Photoelectric Effect and Pair Annihilation with Large Momentum Transfer*

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Photoelectric effect and pair annihilation in hydrogen with large momentum transfer are studied, taking into account the recoil and anomalous magnetic moment of proton, in an effort to see whether these processes can be used to probe quantum electrodynamics at small distances. A negative result is obtained. It turns out that for an incident energy of 100 Mev the important term containing the electron propagator, which is sensitive to small distance modifications, is about 0.5% of the term, which is insensitive to them.

The proton structure is described by two covariant form factors determined by the electron-proton scattering. The differential cross sections are calculated in the Born approximation in the laboratory system, neglecting the binding energy of the hydrogen atom. The results are analyzed in the extreme relativistic energy range and in the special case that the outgoing electron (photon) comes off perpendicular to the incident beam. The total cross sections are calculated in the high-energy approximation simply to check the present method of calculations.

The differential cross sections are very small $\sim 10^{-42}$–10^{-40} cm^2/sr. The same calculations, if applied to an atom with higher atomic number Z, give the differential cross sections for the above processes larger by a factor $2Z^5$, if one neglects screening. For a Au target the differential cross sections are then of the order of 10^{-33}–10^{-31} cm^2/sr. However, this extension of the calculations introduces a considerable error in the differential cross sections, because the influences of the Coulomb field and anomalous magnetic moment of a nucleous to the electron wave functions are neglected. It will serve only as an estimate of the order of magnitude of the differential cross sections.

I. INTRODUCTION

It is expected that large angle photoelectric effect and pair annihilation in hydrogen at high energies might provide a test of the validity of quantum electrodynamics at small distances.[1–4] In particular, these processes may reveal a small-distance ($\sim 10^{-13}$ cm) behavior[2,3] of the electron propagator. This happens because ambiguities arising from the proton vertex (Figs. 1 and 2) are removed by the use of two covariant form factors[5] which can be obtained in the elastic electron-proton scattering experiments.[6,7] With ambiguities arising from interactions of nonelectromagnetic origin removed, the processes in Figs. 1(a) and 2(a) yield new information on the electron and positron propagators at small distances if the virtual intermediate electron and positron lines in Figs. 1(a) and 2(a) are far off the mass shell, as is the case for a high-energy interaction with large momentum transfer.

In this paper, large-angle photoelectric effect and pair annihilation in hydrogen are investigated in an attempt to see if these processes can be used to probe the behavior of the electron and positron propagators in quantum electrodynamics at small distances. Feynman diagrams in Figs. 1(a) and 2(a) show how quantum electrodynamics is probed in such an experiment. In Fig. 1(a) [Fig. 2(a)] the photon is directly absorbed (emitted) at a large angle with the direction of the outgoing electron (incident positron); and the intermediate electron (positron) lines are ~ 69, 134, and 197 Mev off the mass shell for incident photon (positron) energies 50, 100, and 150 Mev, respectively. These diagrams are sensitive to small-distance modifications of the propagators. On the other hand, in Figs. 1(b) and 2(b) the intermediate electron lines are not too far off the mass shell, e.g., 10 Mev for an incident

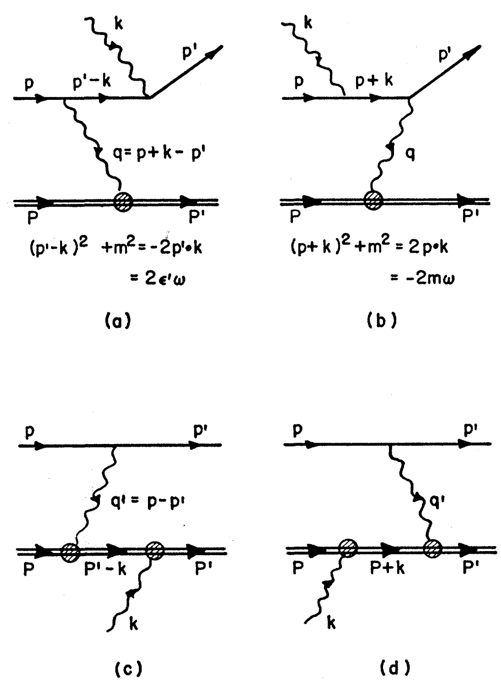

Fig. 1. Feynman diagrams for photoelectric effect.

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[1] R. P. Feynman, Anais acad. brasil. cienc. **26**, 51 (1954).

[2] S. D. Drell, Ann. Phys. **4**, 75 (1958).

[3] J. D. Bjorken, S. D. Drell, and S. C. Frautschi, Phys. Rev. **112**, 1409 (1958). According to the above paper, the radiative corrections amount to a few percent for the pair production.

[4] J. D. Bjorken and S. D. Drell, Phys. Rev. **113**, 1368 (1959).

[5] D. R. Yennie, M. M. Lévy, and D. G. Ravenhall, Revs. Modern Phys. **29**, 144 (1957).

[6] R. Hofstadter, Ann. Rev. Nuclear Sci. **7**, 231 (1957).

[7] R. Hofstadter, F. Bumiller, and M. R. Yearian, Revs. Modern Phys. **30**, 482 (1958).

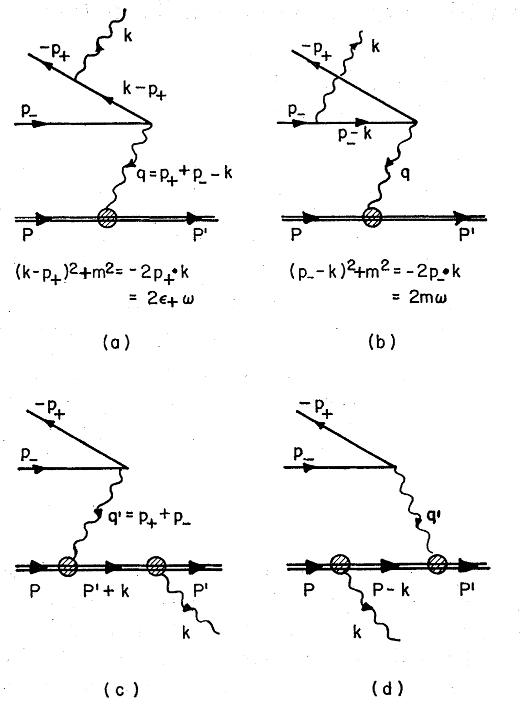

FIG. 2. Feynman diagrams for pair annihilation.

energy 100 Mev. Thus, these diagrams are not too sensitive to small distance modifications. A simple cutoff model[2] suggests that an experiment measuring the propagators to 40% probes the propagators at distances $\sim 1.8\times 10^{-13}$ cm, 0.9×10^{-13} cm, and 0.6×10^{-13} cm for incident energies 50, 100, and 150 Mev, respectively.

Unfortunately, it is shown here that the significant term of Fig. 1(a) [or Fig. 2(a)], which is sensitive to small distance modifications, is down by a factor of mc^2/ω (or $mc^2/\epsilon_+)\sim 10^{-2}$ compared with that of Fig. 1(b) [or Fig. 2(b)], so that the experiment must in fact measure the propagator to less than 1%.

Photoelectric effect and pair annihilation in hydrogen at high energies are just the reverse processes of bremsstrahlung[8] and pair production.[3] In bremsstrahlung and pair production, it is necessary to expand each form factor into a power series[9] in q^2 in order to integrate the differential cross sections over a solid angle of one of the outgoing particles, from which one can derive the expressions required for coincidence experiments. This is due to the fact that there are three particles in the final state. This integration is unnecessary for the processes under consideration, because there are only two particles in the final state.

The present calculations are made in the Born approximation. Radiative corrections[3] and the binding of the hydrogen atom are disregarded. The recoil and anomalous magnetic moment of the proton are taken into account. The differential cross sections for both processes are evaluated for the case in which the outgoing electron and photon emerge at right angles to the incident beam. The total cross sections are calculated in the extreme relativistic approximation simply to check the present method of calculation.

II. CALCULATION

The calculations for these processes are made in the Born approximation with the help of the standard technique[10] in field theory, taking into consideration the recoil and anomalous magnetic moment of the proton. Neglecting the binding of the hydrogen atom, the ground-state wave function can be taken as

$$\Psi(x_1,x_2)=[\pi a_0^3(2\pi)^3]^{-\frac{1}{2}}e^{i(p\cdot x_1+P\cdot x_2)},$$

where a_0 in the normalization factor is the first Bohr radius, $p=(\mathbf{0},m)$ and $P=(\mathbf{0},M)$ are the 4-momenta of the electron and proton and x_1 and x_2 are the space-time coordinates of the electron and proton. It is believed that the use of the free propagator for a bound electron in a hydrogen atom (thus neglecting the binding of the hydrogen atom) is allowed because the incident particles have extremely high energies. In fact, in the extreme relativistic approximation, the use of the above wave function for the hydrogen atom (the electron-proton system) introduces an error of less than 3% to the more accurate value[11] of differential cross section for photoelectric effect which is calculated by using the relativistic Coulomb wave function for the bound and ejected electrons.

The photoelectric effect (Fig. 1) and pair annihilation (Fig. 2) in the hydrogen atom at high energies are very closely related to each other. The matrix element for pair annihilation is easily obtained by the simple substitution

$$p'\rightarrow -p_+,\quad k\rightarrow -k,$$

into the photoelectric effect.

The differential cross section for photoelectric effect is computed below. The matrix element for the above process is

$$\langle p',P'|\mathfrak{M}|p,P,k\rangle=(2\pi)^{-2}(mM/\epsilon'E'2\omega)^{\frac{1}{2}}\frac{e^3}{(\pi a_0^3)^{\frac{1}{2}}}\times\bigg\{\bar{u}(p')C^\mu(p',p)u(p)\frac{1}{q^2}\bar{U}(P')\Gamma_\mu(q)U(P)-\bar{u}(p')\gamma^\mu u(p)\frac{1}{(q')^2}\bar{U}(P')D_\mu(P',P)U(P)\bigg\},\quad (1)$$

[8] R. A. Berg and C. N. Lindner, Phys. Rev. **112**, 2072 (1958).

[9] The expansion of form factors into power series in q^2 may lead to a theoretical difficulty due to the divergence of perturbation calculations in meson field theory.

[10] J. M. Jauch and F. Rohrlich, *Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1955).

[11] R. H. Pratt, Phys. Rev. **117**, 1017 (1960).

where

$$C^{\mu}(p',p)=\gamma\cdot e\frac{1}{i\gamma\cdot(p'-k)+m}\gamma^{\mu}+\gamma^{\mu}\frac{1}{i\gamma\cdot(p+k)+m}\gamma\cdot e, \quad (2)$$

$$D_{\mu}(P',P)=\Gamma\cdot e\frac{1}{i\gamma\cdot(P'-k)+M}\Gamma_{\mu}(q')+\Gamma_{\mu}(q')\frac{1}{i\gamma\cdot(P+k)+M}\Gamma\cdot e, \quad (3)$$

$$\Gamma_{\mu}(q)=\gamma_{\mu}F_1(q^2)+\frac{\kappa}{2M}\sigma_{\nu\mu}q^{\nu}F_2(q^2), \quad (4)$$

and

$$\Gamma\cdot e=\gamma\cdot eF_1(k^2=0)+\frac{\kappa}{2M}\sigma_{\nu\mu}k^{\nu}e^{\mu}F_2(k^2=0). \quad (5)$$

The notation used in this paper is almost the same as that in reference 10. A glossary of symbols is given in the Appendix. The rationalized natural unit system,

$$\hbar=c=1,$$

is employed. The relative sign difference in the first and second terms in Eq. (1) is due to the relative sign difference in electron and proton charges. The first term in Eq. (1) corresponds to Figs. 1(a) and 1(b), and the second to Figs. 1(c) and 1(d).

By the standard technique,[10] the matrix element can be squared, averaged over the initial states (i.e., electron and proton spins and photon polarization) and summed over the final states (i.e., electron and proton spins) to yield the differential cross section

$$\frac{d\sigma}{d\Omega_e}=\alpha r_0^2\frac{|\mathbf{p}'|Mm^3}{\omega E'a_0^3}\frac{d\epsilon'}{d(\epsilon'+E')}\bigg|_{\mathbf{k}=\mathbf{p}'+\mathbf{P}'}[A+B+C], \quad (6)$$

where

$$m^2M^2q^4A=q^2\left\{\frac{\lambda}{\lambda'}+\frac{\lambda'}{\lambda}+\frac{q^2}{2}\left[-\frac{m^2}{\lambda'^2}-\frac{m^2}{\lambda^2}+\frac{r^2}{\lambda\lambda'}\right]+m^2\left[\frac{2}{\lambda'}-\frac{2}{\lambda}+m^2\left(\frac{1}{\lambda'}-\frac{1}{\lambda}\right)^2\right]\right\}(F_1+\kappa F_2)^2$$
$$+\left\{\frac{Q^2}{4}\left[2\frac{\lambda}{\lambda'}+2\frac{\lambda'}{\lambda}+q^2\left(-\frac{m^2}{\lambda'^2}-\frac{m^2}{\lambda^2}+\frac{r^2}{\lambda\lambda'}\right)\right]-\frac{m^2(p\cdot Q)^2}{\lambda'^2}-\frac{m^2(p'\cdot Q)^2}{\lambda^2}+\frac{q^2}{2\lambda\lambda'}[(p\cdot Q)^2+(p'\cdot Q)^2]\right.$$
$$\left.+\frac{m^2}{2\lambda\lambda'}[q^2Q^2+2(p\cdot Q)^2+2(p'\cdot Q)^2-2(k\cdot Q)^2]\right\}\left(F_1^2+\frac{\kappa^2}{4M^2}q^2F_2^2\right), \quad (7)$$

$$m^2M^2q^2r^2B=\sum_{\text{polarization}}\left\{ab(q\cdot r+2m^2)\left[q\cdot rG_1(F_1'+\kappa F_2')-\frac{\kappa^2}{M^2}\Lambda\Lambda'G_2F_2'\right](F_1+\kappa F_2)\right.$$
$$+2a\left[\frac{p\cdot e}{\lambda}[M^2q\cdot r+p'\cdot P(P\cdot R+\lambda')+P\cdot(p+k)(P\cdot R+\lambda)-\tfrac{1}{4}q^2r^2]-\frac{p'\cdot e}{\lambda'}[M^2q\cdot r+p\cdot P(P\cdot R+\lambda)\right.$$
$$\left.+P\cdot(p'-k)(P\cdot R+\lambda')-\tfrac{1}{4}q^2r^2]-P\cdot eQ\cdot R\right]\left[G_1F_1F_1'-\frac{\kappa^2}{4M^2}q\cdot rG_1F_2F_2'+\frac{\kappa^2}{4M^2}q\cdot kG_2(F_2F_1'-F_1F_2')\right]$$
$$+b\lambda\lambda'\left[\frac{q\cdot eQ\cdot k}{\Lambda\Lambda'}(G_1+\kappa G_2)-\frac{a\kappa}{2M^2}q^2G_2\right](F_1+\kappa F_2)\kappa F_2'+\left[\frac{p\cdot e}{\lambda}[\Lambda r^2+\lambda\lambda'+\lambda P\cdot(2p'+k)+2\lambda'p\cdot P]\right.$$
$$\left.-\frac{p'\cdot e}{\lambda'}[\Lambda r^2+\lambda\lambda'+\lambda'P\cdot(2p-k)+2\lambda p'\cdot P]-P\cdot eR\cdot k\right]\left[a'G_1F_1F_1'+a'\frac{\kappa^2}{4M^2}q^2[G_1F_2F_2'\right.$$
$$+G_2(F_2F_1'-F_1F_2')]+\kappa\frac{q\cdot eq\cdot k}{2\Lambda\Lambda'}(G_1+\kappa G_2)(F_1F_2'-F_2F_1')-\frac{\kappa^2}{4M^2}aQ\cdot k[G_1F_2+G_2(F_1+\kappa F_2)]F_2'$$
$$\left.+\frac{\kappa^2}{2M^2}q\cdot eG_2(F_1+\kappa F_2)F_2'\right]+[p\cdot e(\lambda'-\Lambda)-p'\cdot e(\lambda+\Lambda)+P\cdot eR\cdot k]\Big[a'\{\kappa F_1[G_1F_2'+G_2(F_1'+\kappa F_2')]$$

$$+(G_1+\kappa G_2)(F_1+\kappa F_2)F_1'\}-a'\frac{\kappa^2}{4M^2}q^2\{G_1F_2F_2'+G_2(F_1F_2'+F_2F_1'+2\kappa F_2F_2')\}$$

$$+\frac{\kappa^2}{4M^2}\{aQ\cdot kG_1F_2+a'q\cdot kG_2(F_1+\kappa F_2)\}F_2'-\frac{Q\cdot eQ\cdot k}{2\Lambda\Lambda'}(G_1+\kappa G_2)(2F_1F_1'+\kappa F_1F_2'+\kappa F_2F_1')\Bigg]$$

$$+\Bigg[-\frac{p\cdot e}{\lambda}\bigg(\frac{\Lambda r^2}{2}+\lambda' p\cdot P+\lambda(p'\cdot P+\Lambda)\bigg)+\frac{p'\cdot e}{\lambda'}\bigg(\frac{\Lambda r^2}{2}+\lambda p'\cdot P+\lambda'(p\cdot P-\Lambda)\bigg)+P\cdot eR\cdot k\Bigg]$$

$$\times a\frac{\kappa^2}{2M^2}Q\cdot kG_2F_2(F_1'+\kappa F_2')+b\lambda\lambda'\Bigg[-\frac{\kappa^2}{M^2}Q\cdot eG_2F_2'+\bigg(-2a\kappa G_2+\frac{Q\cdot eq\cdot k}{\Lambda\Lambda'}(G_1+\kappa G_2)\bigg)(F_1'+\kappa F_2')\Bigg]$$

$$\times(F_1+\kappa F_2)+\Bigg[\frac{(p\cdot e)^2}{\lambda}(4p'\cdot P+\lambda')-\frac{(p'\cdot e)^2}{\lambda'}(4p\cdot P+\lambda)+2p\cdot ep'\cdot e\bigg(\frac{P\cdot(2p+k)}{\lambda}-\frac{P\cdot(2p'-k)}{\lambda'}\bigg)$$

$$+2P\cdot e\bigg(\frac{p\cdot e}{\lambda}(r^2-\lambda')-\frac{p'\cdot e}{\lambda'}(r^2+\lambda)\bigg)-Q\cdot R\Bigg]\Bigg[\bigg(F_1-\frac{\kappa}{4M^2}q^2F_2\bigg)\kappa[G_1F_2'+G_2(F_1'+\kappa F_2')]$$

$$-\frac{\kappa^2}{4M^2}q\cdot rG_2F_2'(F_1+\kappa F_2)+\frac{1}{4\Lambda\Lambda'}[(q\cdot k)^2(F_1+\kappa F_2)F_1'-(Q\cdot k)^2F_1(F_1'+\kappa F_2')](G_1+\kappa G_2)\Bigg]$$

$$+\Bigg[\frac{(p\cdot e)^2}{\lambda}\lambda'-\frac{(p'\cdot e)^2}{\lambda'}\lambda-2\frac{p\cdot ep'\cdot e}{\lambda\lambda'}\Lambda R\cdot k+2P\cdot e\bigg(\frac{p\cdot e}{\lambda}\lambda'+\frac{p'\cdot e}{\lambda'}\lambda\bigg)-\frac{2}{\lambda\lambda'}\bigg(\lambda^2p'\cdot P+\lambda'^2p\cdot P$$

$$+\frac{R\cdot k}{2}(\Lambda r^2+\lambda\lambda')\bigg)\Bigg]\Bigg[-\bigg(F_1-\frac{\kappa}{4M^2}q^2F_2\bigg)\kappa[G_1F_2'+G_2(F_1'+\kappa F_2')]-\frac{\kappa^2}{4M^2}q\cdot rG_2F_2'(F_1+\kappa F_2)$$

$$+\frac{1}{4\Lambda\Lambda'}[(q\cdot k)^2(F_1+\kappa F_2)F_1'+(Q\cdot k)^2F_1(F_1'+\kappa F_2')](G_1+\kappa G_2)\Bigg]+(bb'\lambda\lambda'-R\cdot k)\frac{Q\cdot k}{2}$$

$$\times\Bigg[\frac{\kappa^2}{M^2}G_2F_2'-\frac{q\cdot k}{\Lambda\Lambda'}(G_1+\kappa G_2)(F_1'+\kappa F_2')\Bigg](F_1+\kappa F_2)-\frac{Q\cdot k}{2\Lambda\Lambda'}[q\cdot k(bb'\lambda\lambda'-R\cdot k)\kappa F_2'$$

$$+q^2R\cdot k(F_1'+\kappa F_2')](G_1+\kappa G_2)(F_1+\kappa F_2)\Bigg\},\qquad(8)$$

$$m^2M^2r^4C=\sum_{\text{polarization}}\Bigg\{\Bigg[r^2+4p\cdot ep'\cdot e+\lambda\lambda'\bigg(a'^2-2a'b'+\frac{q^2}{\Lambda\Lambda'}\bigg)\Bigg]\Bigg[(G_1+\kappa G_2)(F_1'+\kappa F_2')-\frac{\kappa^2}{2M^2}\Lambda G_2F_2'\Bigg]$$

$$\times\Bigg[(G_1+\kappa G_2)(F_1'+\kappa F_2')+\frac{\kappa^2}{2M^2}\Lambda' G_2F_2'\Bigg]-\Bigg[\frac{r^2}{2}+2p\cdot ep'\cdot e+\lambda\lambda'\bigg(\frac{a'^2}{2}-a'b'\bigg)\Bigg]$$

$$\times\Bigg[\frac{\kappa Q^2}{2M^2}[G_1F_2'+G_2(F_1'+\kappa F_2')]+\frac{(Q\cdot k)^2}{\Lambda\Lambda'}(G_1+\kappa G_2)(F_1'+\kappa F_2')\Bigg]\kappa[G_1F_2'+G_2(F_1'+\kappa F_2')]$$

$$+a\Bigg[a(r^2-2m^2)\Lambda\Lambda'+\lambda\lambda'\bigg(a\frac{Q^2}{2}+b'Q\cdot k-2Q\cdot e\bigg)\Bigg]\frac{\kappa^2}{2M^2}G_2(F_1'+\kappa F_2')[2G_1F_2'+G_2(F_1'+\kappa F_2')]$$

$$+\lambda\lambda' a\left[(a'-b')\frac{q\cdot kQ\cdot k}{\Lambda\Lambda'}-\frac{a}{2}\left(\frac{\Lambda}{\Lambda'}+\frac{\Lambda'}{\Lambda}\right)\right]\kappa[G_1F_2'+G_2(F_1'+\kappa F_2')](G_1+\kappa G_2)(F_1'+\kappa F_2')$$

$$+\lambda\lambda'\left[a(b'-a')+\frac{Q\cdot k}{2\Lambda\Lambda'}\right]Q\cdot k\frac{\kappa^2}{2M^2}G_1{}^2F_2'^2+\lambda\lambda'\left[\frac{q\cdot k}{M^2}-\frac{(Q\cdot k)^2}{2\Lambda\Lambda'}\right]\frac{\kappa^4}{2M^2}G_2{}^2F_2'^2$$

$$+\lambda\lambda'\left[a(b'-a')Q\cdot k+4-\frac{(q\cdot k)^2}{\Lambda\Lambda'}\right]\frac{\kappa^2}{2M^2}G_2F_2'(G_1+\kappa G_2)(F_1'+\kappa F_2')+(\Lambda r^2+2\lambda\lambda'+2\lambda p'\cdot P+2\lambda' p\cdot P)$$

$$\times\left(aq\cdot e-\frac{Q\cdot kq\cdot r}{2\Lambda\Lambda'}\right)\frac{\kappa^2}{2M^2}G_2F_2'(G_1+\kappa G_2)(F_1'+\kappa F_2')+\left(\frac{r^2}{2}-m^2\right)\left[a^2r^2G_1{}^2(F_1'+\kappa F_2')^2+\Lambda\Lambda'\right.$$

$$\left.\times\left(\frac{q\cdot k}{\Lambda\Lambda'}(G_1+\kappa G_2)(F_1'+\kappa F_2')-\frac{\kappa^2}{M^2}G_2F_2'\right)^2\right]+[(2p\cdot P+\lambda)(2p'\cdot P+\lambda')+r^2(\Lambda-M^2)]$$

$$\times\left[a^2\left(G_1{}^2F_1'^2+\frac{\kappa^2}{4M^2}[r^2G_1{}^2-(\kappa^2/M^2)\Lambda\Lambda' G_2{}^2]F_2'^2\right)+\frac{\kappa^2}{4M^2}\frac{(q\cdot k)^2}{\Lambda\Lambda'}(G_1+\kappa G_2)F_2'\right.$$

$$\left.\times[(G_1+\kappa G_2)F_2'+2G_2F_1']+\frac{\kappa^4}{4M^4}r^2G_2{}^2F_2'^2\right]+(\Lambda r^2+2\lambda\lambda'+2\lambda p'\cdot P+2\lambda' p\cdot P)\frac{\kappa^2}{4M^2}$$

$$\times\left[2aq\cdot e[G_1F_2'+G_2(F_1'+\kappa F_2')]^2-aa'\left((Q^2+2q\cdot k)G_1{}^2+\frac{\kappa^2}{M^2}\Lambda\Lambda' G_2{}^2\right)F_2'^2\right.$$

$$\left.+\frac{q\cdot k}{\Lambda\Lambda'}(G_1+\kappa G_2)F_2'(G_1F_2'+G_2F_1')\right]+2[p\cdot e(2p'\cdot P+\lambda')+p'\cdot e(2p\cdot P+\lambda)+P\cdot er^2]$$

$$\times a\left[G_1(F_1'+\kappa F_2')[(G_1+\kappa G_2)\kappa F_2'-G_1F_1']+\frac{\kappa^2}{4M^2}\{Q^2G_1F_2'+q\cdot k[G_1F_2'-G_2(F_1'+\kappa F_2')]\}\right.$$

$$\times[G_1F_2'+G_2(F_1'+\kappa F_2')]-\frac{\kappa^2}{4M^2}q\cdot kG_2F_2'(3G_1+\kappa G_2)(F_1'+\kappa F_2')+\frac{\kappa^2}{4M^2}\left(q^2G_1(F_1'+\kappa F_2')\right.$$

$$\left.\left.+\frac{\kappa^2}{M^2}\Lambda\Lambda' G_2F_2'\right)G_2F_2'\right]-\lambda\lambda'\left[a^2G_1(F_1'+\kappa F_2')\kappa(G_1F_2'+G_2F_1')+2\frac{q\cdot k}{\Lambda\Lambda'}(G_1+\kappa G_2)^2(F_1'+\kappa F_2')^2\right]\Bigg\}. \quad (9)$$

The expressions in Eqs. (7)–(9) have already been simplified to a certain extent with the help of energy and momentum conservation. In Eqs. (7)–(9) the following abbreviations for the form factors are used:

$$F_{1,2}=F_{1,2}(q^2), \quad F'_{1,2}=F_{1,2}(q'^2),$$

and

$$G_{1,2}=F_{1,2}(k^2=0).$$

The terms A, B, and C in Eq. (6) are the main term (a generalization of the Bethe-Heitler term[12] which includes the effects of recoil and anomalous magnetic moment of the proton), the interference term and the Compton term[3] (the dynamical correction to the proton current), respectively. Equations (7)–(9) can be applied to the problems of bremsstrahlung and pair production by proper changes of symbols. The results of the present calculation agree with those in reference 8, insofar as the two calculations overlap.

Because of the complicated general expression above, one considers the special case in which it has a simpler form; the outgoing electron is ejected perpendicular to the incident photon direction in the laboratory system. Keeping only the lowest order and simplifying the expression in Eq. (6) with the help of the energy and momentum conservation law, one finds in the extreme, relativistic limit, that the differential cross section in

[12] H. Bethe and W. Heitler, Proc. Roy. Soc. (London) **A146**, 83 (1934).

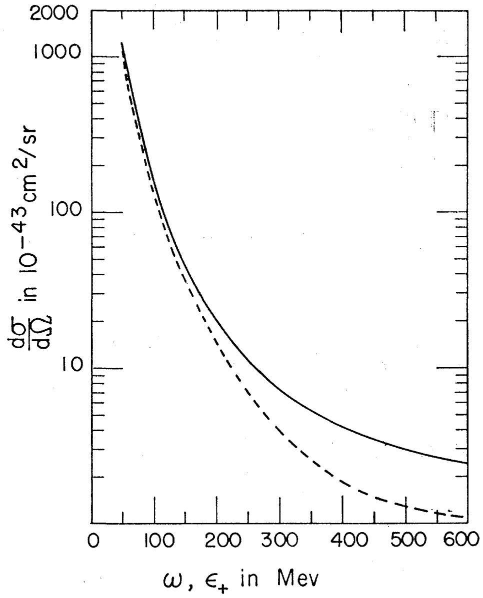

FIG. 3. Differential cross section $d\sigma/d\Omega_e$ for photoelectric effect (solid line) and differential cross section $d\sigma/d\Omega_k$ for pair annihilation (dashed line) are plotted as functions of incident photon and positron energies in the special case that the outgoing electron and photon come off at 90° with respect to the incident beam. The exponential form factors are used for both processes.

the laboratory system is

$$\frac{d\sigma}{d\Omega_e}=\alpha r_0^2\frac{M}{2(M+\omega)(\omega a_0)^3}\left\{F^2\left[1+\frac{\epsilon\omega}{M^2}(1+2\kappa+\tfrac{3}{2}\kappa^2)\right]\right.$$
$$+GFF'\left[\frac{\omega}{M}+\frac{\epsilon\omega^2}{4M^3}(4+8\kappa+9\kappa^2+4\kappa^3)\right]$$
$$+G^2F'^2\left[\frac{\omega^2}{M^2}+\frac{\omega^4}{M^3\epsilon}(1-\kappa^2)+\frac{\epsilon\omega^3}{M^4}\right.$$
$$\left.\left.\times\left(1+2\kappa+\frac{9}{2}\kappa^2+3\kappa^3+\frac{3}{4}\kappa^4\right)\right]\right\}, \quad (10)$$

where

$$\epsilon=M\omega/(M+\omega). \quad (11)$$

In Eq. (10) the form factors are set equal, i.e.,

$$F_1=F_2=F, \quad F'_1=F'_2=F', \quad \text{and} \quad G_1=G_2=G,$$

the initial 4-momenta p and P of the electron and proton are replaced by $(\mathbf{0},m)$ and $(\mathbf{0},M)$, and the prime of the outgoing electron energy ϵ' is omitted. The substitution

$$\omega\rightarrow-\omega, \quad \epsilon\rightarrow-\epsilon_+,$$

in the large curly brackets in Eq. (10) and a simple change of the factor in front yields the corresponding differential cross section for pair annihilation:

$$\frac{d\sigma}{d\Omega_k}=\alpha r_0^2\frac{1}{2(\epsilon_+a_0)^3}\left\{F^2\left[1+\frac{\epsilon_+\omega}{M^2}(1+2\kappa+\tfrac{3}{2}\kappa^2)\right]\right.$$
$$-GFF'\left[\frac{\omega}{M}+\frac{\epsilon_+\omega^2}{4M^3}(4+8\kappa+9\kappa^2+4\kappa^3)\right]$$
$$+G^2F'^2\left[\frac{\omega^2}{M^2}-\frac{\omega^4}{M^3\epsilon_+}(1-\kappa^2)+\frac{\epsilon_+\omega^3}{M^4}\right.$$
$$\left.\left.\times\left(1+2\kappa+\frac{9}{2}\kappa^2+3\kappa^3+\frac{3}{4}\kappa^4\right)\right]\right\}, \quad (12)$$

where

$$\omega=M\epsilon_+/(M+\epsilon_+). \quad (13)$$

The differential cross sections $d\sigma/d\Omega_e$ and $d\sigma/d\Omega_k$ in Eqs. (10) and (12) are plotted in Fig. 3 as functions of the incident photon and positron energies (ω and ϵ_+) from 50 to 600 Mev, respectively. The exponential form factors[5] are used for F and F', and G is set equal to unity.

However, there is a limitation of the incident photon energy, not greater than 140 Mev. This limit is dictated by the necessity of avoiding background from π^0 decays. Also, at higher energies the resonance in the proton Compton effect[13] begins to introduce appreciable uncertainties into the analysis of the virtual Compton matrix elements[3] of Figs. 1(c) and 1(d) [or Figs. 2(c) and 2(d)], because of the polarizability of the proton corresponding to virtual photopion production. However, just as a matter of simple extrapolation, the differential cross sections are evaluated and plotted in Fig. 3 against the incident energies beyond 140 Mev in order to see the effects of the recoil and anomalous magnetic moment of the proton. It can be seen from Eqs. (10) and (12) for the differential cross sections that at high energies, e.g., ω (or ϵ_+)=500 Mev, the terms due to the anomalous magnetic moment of the proton dominate.

It turns out, in the extreme relativistic limit, that only small values of q^2 (forward photoelectric effect and pair annihilation) contribute to the total cross sections. Consequently, in Eq. (6) the interference term $B\sim(m^2/M\omega)A$ and Compton term $C\sim(m^2/M\omega)^2A$ are negligible compared to the main term A for small values of $q^2\sim2m^2$. Setting the form factors F_1 and F_2 equal to unity, integrating over solid angle, and keeping only the lowest order in the main term A, the total cross section for photoelectric effect reads

$$\sigma_{\text{photo}}=2\pi\alpha^4r_0^2(m/\omega). \quad (14)$$

Similarly, the total cross section for pair annihilation reads

$$\sigma_{\text{pair}}=2\pi\alpha^4r_0^2(m/\epsilon_+). \quad (15)$$

[13] T. Yamagata, L. B. Auerbach, G. Bernardini, I. Filosofo, A. O. Hanson, and A. C. Odian, Bull. Am. Phys. Soc. 1, 350 (1956).

Equation (14) is exactly the same as the one with atomic number $Z=1$ for the photoelectric effect[14] in the extreme relativistic energy limit.

III. CONCLUSION

It was the intention of this paper to see whether photoelectric effect and pair annihilation at high energies could be used to obtain new information on the electron and positron propagators, if one could measure small values ($\sim 10^{-42}$–10^{-40} cm^2/sr) of the differential cross sections for these processes. It turns out, however, that the term of Fig. 1(a) [or Fig. 2(a)], which is sensitive to small-distance modifications, is smaller by a multiplicative factor of mc^2/ω [or (mc^2/ϵ_+)] than the term of Fig. 1(b) [or Fig. 2(b)], which is insensitive to them. This can be seen by comparing the first and second terms in Eq. (2), which correspond to Figs. 1(a) and 1(b) [or Figs. 2(a) and 2(b)], after rationalizing the denominators and making use of the experimental arrangement under consideration. For instance, for an incident energy of 100 Mev, the significant term of Fig. 1(a) is about 0.5% of the term of Fig. 1(b). The contribution arising from the term $B+C$ to the term A in Eq. (6) is 13% for the above incident energy and involves effects of polarization of the proton due to the virtual photopion production which is not considered in the present paper. For incident energies of 150 and 500 Mev, these contributions are 24% and 605%, respectively. It is, therefore, in principle impossible to perform experiments to measure the electron and positron propagators to a reasonable percentage; e.g., 40%.

For a rough estimate of the differential cross sections, the whole calculation may be applied to an atom with higher atomic number Z, by neglecting the binding of the atom and by using a vertex operator for the nucleus similar to the proton vertex operator, if a similar interpretation of two covariant form factors is allowed for the nucleus. If such a bold extension of the phenomenological theory is applicable, the following changes in symbols are required:

$$\Gamma_\mu(q) \to Z\Gamma_\mu(q),$$
$$M \to M_{Z,A},$$
$$\kappa \to \kappa_{Z,A},$$

and

$$a_0 \to a_0/Z,$$

where $M_{Z,A}$ and $\kappa_{Z,A}$ are the mass and anomalous magnetic moment of the nucleus, respectively.

Thus the differential cross sections have an additional factor $2Z^5$ (the factor 2 arises from the fact that the K shell contains two electrons), if one neglects screening effects. Consequently, they increase by the factor 6.15×10^9 for a Au target with $Z=79$: to the order of 10^{-33}–10^{-31} cm^2/sr, which can be measurable. For a nucleus with higher atomic number, the interference term B and Compton term C are negligible compared to the main term A because of the large value of the nuclear mass. The effect of the recoil and anomalous magnetic moment in the main term A is also negligible for the same reason.

For more accurate evaluation of the differential cross sections, the Coulomb wave functions for the bound and outgoing electrons including the effects of anomalous magnetic moment of a nucleus must be used.

When the Coulomb correction[11] is made in the K-shell photoelectric effect in a Au atom, in the extreme relativistic limit, the corrected total cross section will be reduced to 20% of the one calculated by the method mentioned above.

[14] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, New York, 1954), 3rd ed., p. 209, Eq. (18).

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APPENDIX

Glossary of Symbols

$p=(\mathbf{p},\epsilon)$: energy-momentum 4 vector of ingoing electron

$p'=(\mathbf{p}',\epsilon')$: energy-momentum 4 vector of outgoing electron

$p_-=(\mathbf{p}_-,\epsilon_-)$: energy-momentum 4 vector of ingoing electron

$p_+=(\mathbf{p}_+,\epsilon_+)$: energy-momentum 4 vector of ingoing positron

$k=(\mathbf{k},\omega)$: energy-momentum 4 vector of ingoing or outgoing photon

$P=(\mathbf{P},E)$: energy-momentum 4 vector of ingoing proton

$P'=(\mathbf{P}',E')$: energy-momentum 4 vector of outgoing proton

$\{e_\mu\}$: polarization 4 vector of ingoing or outgoing photon

m: electron mass

M: proton mass

$\kappa=1.7926$: anomalous magnetic moment of proton in nuclear magneton unit

$\alpha=e^2/4\pi\hbar c=1/137.0$: fine structure constant

$a_0=4\pi\hbar^2/me^2=0.529\times10^{-8}$ cm: first Bohr radius of hydrogen atom

$r_0=e^2/4\pi mc^2=2.818\times10^{-13}$ cm: classical electronic radius

$u(p)$, $\bar{u}(p')$: Dirac spinors (positive energy) of free electron

$U(P)$, $\bar{U}(P')$: Dirac spinors (positive energy) of free proton

$A\cdot B=\mathbf{A}\cdot\mathbf{B}-A_0B_0$: scalar product of two 4-vectors A and B

$q=p+k-p'=P'-P$

$q'=-r=p-p'=P'-P-k$

$Q=P+P'$

$R=p+p'$

$\sigma_{\mu\nu}=(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)/2i$

$\lambda=p\cdot k$

$\lambda'=p'\cdot k$

$\Lambda=P\cdot k$

$\Lambda'=P'\cdot k$

$$a=\frac{P\cdot e}{\Lambda}-\frac{P'\cdot e}{\Lambda'}$$

$$a'=\frac{P\cdot e}{\Lambda}+\frac{P'\cdot e}{\Lambda'}$$

$$b=\frac{p\cdot e}{\lambda}-\frac{p'\cdot e}{\lambda'}$$

$$b'=\frac{p\cdot e}{\lambda}+\frac{p'\cdot e}{\lambda'}$$

$F_1(q^2)$, $F_2(q^2)$: covariant form factors of charge and current distributions of proton. For the exponential distributions one has $F_{1,2}(q^2)=[1+(a_{1,2}q)^2]^{-2}$, where $a_1=a_2=(\langle r^2\rangle/12)^{\frac{1}{2}}$ and $(\langle r^2\rangle)^{\frac{1}{2}}=0.8\times10^{-13}$ cm.

$d\Omega_e$: differential solid angle along the direction of outgoing electron

$d\Omega_k$: differential solid angle along the direction of outgoing photon.

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Effect of Virtual Excitations on the Elastic Scattering of Electrons and Positrons by Atomic Hydrogen*

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Total elastic cross sections for the scattering of electrons and positrons (neglecting positronium formation) from the $1s$ state of atomic hydrogen have been calculated allowing for virtual excitation to the $2s$ and $3s$ states. The S-, P-, and D-wave contributions to $Q_{1s\text{-}1s}$ have been computed for incident energies below 10 ev. The results for positron scattering show that virtual excitation to the $2s$ and $3s$ states only slightly affects the phase shifts calculated in the static approximation. The influence of the $2s$ state appears to be much more important for electrons. The scattering lengths of these exploratory calculations are compared with the results of other calculations.

INTRODUCTION

In low-energy positron or electron scattering from atomic hydrogen, it is to be expected that the charge distribution of the hydrogen atom will be appreciably distorted due to the impinging particle spending a great deal of time in its vicinity. It is possible to allow for this distortion, theoretically, in a variety of ways.

We shall be interested in incident energies too low for excitation of the target hydrogen atom and too low for positronium formation. However, virtual excitation to higher atomic states and also virtual positronium formation could be used to allow for the distortive effects. The latter process has been considered by Moussa[1] and Spruch and Rosenberg.[2] These authors

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[1] A. H. A. Moussa, Proc. Phys. Soc. (London) 74, 101 (1959).
[2] L. Spruch and L. Rosenberg, Phys. Rev. 117, 143 (1960).