Invalidity of the Raphael Analysis for Moderate Energy *NN* Scattering*

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Raphael's modified effective-range expansion for nucleon-nucleon scattering is usually truncated at the second term. It is explicitly shown that the resulting series is not a good representation of the predictions of reasonable potential models for moderate-energy nucleon-nucleon scattering. In particular, it can lead to the wrong sign for the shape parameter.

THE usual effective-range expansion, $\gamma(k^2)=k\cot\delta=-(1/a)+\frac{1}{2}rk^2-r^3Pk^4+\cdots$, has been of limited use for describing nucleon-nucleon scattering in the $^1S_0$ state, as the phase shift is known to have a zero somewhere between 200 and 300 Mev. Raphael[1] has therefore introduced an origin-shifted expansion $\Gamma(k^2)=k\cot(\delta+k\bar{r})=a_0+a_1k^2+a_2k^4+\cdots$, where the additional parameter $\bar{r}$ is to be determined by requiring a_1 to be zero. It was supposed that setting $a_1=0$ would make Γ only very weakly energy-dependent in the region 0–300 Mev (lab energy of incident nucleon), justifying a termination of the series at the term a_2k^4. Indeed, boundary value models give $a_2=0$, and Raphael found that reasonable potential models gave quite small values for a_2.

Recent analyses of the experimental data have yielded plausible values of the $^1S_0$ phase shift at a number of energies. The phases at (roughly) 1, 2, and 300 Mev were the first to become available; they were used by Noyes[2] to determine $\bar{r}$, a_0, and a_2. Comparison with the γ series produced a negative shape parameter P. Since experimental indications are that the shape parameter is positive, Noyes concluded that no potential, and *a fortiori* no boundary-value model, could satisfy the data. Noyes and others concluded that one had no choice but to take a dispersion-theoretic approach.[2,3]

We have tested the truncated Raphael series, so crucial to the above argument, in several ways. First, we computed the exact predictions, at 1, 2, 95, and 300 Mev, of several reasonable potentials[4] which fit the data quite well at those energies. In no case could the four points be fit by the 3-parameter Γ series. We tried expanding the series to include a_3k^6 and a_4k^8, and found that the series was not yet converging at the highest energy used, 300 Mev. Since it was then obvious that the Raphael analysis is invalid for that high an energy, we attempted to determine the energy range for which it is valid. Below is shown the results of our computations for one of the potentials found by Perring and Phillips[5] to have a positive shape parameter. For this 3-parameter Raphael-type analysis the lower energy points were fixed at 1 and 2 Mev, while the higher point was varied as shown:

Highest energy (Mev)	300	50	10	exact
Predicted shape P	-0.03	-0.02	0.00	$+0.02$

It is apparent that the 3-point Raphael expansion has an even smaller range of validity than the 3-parameter effective range series; the latter is a good representation up to ≈ 25 Mev.

After we found that the Raphael analysis was no basis for an argument against boundary value and potential models, we re-examined the boundary-value-plus-potential model recently communicated by Saylor, Bryan, and Marshak.[6] On the basis of the Noyes argument, Saylor, Bryan, and Marshak assumed that their model predicted "*a fortiori* a negative shape parameter." On the contrary, by direct calculation we found $P=+0.025$ for their model (a reasonably good value).

We note in concluding that since the Γ and γ expansions really depend only on time-reversal invariance and the short range of nuclear forces, they are surely just as valid for dispersion-theoretic as for potential models so can hardly be used to differentiate between the two approaches.

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[1] R. B. Raphael, Phys. Rev. **102**, 905 (1956).

[2] H. P. Noyes, University of California Radiation Laboratory Report, UCRL-5521-T, 1959 (unpublished); and International Conference on Nuclear Forces and the Few Nucleon Problem, University College, London, July, 1959 (to be published).

[3] M. H. MacGregor, M. J. Moravcsik, and H. P. Noyes, University of California Radiation Report, UCRL 5582-T, 1959 (unpublished).

[4] The potentials used had hard cores and one-pion-exchange tails. The intermediate region was arbitrarily specifiable at net points but was quite smooth.

[5] J. K. Perring and R. J. N. Phillips, Nuclear Phys. (to be published). We are indebted to Dr. J. Iwadare for supplying us with a preprint. The potential we have chosen for illustration is labeled "1a" in the preprint, but the hard core has been changed from 0.353 $\hbar/\mu c$ to 0.35 $\hbar/\mu c$ (μ is the pion mass) for ease of computation.

[6] D. P. Saylor, R. A. Bryan, and R. E. Marshak, Phys. Rev. Letters **5**, 266 (1960).