Interactions of 7.5-Mev Protons with Copper and Vanadium*†

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The separate cross sections for interaction of 7.5-Mev protons have been measured for natural copper and vanadium. Angular distributions for elastic scattering are presented. The (inelastic scattering+alpha emission) cross sections are: copper, 266 mb, and vanadium, 134 mb. The (p,n) reaction cross sections are: Cu^{65}, 537 mb, and V^{51}, 555 mb. These data, along with separate measurements of polarization from copper, are compared with optical-model computations. The results indicate a volume-absorption potential rather than a surface-absorption potential.

INTRODUCTION

THE last few years have seen extensive qualitative and quantitative success of the optical model for the interpretation of nuclear interactions.[1–3] In the present paper, we are concerned with the optical potential at low bombarding energies (i.e., less than 30 Mev) where for nonbound phenomena the potential can, in principle, be derived from an averaging over the resonance structure of the compound nucleus.[2] Phenomenological analyses of experimental data have not hitherto provided an unambiguous form for the potential, nor have they differentiated conclusively between a concentration of the imaginary (absorptive) part of the potential on the surface of,[4–6] or its distribution throughout,[7–9] the nucleus. Both types of potential have succeeded in fitting experimental data, although from theoretical arguments a surface absorption is expected at low bombarding energies.[10–14]

The proton energy region of 5–10 Mev is of particular interest in medium weight nuclei, since at this energy the proton is reaching the top of the Coulomb barrier and the interaction should therefore be more sensitive to the form of the nuclear potential (see Energy Dependence section). Elastic scattering experiments on medium weight nuclei have been reported at 5.7 Mev,[15] at 6.4 Mev,[8] at 7.5 Mev,[16] at 9.4 Mev,[17] and at 9.8 Mev.[18] Polarization studies have been made at 6 and 7 Mev[19] and at 7.5 Mev.[20] These data must be supplemented with measurements of the nonelastic cross section for unambiguous optical model analyses. At proton energies above 9 Mev, this cross section has been obtained from beam attenuation measurements,[21–24] but at lower energies, charged particles energy loss is primarily an atomic effect. Thus, the attenuation method is not applicable, and the cross section for each of the possible reactions must be separately measured.[25] Extensive studies of (p,n) reaction cross sections have been reported for medium weight nuclei at 5.5 Mev,[26] at 4–6.5 Mev,[27] and at 6.8 Mev.[28,29] Charged-particle emission cross sections were recently measured near 10 Mev for several elements.[30] The (p,n) cross sections near this energy have subsequently been reported for Cu^{63} and Cu^{65},[31] and were combined with the preceding work[30] to obtain total reaction cross sections for Cu^{63} and Cu^{65}.[31] Inelastic scattering to first excited states has also been reported.[32]

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It is important that the measurement of these various cross sections be made at one energy and on one nucleus in view of the possible dependence of the optical potential on target spin,[2] and neutron excess,[33,34] and of the known dependence upon mass number and bombarding energy. Within this framework, the present studies of copper and vanadium were undertaken.

Elastic scattering of 7.5-Mev protons from natural copper and vanadium was measured to give the ratio of elastic to Rutherford scattering cross sections. The total nonelastic cross sections were obtained from measurements of the $[(p,p')+(p,\alpha)]$ cross section for the natural elements and the (p,n) cross sections for Cu^{65} and V^{51}. Q values[35] for other particle-emission reactions are sufficiently negative to preclude them. Proton capture has been neglected as the cross section for (p,γ) is estimated to be at most 1 mb for these elements.[36] An estimate is included for compound-elastic scattering. Earlier polarization measurements[20] are included for the copper analysis.

EXPERIMENTAL PROCEDURE

Elastic Scattering

The scattering targets were 0.25-mil copper and 0.10-mil vanadium foil, 0.3 cm wide by 3 cm high. The method of measuring elastic scattering was developed previously by Waldorf and Wall[16,37]: The flux of protons scattered elastically at some angle θ from a copper or vanadium target is compared with the flux of protons scattered elastically from a gold target at the same angle.

The scattered protons were observed in a movable NaI(Tl) scintillation detector inside a scattering chamber. A measure of the flux of incident particles came from the number of protons scattered elastically at a fixed angle (25°) into a CsI(Tl) monitor detector. The spectrum of pulses from the movable detector was obtained using a twenty channel pulse-height analyzer, and the number of elastically scattered protons was obtained by summing the counts in the appropriate channels. The pulses from the monitor went to an integral discriminator set to accept only pulses corresponding to elastically scattered protons. The pulse-height analyzer and the monitor scaler were turned on simultaneously and were automatically turned off when a prescribed number of counts had been recorded in the monitor counter. Thus, the data which were obtained represent the flux of protons scattered elastically into the movable detector at angle θ per proton scattered elastically into the monitor. If the scattering obeys the Rutherford formula at the monitoring angle, then the ratio of the flux from a target of element X to the flux from a gold target will be the ratio $f_x(\theta)$ of the experimental elastic scattering cross section to the Rutherford scattering cross section.[37] Should the differential cross section at the monitor angle, 25°, not obey the Rutherford formula, the experimental ratios must all be divided by the factor $f_x(25°)$. In the present work, the value for $f_x(90°)$ was independently determined [see section on (p,n) reaction], and the curve of $f_x(\theta)$ vs θ was normalized to this point.

The rms standard error in the precision of the elastic scattering cross section, as determined by repeated measurement, is $\approx 2\%$ for vanadium and $\approx 3\%$ for copper. A possible systematic error of as much as 1% may be present in the selection of elastic scattering from the recorded spectrum. The angles are in error by less than 1%. The normalization to $f_x(90°)$ is accurate to an estimated 4% for vanadium and an estimated 2% for copper.

Inelastic Scattering

The observed inelastic scattering in the present experiments was the sum of (p,p') and (p,α) processes, since the detector did not distinguish alpha particles from protons. For convenience, we refer to both types of particles as *inelastic* particles. Targets were the same as for the elastic scattering studies.

The inelastic particles were observed in the movable detector used for elastic scattering studies. The output from the detector went into a channel-256 pulse-height analyzer. The number of particles scattered inelastically C_i, and elastically, C_e, were computed from the pulse-height distribution spectra making two background corrections. A correction for gamma rays and electrons, $\approx 20\%$, was determined from the spectra observed when the detector was covered by an aluminum absorber sufficiently thick to stop the protons. Particles scattered into the detector crystal from the aluminum detector-housing and lead defining aperture contribute to the inelastic region of the spectra. A correction for these, $\approx 10\%$, was evaluated at small angles where, because of the intense forward peaking of elastic scattering, these background particles predominate over inelastic scattering.

Observations were made of five angles from 70° to 140° and at each angle the differential cross section $d\sigma_{\text{in}}/d\Omega$ for scattering into the solid angle $d\Omega$ was determined from:

$$d\sigma_{\text{in}}/d\Omega = (C_i/C_e) f_x(\theta) d\sigma_R/d\Omega, \quad (1)$$

where $d\sigma_R/d\Omega$ is the differential cross section for Rutherford scattering. It was assumed that the inelastic-scattering differential cross section is isotropic,[30,32] and the total inelastic cross section was thus obtained by multiplying the average differential cross section by 4π.

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[36] B. L. Cohen, Phys. Rev. **100**, 206 (1955).
[37] W. F. Waldorf, Ph.D. thesis, Massachusetts Institute of Technology, 1956 (unpublished).

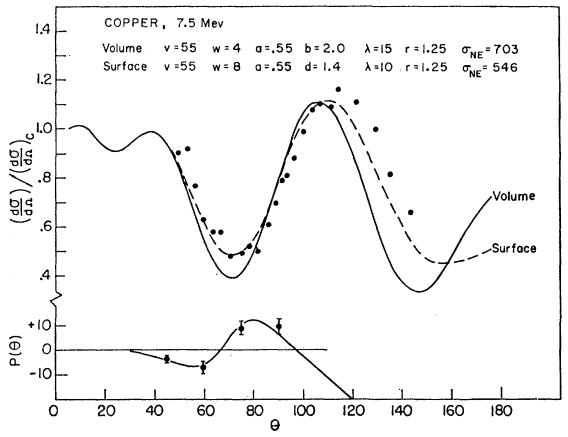

FIG. 1. Elastic scattering from copper. The curves show typical best fits for VA- (solid line) and SA- (broken line) type potentials to the experiments (solid circles). Here v and w are in Mev; a, b, d, and r in fermis ($r=r_{re}=r_{im}$); and σ_{NE} in mb. The σ_{NE} are the nonelastic cross sections predicted by the models. Lower portion of figure shows the fit of these same potentials to the polarization data (solid circles) of Darden (reference 20).

An rms error of 10% is calculated for the inelastic cross section from the following estimates: (a) the gamma-ray background correction is accurate to 2%, (b) the housing- and aperture-scattering correction is accurate to 2%, (c) the dividing point between elastic and inelastic scattering pulses is accurate to 2%, (d) the uncertainty in $f_x(\theta)$ is 4%, (e) the uncertainty in $d\sigma_R/d\Omega$ is 3% from uncertainties in beam energy and detector angle, and (f) the standard error in the average value of the differential cross section, determined from the cross sections at several angles, is 8%.

(p,n) Reaction

The cross sections for the (p,n) reaction on V^{51} and Cu^{65} were determined by measuring the three quantities occurring in the definition of the cross section:

$$\sigma_{p,n}=N/(QS), \qquad (2)$$

where N is the number of (p,n) reactions occurring during a bombardment by Q protons on a target of S atoms/cm^2. For both of the nuclides studied, the product nuclide of the (p,n) reaction is radioactive (27.75-day Cr^{51} and 245-day Zn^{65}),[38] and a few hours after bombardment they constitute the only observable activity induced by 7.5-Mev protons on vanadium and copper. Since the half-life of each of these species is long compared to the usual 1-hr bombardment time, N is equal to the disintegration rate at the end of bombardment divided by the decay constant.

Targets were rectangular metal foils of ≈ 6 cm^2 area held in a target frame in a Faraday cup.[39] All targets were weighed and measured; within the accuracy of this measurement ($\approx 1\%$) the foils were of uniform thickness. The beam of hydrogen-molecule ions emerging from the cyclotron was broken up into protons and electrons by a thin aluminum foil located ≈ 1 m ahead of the target in a region of sufficiently strong magnetic field to sweep the electrons from the beam. A collimator of lead was inserted into the beam several centimeters before the target to ensure that the entire beam entering the Faraday cup would pass through the target foil. The Faraday cup was kept at ground potential, and guard electrodes, at a negative potential to eliminate spurious electron currents, were positioned around it. The current reaching the Faraday cup was electronically integrated with respect to time to obtain the salt charge passing through the target. The integration was done by charging a 0.02 μf polystyrene capacitor with the target current and measuring the voltage across the capacitor to give the charge collected. In the integrating circuit,[40] the positive input current, by means of a positive feed-back amplifier, drives the lower plate of the capacitor negative by the appropriate amount so that the input terminal is held at nearly ground potential. When the capacitor is charged to -50 v (1 μcoul) a Schmitt trigger fires, operating a relay which discharges the capacitor, and a register records the collection of 1 μcoul. The integrator was calibrated to better than 1%. The Faraday cup was provided with a 2.5-cm diam hole in one side, at 90° to the incident beam direction, and the targets were oriented at 45° to the incident beam. The flux of protons elastically scattered out of the cup at a mean angle of 90°, negligible compared with the current into the cup, was measured with a CsI(Tl) scintillation detector to provide an independ-

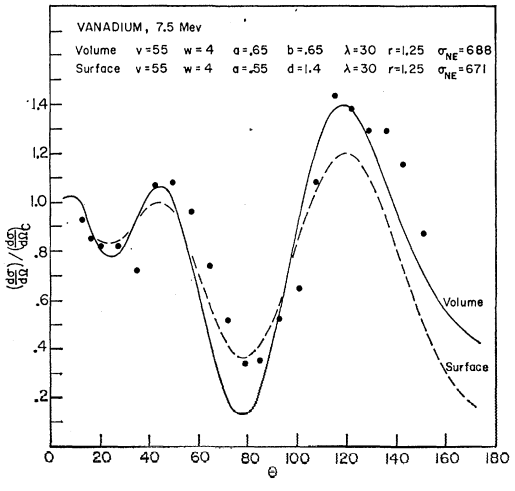

FIG. 2. Elastic scattering from vanadium. Curves show typical best fits for VA- and SA-type potentials to experiments (notation and units same as Fig. 1).

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[39] The authors are indebted to F. Fay for the construction of this cup and the associated apparatus.

[40] Designed by H. A. Enge, Massachusetts Institute of Technology (unpublished). The integrator was made available through the courtesy of Professor Enge.

ent check on current measurements. These scattering observations were used to determine $f_x(90°)$, the normalization point for the elastic scattering data. Deuteron contamination of the beam was less than 0.5%, as determined from the spectra of scattered particles.

Following bombardment, the target foils were removed from the Faraday cup, placed in glass vials, and dissolved in 2.0 ml of a suitable acid. The activity was then compared with the activity of calibrated Cr^{51} or Zn^{65} sources under identical conditions. The standard sources, prepared during the course of this work,[41] were calibrated using x-ray—gamma-ray coincidence-counting techniques. The Zn^{65} source also was compared with Zn^{65} standard sources from the National Bureau of Standards,[42] and further intercomparison of the coincidence-counting techniques with the NBS was done with a Mn^{54} source.[42] The activity of each target was corrected for decay to the end of bombardment.

Note added in proof. J. R. Huizenga has recently raised a question as to the branching ratios in the Zm^{65} decay [Bull. Am. Phys. Soc., Ser. II, **6**, 260 (1961)]. Our coincidence technique would be in error only if the K-capture to total electron capture ratio was different for the ground state and excited state capture processes.

From the repeatability of results, the precision (standard error) of the present work was calculated as 1.3% for copper and 4.0% for vanadium. The accuracy is limited by the following errors: The standard error in comparing the targets to the standards is 1% (counting statistics and imprecision in the preparation of identical standards); the standard error of the coincidence-counting is 3% (counting statistics, background corrections, imprecision in preparing identical sources, and comparison with NBS); neglecting electron and deuteron contamination of the cyclotron beam and error in integrator calibration gives an estimated error of 1%; the uncertainty of the isotopic abundance of Cu^{65} is estimated as 1%[38]; the uncertainty in half-lives is estimated as 0.5%[38]; the target angle of 45° used for calculating the effective target thickness is correct within 1%.

EXPERIMENTAL RESULTS

Elastic Scattering

The angular distribution of elastically scattered protons is shown in Fig. 1 for copper and Fig. 2 for vanadium. These figures also show our best optical-model fits (see Optical Model Analysis section). The copper data differ around 90° from earlier results by Waldorf,[37] shown in Fig. 3. A possible cause for this is discussed under Energy Dependence.

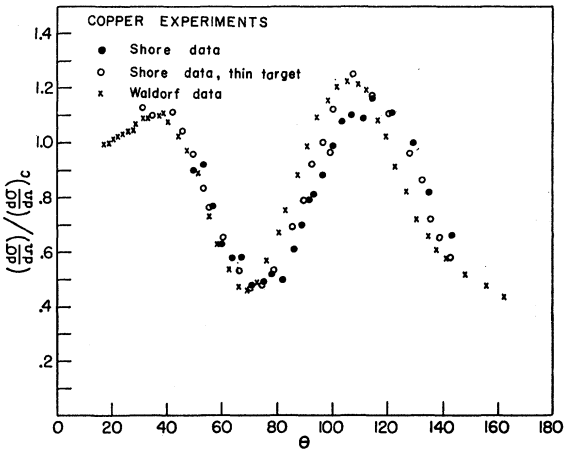

FIG. 3. Observed variations in elastic-scattering from copper. The data used in present analysis are shown as solid circles. Ordinate is ratio of elastic scattering to classical Rutherford-law scattering.

Inelastic Scattering

The total $[(p,p')+(p,\alpha)]$ cross sections are 266±27 mb for copper and 134±13 mb for vanadium.

The vanadium cross section can be assigned entirely to V^{51} (99.75% abundance)[38] and therefore can be added directly to the (p,n) cross section for V^{51} for comparison with the optical model. The (p,α) contribution to the vanadium cross section is ≈1 mb,[43] and the remaining (p,p') contribution is in agreement with an extrapolation of the estimates at 5.0–6.5 Mev by Taketani and Alford.[27]

The contribution of the Cu^{65} isotope to the inelastic cross section, needed for our optical-model comparison, was estimated from two sources: First, the Cu^{65} charged-particle emission cross section measured by Meyer and Hintz[30] for 10-Mev protons; and second, the data of Buechner *et al.*,[44] giving the relative intensities of scattering to individual levels in the separated isotopes, Cu^{63} and Cu^{65}, for 6.5-Mev protons. In calculating the inelastic cross section at 6.5 Mev, a (p,α) cross section of 23 mb is included, extrapolating from the data of Fulmer and Goodman[43] for 8- to 23-Mev protons. Interpolating between the stated 155 mb at 10 Mev and our estimate of 105 mb at 6.5 Mev gives a cross section of 120±30 mb at 7.5 Mev for Cu^{65} $[(p,p')+(p,\alpha)]$. The error represents our estimate of the uncertainty in the interpolation.

(p,n) Reaction

The (p,n) cross sections are 537±21 mb for Cu^{65} and 555±30 mb for V^{51}. The error values are rms standard errors discussed earlier.

[41] B. W. Shore, Ph.D. thesis, Massachusetts Institute of Technology, 1960 (unpublished).

[42] Intercomparisons were conducted through the courtesy of S. B. Garfinkle of the National Bureau of Standards Radioactivity Section.

[43] C. B. Fulmer and C. D. Goodman, Phys. Rev. **117**, 1339 (1960).

[44] M. Mazari, W. W. Buechner, and R. P. de Figueiredo, Phys. Rev. **108**, 373 (1957).

Few precise measurements of (p,n) reaction cross sections in the region of 5- to 10-Mev bombarding energy have been reported. The excitation functions for 60 nuclei determined by Blaser *et al.*[28,29] give cross sections for proton energies up to 6.8 Mev, with a stated accuracy of 10–20%. At 6.8 Mev the cross sections for elements of atomic number between 28 and 32 are all within 40% of 400 mb. The only results at 7.5 Mev are those of Howe,[45] giving the relative $Cu^{65}(p,n)Zn^{65}$ excitation function up to a proton energy of 12 Mev. Normalized to the data of Blaser *et al.*[28] at 6.5 Mev, this work[45] gave a value of 600 mb at 7.5 Mev. Howe[45] also studied the (p,n) excitation function for Cu^{63} and Zn^{66}. At 9.85 Mev, Albert and Hansen[31] have recently reported (p,n) cross sections for Cu^{63} and Cu^{65} of 510 mb and 700 mb respectively, values compatible with our data. A study[26] of 18 medium weight nuclei at energies of 3.5–5.5 Mev showed (p,n) cross sections of $\approx$300 mb at 5.5 Mev for elements with atomic weight from 45 to 70. An extrapolation of these data to 7.5 Mev gives an estimated cross section of $\approx$500 mb for vanadium and copper. The recent work by Taketani and Alford[27] on V^{51} from 4.5 to 6.5 Mev extrapolates to a value of $\approx$450 mb at 7.5 Mev. The present value of 555 mb is not outside the experimental error of the extrapolation and the stated error in reference 27. The extensive data at 12 Mev[46] cannot be extrapolated with confidence to 7.5 Mev.

Compound Elastic-Scattering Estimate

Computations based on the optical model predict: (a) potential scattering, (b) total nonelastic processes, including compound-elastic (CE) scattering, and (c) the polarization of shape-elastically scattered particles. An estimate of the CE cross section is therefore needed before optical model predictions can be compared with experimental data. If one considers the first excited state of the compound nucleus to be the same as the ground state with respect to breakup (the first excited state exit channel may be less favorable than CE scattering due to the energy difference), an estimate of CE scattering can be made using the data of Seward[32] for the inelastic proton scattering from the first excited states of the even-A nuclides Ti^{48}, Cr^{52}, and Fe^{56}. These cross sections are all estimated to be $\approx$120 mb at 7.5 Mev. Also from these data, the cross section for inelastic scattering to the first excited state of V^{51} is slightly over half the similar cross section for Cr^{52}, or 75 mb. As the energy is increased, competition from other reactions makes the compound-elastic exit mode less important; therefore, a better estimate at our energy comes from the 7.5-Mev experiments of Waldorf and Wall,[16] who observed the intensity of scattering to a single excited state of copper to be 10–20% of the scattering to the first excited state of Ni^{58}, or 12–25 mb.

TABLE I. 7.5-Mev proton cross sections.

Process	Cu^{65} (mb)	V^{51} (mb)
$(p,p')+(p,\alpha)$	120±30	134±13
(p,n)	537±20	555±30
CE	25±10	25±10
Total nonelastic	682±40	714±40

Adding the various contributions to the total cross sections we have the results of Table I. The errors are the rms values of the separate errors.

The CE scattering must be subtracted from the observed elastic scattering for comparison with the theoretical curves. Our estimate above of 25 mb, if distributed isotropically, would produce a correction at 160° of $0.15\sigma_R$ for vanadium and $0.10\sigma_R$ for copper. The correction decreases toward smaller angles, becoming negligible below $\approx$90°. Our primary emphasis in fitting the elastic scattering data has been on the angles less than $\approx$100°, and the positions of the maxima and minima in the angular distribution. Parameter selections which produce reasonable fits to our data using these criteria invariably predict less scattering at large angles than we observe. Any correction for CE scattering should tend to improve the fitting of all optical model curves at back angles.

OPTICAL MODEL ANALYSIS

Form of the Potential

Optical model computations were carried out on an IBM-704 computer, using the program of Bjorklund.[47,48] The local potential used in our analyses was:

$$V(\rho)=V_c(\rho)+vF_{\rm re}(\rho)+iwF_{\rm im}(\rho)+\left(\frac{h}{2Mc}\right)^2\frac{\lambda v}{\rho}\frac{dF_{\rm re}}{d\rho}\boldsymbol{\sigma}\cdot\mathbf{l}, \tag{3}$$

where ρ is the radial variable; $V_c(\rho)$ is the Coulomb potential of a sphere, radius $R_{\rm re}$ of uniform charge density; $F_{\rm re}(\rho)$ and $F_{\rm im}(\rho)$ are the radial form factors for the real and imaginary potentials, respectively, with well-depth parameters v and w; and the last term is a (real) spin-orbit interaction term; M is the proton mass, $\boldsymbol{\sigma}$ is the incident proton spin, and $\mathbf{l}$ is the incident proton orbital angular momentum. The form used for $F_{\rm re}(\rho)$ was

$$F_{\rm re}=[1+\exp(\rho-R_{\rm re})/a]^{-1}, \tag{4}$$
$$R_{\rm re}=r_{\rm re}A^{\frac{1}{3}}.$$

Here $r_{\rm re}$ is the real radius parameter, and a is the real diffuseness parameter which determines the tapering of the potential-well edge. Two forms were used for $F_{\rm re}(\rho)$:

[45] H. A. Howe, Phys. Rev. **109**, 2083 (1958).
[46] H. G. Blosser and T. H. Handley, Phys. Rev. **100**, 1340 (1955).
[47] F. Bjorklund and S. Fernbach, Phys. Rev. **109**, 1295 (1958).
[48] The authors are indebted to Dr. Bjorklund (University of California Radiation Laboratory, Livermore) for the use of this program.

First, $H_{\rm im}(\rho)$, corresponding to an absorption of particles throughout the nuclear volume (volume absorption, VA):

$$H_{\rm im}=[1+\exp(\rho-R_{\rm im})/b]^{-1}, \quad (5)$$
$$R_{\rm im}=r_{\rm im}A^{\frac{1}{3}},$$

where $r_{\rm im}$ is the imaginary radial parameter and b is the imaginary diffuseness parameter. Second, $G_{\rm im}(\rho)$, corresponding to absorption of particles at the nuclear surface (surface absorption, SA):

$$G_{\rm im}=\exp\{-[(\rho-R_{\rm im})^2/d^2]\}. \quad (6)$$

In this case, d is the imaginary potential-well surface-thickness parameter. Several computations were also made using a combination of SA and VA potentials of the form

$$F_{\rm im}=c_1H_{\rm im}+c_2G_{\rm im}. \quad (7)$$

Our adjustable parameters were thus: the well depths v and w, the radial parameters $r_{\rm re}$ and $r_{\rm im}$, the diffuseness parameters a and b (or d), the spin-interaction parameter λ, and, when applicable, c_1 and c_2.

Energy Dependence

Many of the previous observations of data fitting[7,9,18,47] such as the evidence for a constant vR^n, are limited in application at 7.5 Mev for copper and vanadium. Predictions here appear more sensitive to parameter details than predictions at higher energies. Since our incident energy is close to the Coulomb barrier height for these elements, the barrier penetrability will be sensitive to the exact shape of the nuclear potential and hence to optical-model parameters. We therefore anticipate that predictable quantities which depend critically upon barrier penetration, the nonelastic cross section in particular, may display rapid changes with slight parameter changes.

We experimented with thinner copper targets (down to 0.05 mil) and found the elastic scattering around 90° to be quite sensitive to thickness (see Fig. 3). We found no evidence for a similar dependence of the (p,n) cross section. These results indicate the scattering is strongly sensitive to either incident beam energy or energy-spread; both the mean projectile energy and the range of energies of scattered particles (the difference between particles scattered at the front and the rear of the target) vary with target thickness.

The sensitivity of optical-model predictions to incident energy for a typical set of fixed parameters is shown in Fig. 4. The elastic scattering predictions differ only at large angles, in contrast to the experimental differences in data (Fig. 3) which are most pronounced at intermediate angles. We conclude that the energy dependence of our model, by itself, will not explain the varied scattering observations. The predicted variation of the nonelastic cross section with incident energy,

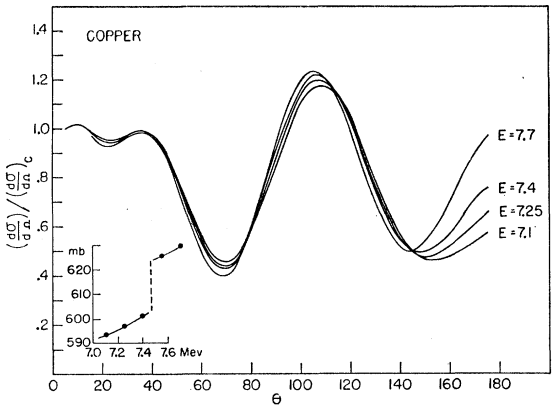

FIG. 4. Energy dependence of elastic-scattering from copper predicted by optical model. Potential is VA type, with $v=57$ Mev, $w=3.5$ Mev, $a=0.52$ f, $b=2.0$ f, $\lambda+25$, and $r_{\rm re}=r_{\rm im}=1.25$ f.

using the same parameters, shows a small but marked discontinuity around 7.5 Mev. An examination of the complex phase shifts obtained as part of the computer output showed a simultaneous discontinuity in the $l=0$ and $l=2$ partial waves. We therefore attribute the cross-section discontinuity to a sudden change in the penetration of these partial waves around 7.5 Mev.

A more likely explanation of the variations of scattering from copper may be in terms of a sensitivity of scattering to energy spread: This varies from 58 to 270 kev for the targets used. This would imply a nonuniform distribution of energy levels, affecting the results of averaging over energy levels. Such a distribution is not unexpected; the excitation energy of the compound nucleus is around 15 Mev in our experiments, close to the energy of the photonuclear giant resonance in Zn.

Analysis of Copper Data

Table II lists several typical sets of parameters which gave satisfactory results for elastic scattering, along with the nonelastic cross section predicted by each set. The range of values indicates the inherent ambiguity of the model. In comparing optical model computations with our experimental data, we found neither a unique set of optimum parameters, nor an indication that such a set exists. A variety of parameter sets gave similar fits to the elastic scattering data, specifically the minimum near 80° and the maximum near 110°. However, this variety is greatly reduced when the nonelastic

TABLE II. Optical model parameters, copper.

	v (Mev)	w (Mev)	a (fermi)	b (fermi)	r (fermi)[a]	λ	$\sigma_{\rm NE}$ (mb)
VA type:	54	4	0.55	1.8	1.25	15	709
	55	4	0.55	1.8	1.25	15	680
	55	4	0.55	2.0	1.25	15	703
SA type:	55	8	0.55	1.4	1.25	15	546

[a] $r=r_{\rm re}=r_{\rm im}$.

TABLE III. Optical model parameters, vanadium.

	v (Mev)	w (Mev)	a (fermi)	b (fermi)	r (fermi)[a]	λ	σ_{NE} (mb)
VA type:	55	4	0.55	0.55	1.25	20	657
	55	4	0.55	1.2	1.25	10	797
	55	4	0.65	0.65	1.25	30	688
SA type:	55	8	0.55	1.4	1.25	30	671

[a] $r=r_{re}=r_{im}$.

cross section must also be predicted by the model. The parameters which produce a large nonelastic cross section also damp out the amplitude of the elastic scattering maxima and minima. This correlation has enabled us to draw several qualitative conclusions:

First, the imaginary potential must be the volume absorption (VA) type, Eq. (5), rather than the surface absorption type (SA), Eq. (6). Figure 1 shows our most satisfactory computations for elastic scattering for the two potential types, along with the nonelastic cross sections predicted from each. For the elastic scattering alone, SA computations gave excellent fits to the experiments, and VA computations were satisfactory. However, our experimental nonelastic cross section is 100–200 mb larger than those predicted by any SA type potential which fits our angular distribution. Similar low nonelastic cross sections are also predicted by linear combinations of VA and SA potentials, Eq. (7), proposed by Easlea and Brown.[49]

Second, using the VA potential, our most satisfactory fits used identical radii for the real and imaginary potentials, but with the imaginary diffuseness parameter b several times as large as the real diffuseness parameter a. This large b is needed to give our large experimental nonelastic cross section. An increased imaginary well depth w or an imaginary potential radius ≈ 1 fermi larger than the real potential radius can also give a larger cross section, but only at the expense of the elastic scattering fit.

Third, the spin-orbit parameter λ, while affecting the large-angle scattering predictions (λ is not the only parameter to produce such effects, however), was primarily determined from the polarization data. The limited experimental data do not justify the inclusion of an imaginary term.[9] A value comparable to that of Nodvick and Saxon[9] gave satisfactory fits to the Darden data[20] (see Fig. 1).

Analysis of Vanadium Data

Table III lists several of the sets of parameters which are satisfactory for vanadium. As with copper, a variety are possible. The parameters which provided the best copper fit do not give reasonable vanadium fits: The predicted nonelastic cross section, for a fixed set of parameters and potential type, is ≈ 200 mb larger for vanadium than for copper. Our experiments give nearly the same value for the two cross sections. The simplest compensation for this is to use a smaller imaginary diffuseness for vanadium. Except for this change, identical parameters can be used for the two elements to give satisfactory fits. Our most satisfactory computations for both SA and VA potentials are shown in Fig. 2. The vanadium data do not provide a clear mandate for either the SA- or the VA-type potential, although the SA type gives a better fit to the experimental maximum and minimum in the angular distribution. As with copper, the most satisfactory VA computations used identical radii for the real and imaginary potentials.

CONCLUSIONS

The oft repeated homily that a more extensive search for optical model parameters is generally fruitful has been confirmed by our experience. The problem of optimizing the seven parameters used in present computations has neither a unique solution nor assurance that such an optimization, restricted only to seven variables, exists.

Our most significant observation, in agreement with Greenlees[24] and Meyer and Hintz,[30] is that nonelastic cross sections are notably larger than predicted using parameters of earlier optical model analyses.[1,2,7–9,47,50] We met this requirement by using a more diffuse imaginary potential than previously reported, thus cutting the reflectivity of the nuclear surface. Our analyses indicate the imaginary potential to be proportional to the real potential (VA type with $r_{re}=r_{im}$).

Note added in proof. To obtain less reflection at the nuclear surface while simultaneously keeping the real and imaginary rounding parameter equal we have recently attempted to fit the upper data with a lower real well depths. Such fits generally give the requisite σ_{NE} but give consistently poorer fits to the angular distribution data.

While it has been mentioned[33,34] that the central well depth of the potential depends upon the neutron excess, the difference between copper and vanadium potentials from this effect is estimated as less than 0.5%, using values from Wyatt *et al.*[34]

An important difference between copper and vanadium analyses at 7.5 Mev originates with the Coulomb barrier; dependence of observables on barrier penetration makes them very sensitive to the details of the potential near the nuclear surface.

Our observations of a possible energy-spread dependence of experiment is related to the formulation of the optical model in terms of the averaging over many levels in the compound nucleus. We question the application of too detailed an optical model analysis without more information on energy level density. Clearly, detailed

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experimental investigation of energy and energy-spread dependence are needed. Our work further shows the need for experiments on separated isotopes for information on nucleus spin and angular momentum interactions. Polarization data, especially at large angles, are also needed.

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Survey of Inelastic Scattering of Deuterons by Heavy Elements*

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Energy spectra of inelastically scattered deuterons from approximately 30 heavy elements are measured with about 80-kev resolution. Many new levels are reported, including a level in Pr^{141} whose discovery substantially alters the decay scheme of Nd^{141}. The gross structure of the spectra is studied and several regularities are noted. Angular distributions in Zr and the even isotopes of Sn indicate that the parity of the strongest levels in the anomalous peaks (~2.5 Mev) are negative, in agreement with the popular assumption that they are the 3^- collective vibrational level; however, there are also several strongly excited positive-parity levels in that region. The correlation between cross sections for exciting given levels by (d,d') and (d,p) or (d,t) reactions is studied. The correlation coefficients are generally slightly negative, but there are several cases where the same levels are strongly excited by all three reactions, including one case (in Sn^{117}) where the principal $d_{\frac{1}{2}}$ single quasi-particle level is also the principal 2^+ vibrational level based on the $s_{\frac{1}{2}}$ ground state. A very strong positive correlation is found between cross sections for exciting given levels by Coulomb excitation and by direct-interaction inelastic scattering. The large peaks reported by Yntema and Zeidman in inelastic deuteron scattering from Rh, Ag, and Sn at 4–5 Mev and from Ta and Pt at about 3 Mev are not found here; explanations for this are offered.

INTRODUCTION

THERE has now accumulated abundant evidence that there are vast differences between (p,p') and (p,n) reactions[1,2] leading to low-lying states of the final nucleus. The former strongly excite the well-known collective levels while the latter excite single-particle levels, the former have an order-of-magnitude larger total cross section, and there are vast differences between the dependences of their cross sections on bombarding energy and target mass. It has further been shown[1,3] that other inelastic scattering processes such as (d,d') and (α,α') are markedly similar to (p,p') in these respects.

A tentative explanation for these facts[4] emerges from the recent work of Baranger,[5] Ferrell *et al.*,[5] Brown *et al.*,[6] Mottelson,[7] and others. They have shown that the collective 2^+ and 3^- states may be expressed as a $T=0$ (relative to the ground state) coherent superposition of particle-hole pairs; i.e., proton particle—proton hole and neutron particle—neutron hole pairs. States consisting of a superposition of these pairs are obviously those excited in inelastic scattering, and the coherent mixture (i.e., all signs positive) will clearly have by far the largest cross section, and indeed will have a large cross section on an absolute basis compared to any sort of single particle reaction. A (p,n) reaction, on the other hand, excites states which are a superposition of proton particle-neutron hole states; the most strongly excited states of this type are coherent mixtures, which then form $T=1$ collective states. These states should be about as strongly excited in (p,n) reactions as $T=0$ collective states are excited in (p,p') reactions; one example of such a state is the giant dipole resonance well known from photonuclear experiments.

However, as shown by Brown[6] and others, the particle-hole interaction is attractive in $T=0$ states, but repulsive in $T=1$ states. Thus, the $T=0$ collective

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