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Theory of the Slow Pinch Discharge. I. Magnetohydrodynamic Stability of the Discharge Core*

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Among the unexplained properties of the slow pinch discharge observed in Zeta, Sceptre, and other experiments is the gross magnetohydrodynamic stability of the discharge. Since the pinch of the magnetic lines of force is approximately constant within the main core of these discharges, the energy principle is applied to such a configuration. The stability condition for interchange modes is $dp/dr \geq -2\gamma p B_\theta^2/r(B^2+\gamma p)$ and, for kink modes, dp/dr should be greater than an undetermined positive quantity. Comparison with experimental results suggests that interchange instabilities are occurring and limiting the negative pressure gradient to the above value, whereas the kink modes can be occurring with, at most, only small amplitude. Possible reasons are given to explain why the kink instability is not observed.

1. INTRODUCTION

IN the toroidal discharge experiments Zeta[1] and Sceptre,[2] which use low electric fields, a type of pinch discharge is produced which persists in an approximately steady state for as long as the electric field is applied, i.e., for milliseconds. Similar discharges have been observed in other toroidal discharge tubes[3,4] and also in straight tubes.[5] In the Perhapsatron, where a considerably higher electric field is applied, the initial magnetic field configuration with skin currents can be seen to degenerate fairly rapidly to the slow-discharge configuration.[6,7] These discharges have been extensively studied over the past few years and it is possible to tabulate many of their characteristic properties. This is done below in Sec. 2.

Several of the properties of these discharges are unusual and have not been explained theoretically. The most notable of these are the gross magnetohydrodynamic stability of the discharge, the large flux of electrons traversing the magnetic field and reaching the wall, the high positive-ion temperatures, and the nuclear reactions. This paper is concerned with the gross stability of the discharge. Subsequent papers will deal with other aspects of the discharge.

One of the experimental properties of these discharges is that within the main core of the discharge the magnetic field has approximately the same pitch at all radii (see Sec. 2). The well-known magnetohydrodynamic energy principle[8] is therefore applied to a discharge with a constant-pitch magnetic field (**B**). Markedly different stability criteria are obtained depending on whether or not the pitch of the instability modes coincides with the pitch of **B**. The modes which have the same pitch as **B** are "interchange" instabilities,[9] since any particular line of force, with the plasma frozen to it, will undergo the same relative displacement at all the points along its length. In this way each line of force will accurately match the line it has displaced and, although **B** may change in magnitude at any point in space, it will not change direction. When the discharge has a constant pitch magnetic field at all radii, the interchange instability can extend throughout the whole discharge. On the other hand, the instability modes which do not have the same pitch as **B** bend the lines of force. For brevity these modes will be referred to as "kink" instabilities, following Johnson *et al.*[9] (It should be noted that the term "kink" as used here includes modes with all values of m and not just $m=1$.)

It is not surprising that different stability criteria are obtained for the two cases. What is at first surprising is that the stability condition is much less severe for the interchange modes than the kink modes. Whereas a substantial negative pressure gradient can be built up before the interchange modes become unstable, the kink modes require a positive pressure gradient. The comparison between theory and experiment in Sec. 4 suggests that only the interchange modes can be occurring in practice.

The outer region of the discharge, which has a varying

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[2] T. E. Allibone, D. R. Chick, G. P. Thomson, and A. A. Ware, reference 1, Vol. 32, p. 169.

[3] G. G. Dolgov-Saveliev, D. P. Ivanov, V. S. Mukhovatov, K. A. Razumova, V. S. Strelkov, M. N. Shepelyev, and N. A. Yavlinsky, reference 1, Vol. 32, p. 82.

[4] C. Etievant, P. Ginot, P. Hubert, P. Rebut, B. Taquet, and A. Torossian, *Proceedings of the Fourth International Conference on Ionization Phenomena in Gases, Uppsala, Sweden, 1959* (North-Holland Publishing Company, Amsterdam, 1960), p. 967.

[5] I. N. Golovin, D. P. Ivanov, V. D. Kirillov, D. P. Petrov, K. A. Razumova, and N. A. Yavlinsky, reference 1, Vol. 32, p. 72.

[6] J. Honsaker, H. Karr, J. Osher, J. A. Phillips, and J. L. Tuck, Nature **181**, 231 (1958).

[7] J. P. Connor, D. C. Hagerman, J. L. Honsaker, H. J. Karr, J. R. Mize, J. E. Osher, J. A. Phillips, and E. J. Stovall, reference 1, Vol. 32, p. 297.

[8] I. B. Bernstein, E. A. Frieman, M. D. Kruskal, and R. M. Kulsrud, Proc. Roy. Soc. (London) **A244**, 17 (1958).

[9] J. L. Johnson, C. R. Oberman, R. M. Kulsrud, and E. A. Frieman, reference 1, Vol. 31, p. 198.

magnetic pitch, will not be considered. It is worth noting, however, that the positive pressure gradients observed for most of this region are of the right order of magnitude to satisfy the sufficient stability criterion[10] $dp/dr \geq 2B_\theta^2/3r$.

2. MAIN EXPERIMENTAL PROPERTIES OF THE SLOW PINCH DISCHARGE

The following results refer in general to discharges for which the initial value of the axial field B_z (i.e., the component parallel to the discharge tube) are in the approximate range from about a quarter to unity times the maximum value of B_θ generated at the tube wall. Above this range little pinch effect occurs and with B_z much below this range the discharge is grossly unstable.[1]

a. Magnetic Field Configuration

The axial magnetic field B_z is enhanced in the discharge and its radial profile exhibits a characteristic bell shape[1,4,11] with a maximum at the center of the discharge. (Since the enhancement persists for such long times with little change, it cannot be due to trapping of the field in the initial discharge constriction, and it is not explained by the paramagnetic effect of the anisotropic conductivity.[1]) The component B_θ, when converted to axial current density, shows that j_z is a maximum at the center of the discharge and falls off slowly with radius.[1,4,11] At a radius which is often about half the tube radius, dB_θ/dr changes sign, and j_z is much smaller beyond this radius although j_θ is often still large.

Over the center of the discharge the pitch of the magnetic lines of force is approximately constant[11] or varies only slowly with radius (see the published field profiles for Zeta,[1] Perhapsatron S4,[7] and TA 2000[4]), but near the walls B_z falls too rapidly with radius to maintain this condition (and sometimes goes negative), so that the pitch decreases appreciably in this region.

Among the slow pinch discharges, only in the case of Sceptre III has the plasma pressure deduced from the magnetic probes measurements been published.[11] The pressure is a maximum in the center of the discharge and falls off with radius throughout the main core of the discharge. At approximately half the tube radius the pressure passes through a minimum and then rises towards the walls. A somewhat similar pressure profile was obtained for the faster discharge in Perhapsatron S4, except that the central maximum has a dip in it.[7]

b. Discharge Stability

With zero or low applied axial magnetic fields, the slow pinch discharge exhibits violent kink instability and large magnetic field fluctuations. Within the magnetic field range considered here this large amplitude kink instability no longer occurs.[1,2] However, smaller magnetic field fluctuations remain whose amplitude is usually in the range 10 to 20% of the unperturbed field[1,11] and whose mean frequency is usually in the range 10^4 to 10^5 cps. On streak photographs light intensity variations are observed which sometimes show helical patterns[5] and sometimes "bars,"[2] these variations being correlated with the field fluctuations.[12] The fluctuations in the central core of the discharge have a wave velocity antiparallel to the axial current,[13] whereas those in the outer regions have a different character and move in the opposite direction.[14]

c. Particle Temperature

The electron temperatures are in the range 7×10^4 to 5×10^5 °K. Values in this range have been obtained from the electrical conductivity[7,11] and also by a fairly accurate spectroscopic measurement.[15] On the other hand, the evidence is growing that the observed Doppler broadening of impurity ion spectral lines[1,2] is due to thermal motion,[12] so that the positive-ion temperatures may be in the range 10^6 to 5×10^6 °K.

d. Electron Energy Loss

Most of the energy given to the electrons by Ohmic heating is lost to the walls.[2,7] Since it has been shown with Zeta that radiation is not sufficient to explain this loss,[16] the energy must be carried to the walls by the electrons themselves. Large wall currents of low-energy electrons have been detected.[12,16] Similar considerations apply to the beginning of the pulse in the faster discharges produced in ceramic tubes,[17] but the rapid vaporization of the tube walls raises the impurity level so that radiation soon becomes the main energy loss.[17,18]

e. Nuclear Reactions

For currents of the order 10^5 amps and above in deuterium, nuclear reactions occur at rates up to about 5×10^6 per pulse.[1,2,7] The centers of mass of the reacting particles have a mean velocity parallel to the current in the z direction in the range 4×10^7 to 6×10^7 cm sec^{-1}. (See Rose *et al.*,[19] Jones *et al.*,[20] and Conner *et al.*[7])

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[10] E. W. Laing, Atomic Energy Research Establishment Report T/M 161, 1958 (unpublished).
[11] N. L. Allen and B. S. Liley, reference 4, p. 937.
[12] A. A. Ware, reference 4, p. 931.
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[14] N. L. Allen, Nature **187**, 279 (1960).
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[16] A. Gibson, D. W. Mason, and G. Barsanti, Bull. Am. Phys. Soc. **5**, 342 (1960).
[17] S. A. Colgate, J. P. Ferguson, and H. P. Furth, reference 1, Vol. 32, p. 129.
[18] J. L. Tuck, reference 4, p. 920.
[19] B. Rose, A. E. Taylor, and E. Wood, Nature **181**, 1630 (1958).
[20] W. M. Jones, A. C. L. Barnard, S. E. Hunt, and D. R. Chick, Nature **182**, 216 (1958).

3. MAGNETOHYDRODYNAMIC STABILITY OF A DISCHARGE WITH CONSTANT PITCH MAGNETIC FIELD

In this section the energy principle will be applied to a cylindrical discharge whose magnetic field is assumed to have a constant pitch at all radii, namely,

$$(B_\theta/B_z)=(2\pi r/\lambda_B)$$

or

$$(1/B_z)(dB_z/dr)=(r/B_\theta)(d/dr)(B_\theta/r), \tag{1}$$

where λ_B is the constant pitch. Small normal mode displacements of the plasma of the form

$$\xi(r)\exp[i(\omega t-m\theta-kz)]$$

are considered, and it is necessary to divide the modes into two classes depending on whether or not their crests (and troughs) are parallel to the magnetic field. The modes whose crests are parallel to the field, the interchange modes, are those which satisfy

$$mB_\theta+krB_z=0, \tag{2}$$

and the kink modes are those for which Eq. (2) becomes an inequality.

The energy principle[8] is derived from Maxwell's equations, the continuity equation and the commonly used magnetohydrodynamic approximations:

$$nM(d\mathbf{V}/dt)=\mathbf{j}\times\mathbf{B}-\nabla p, \tag{3}$$

$$\mathbf{E}+\mathbf{V}\times\mathbf{B}=0, \tag{4}$$

$$(1/p)(dp/dt)=(\gamma/n)(dn/dt), \tag{5}$$

where n is the particle density, M the positive ion mass, $\mathbf{V}$ the plasma velocity, p the plasma pressure, $\mathbf{j}$ the current density, $\mathbf{E}$ the electric field, and γ is the ratio of the specific heats. Rationalized electromagnetic units are used. (The simplifying assumptions associated with these equations are considered in Sec. 4.c.)

a. Interchange Modes

When the relation (2) is satisfied, the change in the potential energy of a cylindrical discharge caused by the displacement ξ reduces to the form given by Tayler.[21]

$$\delta W=\int_0^r \frac{\pi r}{2}dr\left\{[B_\theta^2+B_z^2+\gamma p]|\nabla\cdot\xi|^2 -\left[2B_\theta\frac{d}{dr}\left(\frac{B_\theta}{r}\right)\right]\xi_r^2-\frac{4B_\theta^2}{r}(\xi_r\nabla\cdot\xi)\right\}. \tag{6}$$

When this integral is minimized with respect to ξ_θ and ξ_z, the most unstable modes are found to satisfy

$$(B^2+\gamma p)\nabla\cdot\xi=2(B_\theta^2/r)\xi_r, \tag{7}$$

and the energy increment becomes

$$\delta W=-\int_0^r \pi r dr\left\{B_\theta\frac{d}{dr}\left(\frac{B_\theta}{r}\right)+\frac{2B_\theta^4}{r^2(B^2+\gamma p)}\right\}\xi_r^2. \tag{8}$$

The condition for stability is thus

$$\frac{d}{dr}\ln\left(\frac{B_\theta}{r}\right)\leqslant\frac{-2B_\theta^2}{r(B^2+\gamma p)}. \tag{9a}$$

This relation has been given previously by Tayler,[21] and for the particular case where $B_z=0$ and $m=0$ by Kadomtsev,[22] who points out the convective character of the interchange modes.

With the aid of Eq. (1), the initial equilibrium condition $\mathbf{j}\times\mathbf{B}=\nabla p$ can be written in the form

$$\frac{d}{dr}\ln\left(\frac{B_\theta}{r}\right)+\frac{2B_\theta^2}{rB^2}=-\frac{1}{B^2}\frac{dp}{dr}, \tag{10}$$

so that an alternative form of Eq. (9a) is

$$-(dp/dr)\leqslant[2\gamma pB_\theta^2/r(B^2+\gamma p)]. \tag{9b}$$

Since all the quantities on the right-hand side of (9b) are positive, it follows that in addition to all positive pressure gradients, negative pressure gradients up to this magnitude are stable against interchange instabilities.

In the case of the plasma configuration for marginal stability which is given by the equality in Eqs. (9), a small displacement of the plasma produces no change in the magnetic field. Thus, by taking the curl of Eq. (4) and using the appropriate Maxwell equation, it follows that the change in $\mathbf{B}$, namely $\delta\mathbf{B}$, is given by

$$\delta\mathbf{B}=\nabla\times(\xi\times\mathbf{B}), \tag{11}$$

so that to first order

$$\delta B_\theta=-\xi_r r\frac{d}{dr}\left(\frac{B_\theta}{r}\right)-B_\theta\nabla\cdot\xi, \tag{12}$$

$$\delta B_z=-\xi_r\frac{d}{dr}B_z-B_z\nabla\cdot\xi. \tag{13}$$

δB_r is zero, and from Eqs. (7), (9a), and (1) it is seen that the right-hand sides of (12) and (13) are also zero. A further property of displacements at marginal stability is that δp is zero.

For configurations away from marginal stability it can be shown that δB is small compared with $\xi_r(dB/dr)$ provided $\omega/\omega_{ic}\ll 1$, where ω_{ic} is the ion cyclotron frequency. (The condition $\omega/\omega_{ic}\ll 1$ is already a necessary condition for the validity of the energy principle.) Hence in discharges whose pressure gradients exceed that given by Eq. (9b) by only a small amount, the

[21] R. J. Tayler, Atomic Energy Research Establishment Report L 103, 1959 (unpublished).

[22] B. B. Kadomtsev, J. Exptl. Theoret. Phys. (U.S.S.R.) 37, 1096 (1959).

interchange instabilities will produce only small fluctuations in $\mathbf{B}$. The second-order effect of these interchange modes is to reduce the pressure gradient and hence to self-stabilize the discharge at the pressure gradient given by Eq. (9b).

b. Kink Modes

For the purposes of this paper it will be sufficient to consider those kink modes whose crests and troughs make only a small angle with the magnetic field. Therefore modes are considered with wave numbers m and $(k_0+\delta k)$ where m and k_0 satisfy Eq. (2). For such an instability, the terms in the energy integral involving the quantity $(mB_\theta+krB_z)$ no longer vanish, and the condition for the integral to be a minimum is the more usual condition $\nabla\cdot\boldsymbol{\xi}=0$. In this case the integral reduces to

$$\delta W=\int_0^r \frac{\pi r}{2}dr\Bigg[\xi_r^2\Bigg\{\left(\frac{mB_\theta}{r}+kB_z\right)^2-\frac{2B_\theta}{r^2}\frac{d(rB_\theta)}{dr} +\left(\frac{mB_\theta}{r}-kB_z\right)^2\Big/(m^2+k^2r^2)\Bigg\} +2r\xi_r\frac{\partial\xi_r}{\partial r}\left(k^2B_z^2-\frac{m^2B_\theta^2}{r^2}\right)\Big/(m^2+k^2r^2) +\left(r\frac{\partial\xi_r}{\partial r}\right)^2\left(\frac{mB_\theta}{r}+kB_z\right)^2\Big/(m^2+k^2r^2)\Bigg] \quad (14)$$

(see for example Newcomb[23]) or expanding in terms of δk:

$$\delta W=\int\frac{\pi r}{2}dr\left\{\left[\frac{(mB_\theta/r-k_0B_z)^2}{m^2+k_0^2r^2}-\frac{2B_\theta}{r^2}\frac{d(rB_\theta)}{dr}\right]\xi_r^2 +A_1\delta k+A_2\delta k^2+\cdots\right\}, \quad (15)$$

where the coefficients A_1, A_2, etc., are functions of m, k_0, B_z, ξ_r, and $(\partial\xi_r/\partial r)$.

For small δk only the first term $A_1\delta k$ need be considered, and the condition for stability is

$$\frac{(mB_\theta/r-k_0B_z)^2}{m^2+k_0^2r^2}-\frac{2B_\theta}{r^2}\frac{d(rB_\theta)}{dr}\geqslant-\frac{A_1\delta k}{\xi_r^2},$$

or using the relations (2) and (10),

$$(dp/dr)\geqslant-(rB^2A_1\delta k/2B_\theta^2\xi_r^2), \quad (16)$$

or

$$(dp/dr)\geqslant\lambda^2, \quad (17)$$

where $\lambda^2=|rB^2A_1\delta k/2B_\theta^2\xi_r^2|$, since in general A_1 will not be zero and the sign of δk can be chosen so that the quantity on the right-hand side of (16) is positive. (In actual fact the sign of δk required is negative; i.e., modes with wavelength longer than the interchange modes are the more unstable.) Hence for stability against the modes which are at a small angle to $\mathbf{B}$ the pressure gradient must be greater than this small positive quantity λ^2. For a given pressure gradient, instability will increase with $|\delta k|$ until the higher order terms become important, when δW will probably go through a minimum. For the purpose of this paper, however, it is not necessary to find the most unstable kink mode.

For these modes if the plasma configuration is sufficiently close to that for marginal stability so that $\omega/\omega_{ci}\ll1$, then the plasma displacement will satisfy approximately $\nabla\cdot\boldsymbol{\xi}=0$ and the changes in B_θ and B_z will be approximately $\xi_r(dB_z/dr)$ and $\xi_r r(d/dr)(B_\theta/r)$.

A simple physical picture may help to explain why a small change in the angle of the instability mode with respect to $\mathbf{B}$ should make such a marked change in the stability condition. Consider a discharge with a negative pressure gradient given by the equality in Eq. (9b). A normal mode displacement with troughs and crests parallel to the magnetic field (an interchange mode) will leave $\mathbf{B}$ and p unchanged at all points in space and will be neutrally stable. A similar displacement, with crests and troughs at a small angle to the magnetic field, will also lead to practically no change in p, but in this case the lines of force will be bent. On any particular line of force, at some positions along its length it will be carried outwards, and the plasma frozen to the line will expand to match the lower pressure at larger radii, and at other points the line will move inwards and the plasma will compress. Thus, regions of differing pressures will be connected by lines of force and the plasma will flow along the lines of force towards the low-pressure regions. This will enhance the outward displacement at the low-pressure region, making the original perturbation unstable. As the angle between the crests (and troughs) of the instability and the magnetic field tend to zero, the growth rate of the instability will tend to zero because the distances between high- and low-pressure regions becomes infinitely great.

4. COMPARISON BETWEEN THEORY AND EXPERIMENT

a. Pressure Gradient

Since the experimental results show a substantial negative pressure gradient for the core of the discharge,[11] the relation (17) predicts that the core should exhibit a gross kink instability. In the case of the interchange mode, to compare the predicted stability criterion [Eqs. (9)] with experiment it is necessary to know the plasma pressure. Since there is an unknown integration constant in the pressure deduced from the observed pressure gradient, it is more convenient to compare observed and predicted pressures in this case.

[23] W. A. Newcomb, Ann. Phys. **10**, 232 (1960).

TABLE I. Comparison between experimental and theoretical pressures.

Radius (cm)	Pressure observed by Allen and Liley.[11] (p_0 is the integration constant) (10^5 d cm^{-2})	Pressure calculated from (9b) using observed pressure gradient, etc. (10^5 d cm^{-2})	Percentage error with $p_0=1.3\times10^5$ dynes cm^{-2}
1	$1.04+p_0$	2.9	+24%
2	$0.84+p_0$	2.19	+ 2%
3	$0.56+p_0$	2.28	+23%
4	$0.30+p_0$	1.32	+17%
5	$0.10+p_0$	0.68	−51%
−1	$1.05+p_0$	2.68	+14%
−2	$0.90+p_0$	2.33	+ 6%
−3	$0.72+p_0$	2.07	+ 2%
−4	$0.52+p_0$	1.66	− 9%
−5	$0.33+p_0$	1.1	−32%

In Table I, the second column shows the plasma pressures obtained experimentally by Allen and Liley[11] for the discharge core. p_0 is the integration constant and is the value of the pressure at $r=6$ cm, the position of the pressure minimum. The third column shows the pressures calculated from the equality in (9b) using the observed values for dp/dr, B_θ, and B_z. The last column shows the percentage difference between the two pressures if p_0 is taken as 1.3×10^5 d cm^{-2}, the value for best fit. Assuming that this is the correct value for p_0, the agreement between theory and experiment is as good as can be expected, since dp/dr will have a large experimental error and no allowance has been made for the toroidal geometry.

The range of other values which could be taken for p_0 is limited from other considerations. Thus, the lowest possible value for p_0 is zero; this would make the observed negative pressure gradients twice (at $r=2$ cm) to six times (at $r=5$ cm) the allowable gradient for interchange stability. The upper limit for p_0 cannot be much higher than the value taken, since the integral $\int 2\pi rp dr$ already leads to a value of N which is several times the initial gas filling[11] even if the mean ion temperature is taken as high as 10^6 °K. (N is the number of particles per unit length of the discharge.) These considerations, therefore, suggest that the plasma pressure must be such that the negative pressure gradient is of the same order as, or somewhat greater than, the value for maginal interchange stability.

b. Magnetic Field Fluctuations

The expected fluctuation of B_z due to the kink instability is approximately $\xi_r dB_z/dr$. In the experimental results[11] for Sceptre III, dB_z/dr is approximately $-B_z/r$ at $r=6$ cm and hence the expected percentage fluctuations are $100\xi_r/r$. Since the observed fluctuations are only 15%, this means ξ_r cannot be greater than about $0.15r\approx1$ cm or about 6% of the tube radius. On the other hand, the interchange instability, which produces only small changes in the magnetic field, could be occurring with large amplitude. Since other workers have observed only small fluctuations these conclusions must be general to all slow pinch discharges.

c. Discussion

The above results indicate that the kink instability is either not occurring in the slow pinch discharge or, at most, is occurring with only small amplitude. This reveals an apparent discrepancy between experiment and the predictions of magnetohydrodynamic stability theory. Secondly, the observed pressure gradient suggests that the interchange instability is occurring and is limiting the negative pressure gradient to the value for marginal stability with these modes.

Since stability theory considers only small displacements and neglects products of quantities proportional to the displacement, the discrepancy between theory and experiment could be due to the higher order terms causing stability at finite amplitude. It seems unlikely, however, that a kink displacement which is 6% of the radius is sufficiently large by itself to make second-order terms important. However, because of the large power being generated in the discharge by Ohmic heating, an interchange instability could be occurring continuously with large amplitude. The interchange displacements might be the cause of nonlinear stabilizing terms for the kink modes. However, because of the simplifying assumptions of magnetohydrodynamic theory, there are other possible causes of the discrepancy which do not invoke these higher-order terms.

The major assumptions which must be made in order to derive Eqs. (3)–(5) from the exact Boltzmann equations are:

(a) The particle collision frequencies are large compared with the perturbation frequency (ω), so that the velocity distributions are approximately Maxwellian.

(b) The electron-ion collision frequency is small compared with the electron cyclotron frequency, so that the resistivity term can be neglected.

(c) $\rho\ll L$, where ρ is the ion Larmor radius and L is a characteristic length in the discharge.

(d) $\omega L\lesssim C_i$, where C_i is the mean ion thermal velocity.

When these assumptions are made, Eqs. (3)–(5) can be obtained from more accurate transport equations provided only terms of zero order in the parameter

ρ/L are retained (see for example Bernstein *et al.*[8]). In applying these equations to derive the energy principle[8] the further assumption is made that the plasma has no mass motion.

In most slow pinch discharges the electron temperature and the magnetic fields have been sufficiently high for condition (b) to be satisfied. However, at the high ion temperatures observed in Zeta and Sceptre the deuteron-deuteron collision frequency is of the same order as, rather than larger than, the instability frequencies as required by assumption (a). This means that the perturbation to the deuteron pressure may not be isotropic, but from the work of Rosenbluth and Rostoker[24] it will still be isotropic at marginal stability and the stability condition will be unaffected.

A further effect of the high ion temperatures is that the ion Larmor radius is not negligible compared with the characteristic lengths in the discharge. For example, in the results of Allen and Liley considered in Table I, the Larmor radius is about a third of the characteristic length $p/(dp/dr)$. The higher order terms in the parameter ρ/L could have a stabilizing effect. In connection with assumption (d) the observed fluctuation frequencies are too small to satisfy the equality $\omega L \sim C_i$, but the kink instability growth rates predicted by Eqs. (3)–(5) are of the order required, so that the prediction is consistent with this condition.

A further experimental property of the slow pinch, which is not consistent with the above assumptions, is the presence in the discharge of deuterons with directed velocities well in excess of their mean thermal velocity. That at least a small fraction of the deuterons have such velocities is shown by the energy shift exhibited by the neutrons and protons from the nuclear reactions. However, due to the presence of moderate concentrations of highly ionized impurity ions in these discharges and the fact that $T_d \gg T_e$, the fraction of deuterons with these high velocities may be large. (The main effect of the impurity ions is to reduce the electron drift velocity.[25] On the other hand, the impurity concentration may be such that many of the deuterons can "run away" from the impurity ions. Such deuterons will reach a terminal velocity with respect to the electrons which is approximately equal to what the electron-deuteron drift velocity would be in the absence of impurities. The observed center-of-mass velocities for the reacting deuterons are in reasonable agreement with this expected terminal velocity.)

The direction of this deuteron motion will be along the magnetic field lines, and hence, if a kink perturbation is present, these fast deuterons will experience an effective frequency $\omega' = k'V_d$, where V_d is the deuteron velocity and $2\pi/k'$ is the wavelength of the perturbation measured along the field lines. Simple considerations suggest that, if $\omega' > \omega_{ic}$, the deuterons will exert a stabilizing effect on instabilities which bend the magnetic lines of force and if $\omega' < \omega_{ic}$ the deuterons have a destabilizing effect. In either case the deuterons close to resonance will be the most effective, so that if the number of fast deuterons is increasing with velocity at the point of resonance, there will be a net stabilizing effect. The observed velocity for the fast deuterons—the center-of-mass velocity—is such as to give the cyclotron resonance at wavelengths $(2\pi/k')$ approximately equal to the tube diameter.

[24] M. N. Rosenbluth and N. Rostoker, reference 1, Vol. 31, p. 144.

[25] A. A. Ware and J. A. Wesson, Proc. Phys. Soc. **77**, 801 (1961).

5. CONCLUSIONS

The application of the magnetohydrodynamic energy principle to a discharge with constant-pitch magnetic field shows that the stability conditions for interchange and kink modes are those given by Eqs. (9) and (17), respectively. Comparison between theory and experiment suggests that only the interchange instabilities occur with appreciable amplitude in practice. These modes are self-stabilizing and limit the negative pressure gradient to the value given by Eq. (9).

The slow pinch configuration should still be grossly unstable to kink modes, but the smallness of the observed magnetic field fluctuations shows that they can be occurring at most with only small amplitude. The cause of the kink stability is not known, but it is pointed out that the slow pinch has properties which are not consistent with the assumptions of magnetohydrodynamic stability theory, and which could have a stabilizing effect on the kink modes. Thus, the ion Larmor radius is not small compared with characteristic lengths in the discharge, and there are deuterons present with high velocities parallel to the magnetic field. A further possible cause of the small amplitude of the kink modes is the presence of a large-amplitude interchange instability making nonlinear terms important.

Since the slow pinch discharges are sufficiently stabilized against the magnetohydrodynamic instabilities to make the observed fluctuation frequencies small compared with C_i/L, the standard magnetohydrodynamic equations (3)–(5) are no longer good approximations. The Hall emf, the electron pressure gradient emf, and heat conduction can no longer be neglected. Kadomtsev[22] has shown that these extra terms lead to new instability modes with lower growth rates. The presence of these modes in the slow pinch discharge will be considered in a future paper.

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