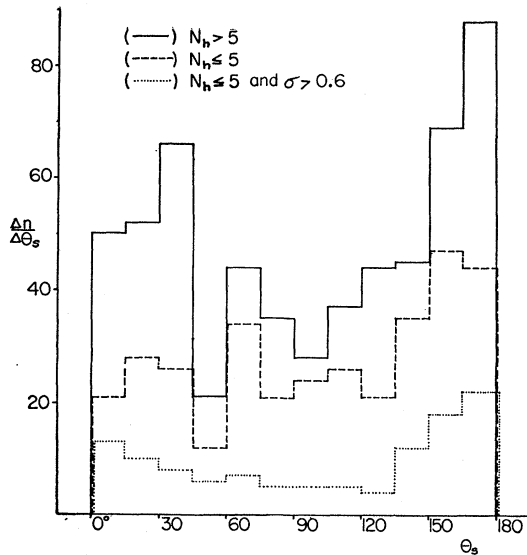


FIG. 4. Angular distribution in the symmetry system. All events with $N_h \leq 5$ and $\sigma > 0.6$ (dotted line); all events with $N_h \leq 5$ (broken line); all events with $N_h > 5$ (continuous line).

i.e., the angular distribution in the c.m. system for both nucleon-nucleon and nucleon-nucleus collisions predicted by hydrodynamical theory, is given by Gaussian distribution curves.

According to these theories the anisotropy σ increases with increase in the energy, but the shape of the distribution remains a Gaussian one irrespective of the primary energy. σ increases with increasing collimation of the shower particles in the direction of the shower axis and for $\sigma > 0.6$ the distribution is a strongly anisotropic one. Values of σ for all the events are shown in the fifth column of Table I, and they are in good agreement with the other published^{9,13} results at about the same primary energy. We may also point out that in the same energy interval the dispersion for a particular event may differ greatly from that for the group, and in such cases of high anisotropy the angular distribution is not normal but may show two separated maxima.

We may compare our experimental values of the anisotropy parameter σ with the theoretical value predicted from the hydrodynamical theory given by Milehin¹⁴ as

$$(2.3\sigma)^2 = 0.56 \ln(E/M) + 1.6 \ln[2/(N+1)] + 1.6, \quad (11)$$

where N is the number of nucleons contained in the column of the target nucleus, E the primary energy, and M the nucleon mass. We have used $N=3.4$ for events with $N_h > 5$ and $N=2$ for the events with $N_h \leq 5$. The theoretical values of anisotropy calculated from Eq. (11) are also shown in the last column of Table I and they are all higher than the experimental values.

¹³ O. Minakawa *et al.*, Nuovo cimento **11**, 125 (1959).

¹⁴ G. A. Milehin, Zhur. Eksper. i Teoret. Fiz. **35**, 1185 (1958).

7. ANALYSIS OF COMPOSITE ANGULAR DISTRIBUTION

It was mentioned earlier that according to hydrodynamical theories the shape of the total differential angular distribution of all the individual normal distributions, due to nucleon-nucleon and nucleon-nucleus collisions, is also a normal one. Therefore one could normalize all distributions in a single composite jet, to one Gaussian distribution.

In order to get a statistical analysis of the angular distribution of α events, we have formed composite jets of different combinations of α events from Table I. The differential angular distribution of all the events in a composite jet is plotted in the c.m. system in terms of a variable y as given by Eq. (7). All the distributions in a single composite jet were normalized to one Gaussian distribution.

In order to investigate the detailed shape of the angular distribution near the center, we have divided the whole area under the normal curve into ten equal parts. The intervals on the abscissa in terms of a variable y correspond to equal areas for the Gaussian distribution and thus one should expect equal numbers of tracks in each of these intervals. For the normal distribution one obtains, when plotting number of tracks in any interval against the y variable, a straight horizontal line and any deviation from the normal shape is thus quite obvious.

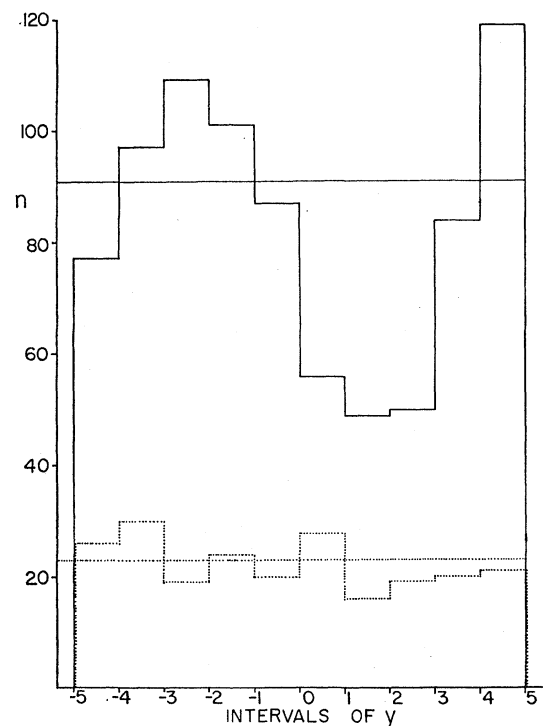


FIG. 5. Histogram of the composite differential angular distribution for the collection of jets. All events together; $P < 0.01$ (continuous line); events with $\sigma < 0.6$; $P = 0.60$ (dotted line). The intervals on the abscissa correspond to equal areas for the normal curve. A horizontal line represents a normal curve.

The continuous-line histograms in Figs. 5, 6, and 7 show the angular distribution for all events, events with $N_h > 5$ and $\sigma > 0.6$, and events with $\sigma > 0.6$, respectively. The dashed-line histograms in the above figures show the angular distribution for all events with $\sigma < 0.6$, events with $N_h < 5$ and $\sigma < 0.6$, and events with $N_h < 5$ and $\sigma > 0.6$, respectively. In order to check the statistical significance of the deviation of the observed distribution from the Gaussian curves, we have applied a χ^2 test to all these curves. Pearson probability (P) values for each curve are given in the captions of the diagrams. In Figs. 5 and 6 the group of events with $\sigma < 0.6$ shows P values < 0.6 and 0.3 , respectively, while the group of events with $\sigma > 0.6$ in Figs. 6 and 7 shows P values ≤ 0.01 .

Several authors⁴⁻⁶ have pointed out that certain features of the angular distribution of jets suggest a model in which the mesons are emitted isotropically from two centers, the "two-center model." This model has been applied to some extent successfully in nuclear events with anisotropy parameter $\sigma > 0.6$.^{15,16} A special feature of this theory is that it shows two distinctive maxima in the angular distribution with a sufficient deficit of particles at 90° in the c.m. system. From the point of view of the two-center model, the angular distribution is a superposition of two separate Gaussian distributions. Therefore, we can expect the appearance of the separate maxima only if there is a sufficient separation of the partial distributions corresponding to a sufficiently high dispersion of the resulting distribu-

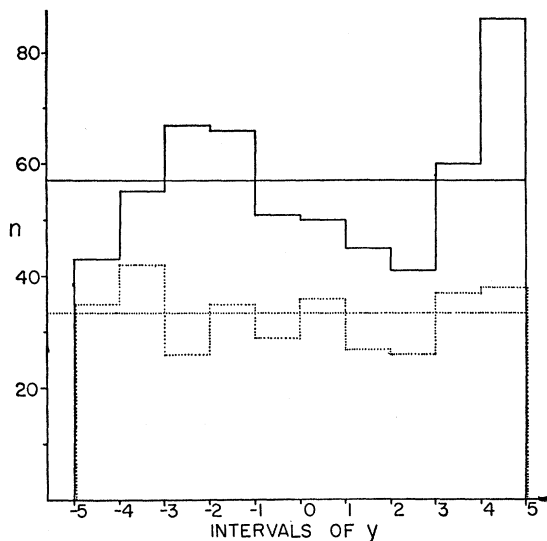


FIG. 6. Histogram of the composite differential angular distribution for the collection of jets. Events with $N_h > 5$ and $\sigma > 0.6$; $P < 0.01$ (continuous line); events with $N_h < 5$ and $\sigma < 0.6$; $P = 0.30$ (dotted line). The intervals on the abscissa correspond to equal areas for the normal curve. A horizontal line represents a normal curve.

¹⁵ J. Bartke, P. Ciok, *et al.*, Nuovo cimento **15**, 18 (1960).

¹⁶ J. Gierula, M. Miesowicz, and P. Zielinski, Nuovo cimento **18**, 102 (1960).

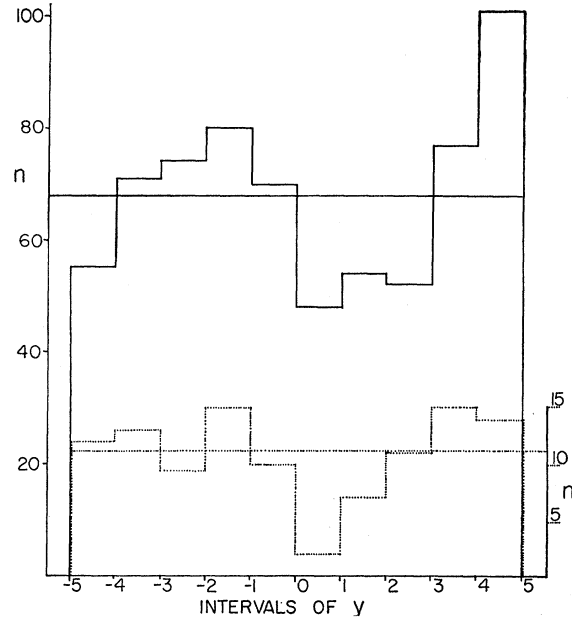


FIG. 7. Histogram of the composite differential angular distribution for the collection of jets. Events with $\sigma > 0.6$; $P < 0.01$ (continuous line); events with $N_h < 5$ and $\sigma > 0.6$; $P \sim 0.1$ (dotted line). The intervals on the abscissa correspond to equal areas for the normal curve. A horizontal line represents a normal curve.

tion. But when the distance between the two centers is small the two maxima may overlap each other and thus smear away the fine structure corresponding to individual centers, with the results that we cannot distinguish the shower particles belonging to one center from the ones belonging to the other.

It has been pointed out earlier that events with $\sigma < 0.6$ do not show distinct maxima and this is shown here in the dashed histograms of Figs. 5 and 6, respectively. The distribution in Fig. 5, for all the events, in which the majority of the events have $\sigma > 0.6$, shows two distinct maxima, and its P value is < 0.01 . The application of the χ^2 test to the composite distribution of the group with $N_h > 5$ and $\sigma > 0.6$ gives a rather high deviation of the normal distribution with P values < 0.01 , but does not indicate two quite distinct maxima. This is due to the fact that the procedure of superposing the particular distributions is only applicable to the groups of events with the same shape of angular distribution. It appears that the groups formed in the distribution may be a mixture of events belonging to some different physical collision phenomenon and consequently they give rise to different shapes of differential angular distribution whose superposition is not allowed. But the groups in Fig. 7 for events with $\sigma > 0.6$ as well as the events with $N_h < 5$ and $\sigma > 0.6$ show two distinct maxima, suggesting not only the relationship in the physical processes but also indicating that the interactions of events included in Fig. 7 may be explained by the "two-fireball" model.

8. GENERAL REMARKS

From Table I we observe that at a given primary energy E_p , the fluctuations in multiplicity n_s are very large. In general, n_s increases very slowly with E_p , but definitely rises with increasing mass of the target nucleus.

The inelasticity for α -particle interactions shows large fluctuations for individual events. Its mean value is smaller by roughly a factor of 2 when compared with the interactions of singly charged or neutral primaries of approximately the same primary energy as the energy per nucleon of α particles.

The general experimental results on the anisotropy parameter σ , for nucleon-nucleon and nucleon-nucleus collisions, as discussed in Sec. 6, are quite in agreement with the results of other authors^{9,15} but they are not in accord with predictions of the hydrodynamical theory.

In Sec. 1, it was mentioned that on the average two nucleons of the α particle undergo interaction. Thus some of the tracks appearing at very small angles in these α jets may correspond to fragments of the primary particle. Sometimes one can identify these tracks if they are separated from the rest of the tracks as shown in Figs. 2 and 3, and in event number 14 of Fig. 1. This effect will cause a more pronounced concentration of tracks at small angles in the laboratory system. As n_s for mesons in α events is high, the increase in n_s due to the presence of (nucleon) fragments of the primary particle will not greatly affect the n_s value. On the other hand, the presence of the fragments of α particles which make very small angles with the primary direction will overestimate the primary energy, and thus one should eliminate these tracks while determining the energy. Also, in cases where the incoming particle collides with a nucleus, the emitted particles may be scattered in the nucleus before escaping, giving thus a broader distribution of angles in the laboratory system.

Some of these scattered particles may have sufficiently great angles to increase appreciably the tail of the distribution at high θ_L values in the y variable, as given by Eq. (7). Some slight effect of this is seen in Fig. 4, for events with $N_h \leq 5$ and $\sigma > 0.6$, where there may be some contamination of nucleon-nucleus interaction in nucleon-nucleon collisions, but is much more pronounced in the same figure, for events with $N_h \leq 5$ and $N_h > 5$, where there is a concentration of tracks at large angles which makes the angular distribution in the s system a bit asymmetric.

The main source of error in our discussion of the symmetric reference system in Sec. 5 and of composite jets in Sec. 7 is due to the fact that only a small number of jets have been used in this analysis. What one needs is good statistics. But in spite of poor statistics, our results do indicate that, irrespective of their type of interaction, particles of primary energy $\sim 10^{12}$ ev per nucleon, with an anisotropy parameter $\sigma > 0.6$, show in their interactions the existence of double maxima in their angular distributions. These results are in agreement with those of other authors.^{15,16} None of the present nuclear theories explains these general results, though the "two-fireball" model explains the existence of double maxima in nucleon-nucleon interactions. In general, the shape of the angular distribution of shower particles is a function of both the primary energy and the number of nucleons struck in the target nucleus, which makes the comparison between the theoretical and experimental results far more complicated.

ACKNOWLEDGMENTS

We appreciate very much the hospitality of the late Professor Marcel Schein and of his group at the University of Chicago, where this work started. We are also thankful to Dr. E. Kerner of the University of Buffalo for a very useful discussion.