

experimental investigation of energy and energy-spread dependence are needed. Our work further shows the need for experiments on separated isotopes for information on nucleus spin and angular momentum interactions. Polarization data, especially at large angles, are also needed.

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## Survey of Inelastic Scattering of Deuterons by Heavy Elements\*

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Energy spectra of inelastically scattered deuterons from approximately 30 heavy elements are measured with about 80-kev resolution. Many new levels are reported, including a level in  $\text{Pr}^{141}$  whose discovery substantially alters the decay scheme of  $\text{Nd}^{141}$ . The gross structure of the spectra is studied and several regularities are noted. Angular distributions in Zr and the even isotopes of Sn indicate that the parity of the strongest levels in the anomalous peaks ( $\sim 2.5$  Mev) are negative, in agreement with the popular assumption that they are the  $3^-$  collective vibrational level; however, there are also several strongly excited positive-parity levels in that region. The correlation between cross sections for exciting given levels by  $(d,d')$  and  $(d,p)$  or  $(d,t)$  reactions is

studied. The correlation coefficients are generally slightly negative, but there are several cases where the same levels are strongly excited by all three reactions, including one case (in  $\text{Sn}^{117}$ ) where the principal  $d_1$  single quasi-particle level is also the principal  $2^+$  vibrational level based on the  $s_1$  ground state. A very strong positive correlation is found between cross sections for exciting given levels by Coulomb excitation and by direct-interaction inelastic scattering. The large peaks reported by Yntema and Zeidman in inelastic deuteron scattering from Rh, Ag, and Sn at 4–5 Mev and from Ta and Pt at about 3 Mev are not found here; explanations for this are offered.

#### INTRODUCTION

THERE has now accumulated abundant evidence that there are vast differences between  $(p,p')$  and  $(p,n)$  reactions<sup>1,2</sup> leading to low-lying states of the final nucleus. The former strongly excite the well-known collective levels while the latter excite single-particle levels, the former have an order-of-magnitude larger total cross section, and there are vast differences between the dependences of their cross sections on bombarding energy and target mass. It has further been shown<sup>1,3</sup> that other inelastic scattering processes such as  $(d,d')$  and  $(\alpha,\alpha')$  are markedly similar to  $(p,p')$  in these respects.

A tentative explanation for these facts<sup>4</sup> emerges from the recent work of Baranger,<sup>5</sup> Ferrell *et al.*,<sup>6</sup> Brown

*et al.*,<sup>6</sup> Mottelson,<sup>7</sup> and others. They have shown that the collective  $2^+$  and  $3^-$  states may be expressed as a  $T=0$  (relative to the ground state) coherent superposition of particle-hole pairs; i.e., proton particle—proton hole and neutron particle—neutron hole pairs. States consisting of a superposition of these pairs are obviously those excited in inelastic scattering, and the coherent mixture (i.e., all signs positive) will clearly have by far the largest cross section, and indeed will have a large cross section on an absolute basis compared to any sort of single particle reaction. A  $(p,n)$  reaction, on the other hand, excites states which are a superposition of proton particle-neutron hole states; the most strongly excited states of this type are coherent mixtures, which then form  $T=1$  collective states. These states should be about as strongly excited in  $(p,n)$  reactions as  $T=0$  collective states are excited in  $(p,p')$  reactions; one example of such a state is the giant dipole resonance well known from photonuclear experiments.

However, as shown by Brown<sup>6</sup> and others, the particle-hole interaction is attractive in  $T=0$  states, but repulsive in  $T=1$  states. Thus, the  $T=0$  collective

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<sup>1</sup> B. L. Cohen, Phys. Rev. **116**, 426 (1959).

<sup>2</sup> B. L. Cohen, *Proceedings of International Conference on Nuclear Structure, Kingston*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, Canada, 1960), p. 835.

<sup>3</sup> J. L. Yntema and B. Zeidman, Phys. Rev. **114**, 815 (1959).

<sup>4</sup> The author is greatly indebted to M. Baranger for explanations of most of these ideas.

<sup>5</sup> M. Baranger, Phys. Rev. **120**, 957 (1960); see also R. A. Ferrell, Phys. Rev. **107**, 1631 (1957), Bull. Am. Phys. Soc. **4**, 59 (1959); S. Fallieros and R. A. Ferrell, Phys. Rev. **116**, 660 (1959).

<sup>6</sup> G. E. Brown and M. Bolsteri, Phys. Rev. Letters **3**, 472 (1959). G. E. Brown, J. A. Evans, and D. J. Thouless (to be published).

<sup>7</sup> B. R. Mottelson, *Proceedings of International Conference on Nuclear Structure, Kingston*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, Canada, 1960), p. 525.

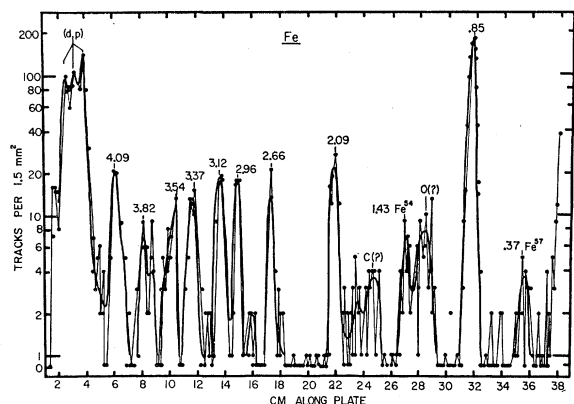


FIG. 1. Data from Fe target at 60°. These are typical of  $(d,d')$  data from light nuclei. Numbers on peaks are excitation energies in Mev. The energy scale is very nearly linear with the abscissa. The standard deviation of ordinates is their square root. Light weight lines connect consecutive data points while heavy line is best estimate of true spectrum. Note that ordinate scale is logarithmic down to 1, but zeros are also shown. Peaks labeled " $(d,p)$ " are from  $(d,p)$  reactions.

states are at low excitation energy (e.g., the  $2^+$  and  $3^-$ ), whereas the  $T=1$  collective states are at high excitation energy (e.g., the giant dipole resonance). Thus, the collective states excited by  $(p,p')$  are at low energy, but the collective states excited by  $(p,n)$  are at high excitation energies; they would not have been observed in 14-Mev  $(n,p)$  experiments, and would not contribute to  $(p,n)$  activation cross sections.<sup>1</sup>

This, then, explains all the differences between  $(p,p')$  and  $(p,n)$ . The  $(p,n)$  reaction leading to low-lying states cannot excite a coherent superposition of particle-hole states, so that the cross section is small and varies irregularly. The  $(p,p')$  excites the coherent mixture and hence has a large cross section which varies slowly and regularly with mass (collective states are well known to occur regularly as a function of  $A$ ). It is also clear that other inelastic scattering processes will also excite these coherent superpositions of  $T=0$  particle-hole states, so that they should exhibit the same general features as  $(p,p')$  reactions.

These ideas suggest that it would be very interesting to tie down more definitely the hypothesis that the so called "anomalous peaks" observed in inelastic scattering<sup>8,9-10</sup> are indeed due to the  $3^-$  collective state, to look for the other predicted collective states,<sup>7</sup> and to test various qualitative and quantitative predictions of the theory for the known collective states. In this paper we present an experimental survey of  $(d,d')$  reactions in heavy elements induced by 15-Mev deuterons. It involves an extensive study of energy spectra with much better resolution than used heretofore in

TABLE I. Absolute differential cross sections (in mb/sr at 60°) for various peaks. No corrections for isotopic abundance or back-ground have been made, so that estimates of cross sections for other peaks in Figs. 3-6 can be made by direct comparison with those listed here. Energies are in Mev.

Element	Energy	$d\sigma/d\Omega$	Element	Energy	$d\sigma/d\Omega$
V	0.30	0.21	Sn <sup>116</sup>	1.27	0.44
Fe	0.85	0.70	Sn <sup>117</sup>	1.02	0.28
Co	1.47	~0.18	Sn <sup>118</sup>	1.22	0.51
Cu	0.97	0.20	Sn <sup>119</sup>	0.92	0.29
Zn	0.99	0.92	Sn <sup>120</sup>	1.17	0.51
Se	0.62	1.8	Sn <sup>122</sup>	1.14	0.54
Y	1.81	0.28	Sn <sup>124</sup>	1.13	0.40
Zr	0.93	0.19	Te	0.46	0.16
Nb	0.96	0.36	Ba	2.83	0.15
Rh	0.33	1.21	Pr	1.14	0.19
Pd	0.44	0.66	Nd	0.46	0.35
Ag	0.42	0.80	Er	0.82	0.23
Cd	0.33	0.24	Ta	0.30	0.63
In	0.59	0.11	Pt	0.34	1.6

any inelastic scattering studies in heavy elements, several angular distribution studies, and elaborate comparisons with other experimental results. Unfortunately, this work was interrupted by a very extended cyclotron breakdown so that the study is not as exhaustive as possible with the techniques used. However, it seems best not to delay further the publication of the results already obtained.

## EXPERIMENTAL

The experiments were done with 15-Mev incident deuterons from the University of Pittsburgh cyclotron. Wedge magnet spectrographs are used to analyze both the incident beam and the reaction products,<sup>11</sup> and the latter are generally detected with photographic plates. In a few angular distribution studies, scintillator detection was employed. Both of these methods have been described previously.<sup>12</sup> Typical data for one light element and one heavy element are shown in Figs. 1 and 2.

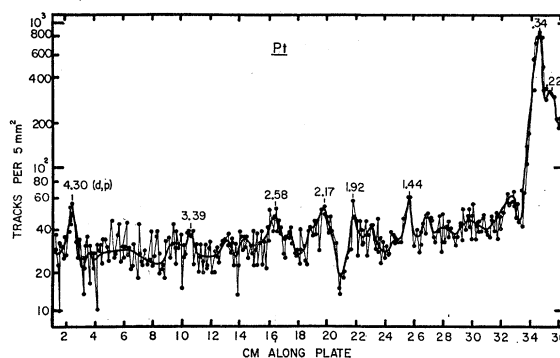


FIG. 2. Data from Pt target at 60°. These are typical of  $(d,d')$  data from heavy nuclei. See caption for Fig. 1.

<sup>8</sup> B. L. Cohen, Phys. Rev. **105**, 1549 (1957).

<sup>9</sup> M. Crut, D. R. Sweetman, and N. S. Wall, Nuclear Phys. **17**, 655 (1960).

<sup>10</sup> J. S. Blair, Phys. Rev. **115**, 928 (1959); D. K. McDaniels, J. S. Blair, S. W. Chen, and G. W. Farwell, Nuclear Phys. **17**, 614 (1960); J. Thirion, Comp. rend. **249**, 2189 (1959).

<sup>11</sup> R. S. Bender, E. M. Reilley, A. J. Allen, R. Ely, J. S. Arthur, and H. S. Hausman, Rev. Sci. Instr. **23**, 542 (1952).

<sup>12</sup> B. L. Cohen, J. B. Mead, R. E. Price, K. S. Quisenberry, and C. Martz, Phys. Rev. **118**, 499 (1960).

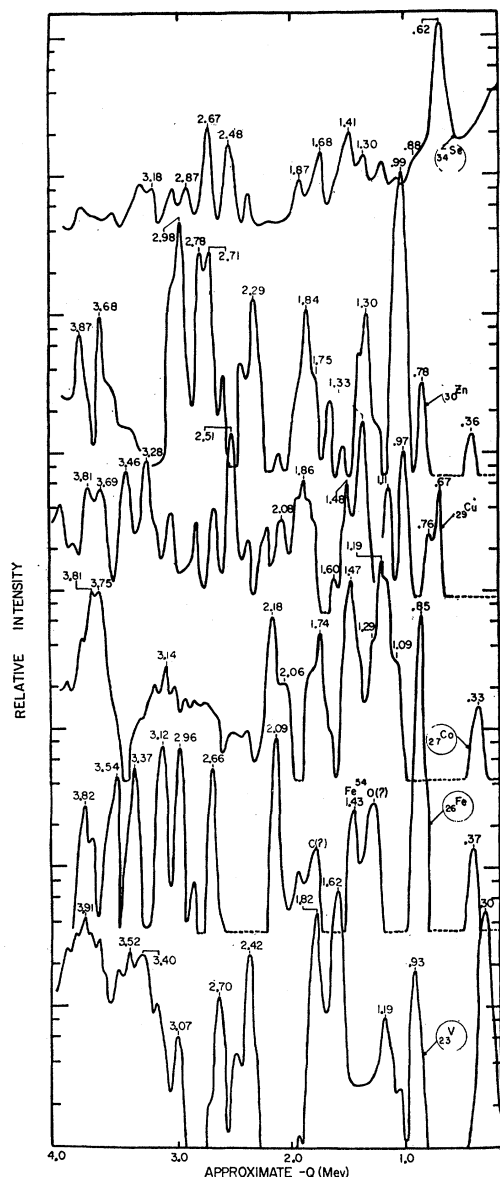


FIG. 3. Energy spectra of deuterons inelastically scattered from nuclei of atomic number 23-34. Angle of observation is  $60^\circ$ . Numbers on peaks are excitation energies in Mev; they are more accurate than energy scale shown on axis. Ordinate scale is logarithmic, but different curves are shifted arbitrarily. "C" or "O" on peaks indicates that they are probably elastic scattering peaks from carbon or oxygen, respectively. Dashed horizontal portions indicate that curves are below background level.

In the lighter elements, individual levels are generally resolved up to about 3-Mev excitation energy, and background is negligible, whereas in the heaviest elements resolution is frequently a limiting factor, and background from elastic scattering introduces at least some difficulties.

The energy resolution is about 80 kev for most targets. In a few cases where target quality is poor, it is considerably worse.

## RESULTS AND DISCUSSION

### A. Good Resolution Survey of Energy Distributions

The results of the survey of energy spectra measured at  $60^\circ$  are shown in Figs. 3-6. Excitation energies for some of the prominent peaks and some of the less prominent peaks reported in other work are shown on the figures; they may be used to establish the energy scale. The absolute differential cross sections for one level in each spectrum are listed in Table I; cross sections for other levels may be estimated from them.

The data for the lighter group of elements, shown in

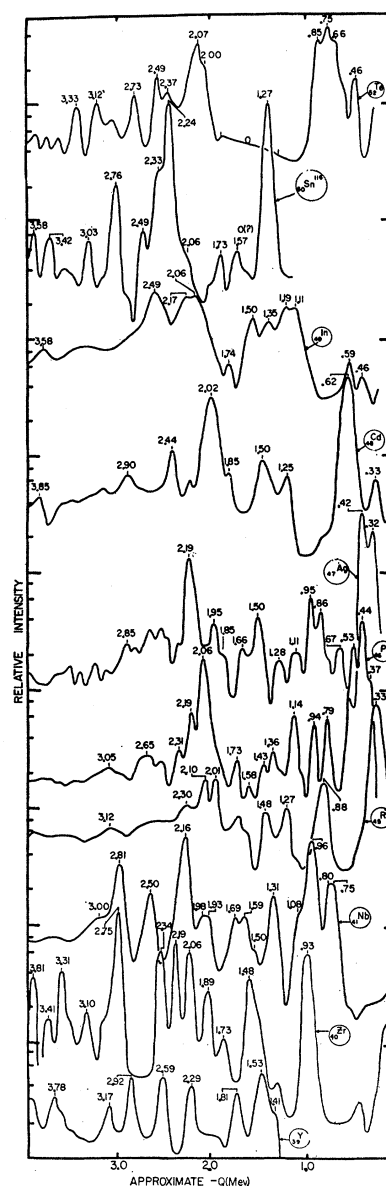


FIG. 4. Energy spectra of deuterons inelastically scattered from nuclei of atomic number 39-52. See caption for Fig. 3.

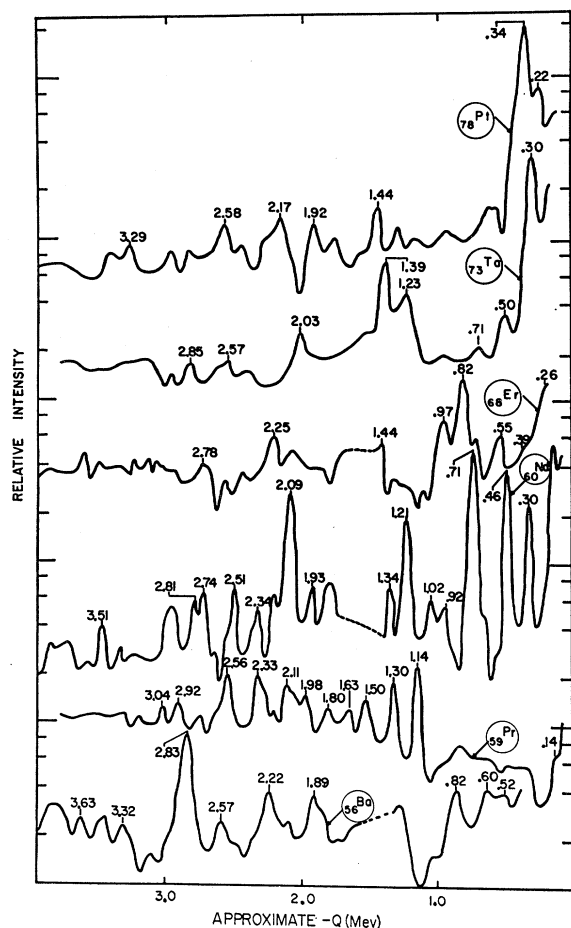


FIG. 5. Energy spectra of deuterons inelastically scattered from nuclei of atomic number 56-78. See caption for Fig. 3. Dashed lines indicate portions of spectra that are obscured by oxygen impurity peaks. Portions of the Pr and Ba spectra were measured at  $45^\circ$  and were arbitrarily normalized to remainder of spectra.

Fig. 3, offer many opportunities for comparison with other data up to about 2.5- or 3-Mev excitation energy. In V, all levels up to 2.42 (all energies are in Mev; this will generally not be stated) are given in reference 13 with the exception of 1.19 which has been reported previously<sup>14</sup> but was not accepted in reference 13. All the levels seen in Fe up to 3.12 Mev are known from other work<sup>15</sup> and the energies agree within 5 kev.<sup>16</sup> The levels in Co are also in good agreement with known levels<sup>15</sup> up to 2.09 Mev, although the peaks at 1.47, 2.06, and 2.18 are known to consist of several individual closely spaced levels; the apparent complexity

of the levels at 1.19 and 1.74 can be explained by impurities. In Cu, there is a one-to-one correspondence between observed and previously known levels up to 2.08 Mev, except for the structure on the sides of the 1.86-Mev peak which could be due to carbon, and lack of evidence for a 2.01-Mev level which could easily be lost in the background.

Thus, the data for nuclei between V and Cu up to about 2.5 or 3 Mev compare favorably with other accurately known data. The average discrepancy is about 5 kev, and there are essentially no discrepancies larger than 20 kev; this may serve as an index of the accuracy of other data given in this paper. For higher excitation energies, the known levels form essentially a continuum of levels with the experimental resolution used here. The sharp structure observed in many cases may therefore be considered as due to intensity variations rather than the occurrence of levels.

For nuclei heavier than copper, the data on known levels are much less abundant, and many of the data presented here are the best available. In Zn, the levels at 0.78 and 1.30 are not known; the strongly excited levels at 2.71, 2.78, and 2.98 are very probably the anomalous levels<sup>16</sup> in the isotopes of mass 68, 66, and 64, respectively. In Se, the level shown at 0.62 is a combination of known first excited states at about this energy in each of the major isotopes; the other observed levels agree well with those from reference 16, but assignments of levels to specific isotopes cannot be made.

Figure 4 shows the results for nuclei between Y and Te. In Y, each of the levels reported in reference 16 is found; where there are discrepancies, the present data should be considered much more reliable. The levels shown are levels in  $Y^{89}$ ; it is interesting to note that the known 14 sec isomeric level at 0.91 is not excited observably here. All levels observed in Zr up to 2.34 are known from decay scheme work, and 2.75 is known from reference 16.

Niobium is especially interesting since it has been studied intensively by  $(n, n'\gamma)$  reactions.<sup>17</sup> The correspondence between that data and this is essentially perfect up to 1.31 Mev; all levels above this, however, are about 0.03 Mev higher in the present data, which may indicate that these levels actually decay to the 0.03 Mev first excited state rather than to the ground state. In addition, an extra level is found in the present data at 1.59 Mev. Above 2 Mev, there are 3 strong levels in the anomalous region; they agree roughly with reference 16. In Rh, the two Coulomb-excited levels at 0.30 and 0.36 are strongly excited here but not resolved. The other two known levels at 0.54 and 0.65 are very weakly excited as might be expected from the facts that they correspond to pure single proton excitations, and cannot contribute to the quadrupole collective excitation as their parity is opposite to that of the ground

<sup>13</sup> *Nuclear Level Schemes, A=40-A=92*, compiled by K. Way, R. W. King, C. L. McGinnis, and R. van Lieshout, Atomic Energy Commission Report TID-5300 (U. S. Government Printing Office, Washington, D. C., 1955).

<sup>14</sup> H. J. Hausman, A. J. Allen, J. S. Arthur, R. S. Bender, and C. J. McDole, *Phys. Rev.* **88**, 1296 (1952).

<sup>15</sup> *Nuclear Data Sheets*, National Academy of Sciences, National Research Council (U. S. Government Printing Office, Washington, D. C.).

<sup>16</sup> B. L. Cohen and A. G. Rubin, *Phys. Rev.* **111**, 1568 (1958).

<sup>17</sup> N. Nath, D. M. Van Patter, M. A. Rothman, and C. E. Mandeville, quoted in reference 15.

state. The higher excited states of Rh are not known from other work except for reference 16. The resolution in that work was somewhat poorer, but its correspondence with this is readily discernible.

In Pd, the interpretation is complicated by the multiplicity of high abundance isotopes, but all levels up to 1.73 Mev are known from decay scheme work; the anomalous region shows more structure than reference 16 because of the better resolution. The peaks in Ag at 0.67, 1.11, 1.66, and 1.95 have not been reported previously; the others are known from decay scheme data and reference 16. The Cd data give little information because of the complex isotopic mixture; the low-lying levels are known from other work. For In, the lowest observed level at 0.46 is not known from other work. The peaks observed between 1.11 and 1.50 can perhaps be explained by known levels, but there is some difficulty in understanding the 1.50 peak as such. The anomalous region agrees well with reference 16. The Sn data will be discussed below in connection with Fig. 6. The low-lying levels in Te are all well known; the anomalous region is very much better resolved than in reference 16. The high-energy peaks for Te seem especially interesting since they are quite sharp in spite of the undoubtedly large level density in that region.

The heavy-element data are shown in Fig. 5. For Ba, the levels up to 2.22 are all known with the exception of 0.52. The higher energy region has not been investigated previously; the 2.83 level is almost certainly due to  $\text{Ba}^{138}$  because of its high intensity.

The 0.14 and 1.30 levels in  $\text{Pr}^{141}$  are known from the decay of  $\text{Nd}^{141}$ ; the decay scheme also includes a strong 1.14-Mev  $\gamma$  ray which was assigned as a transition between these states. The appearance of a 1.14-Mev peak in Fig. 5, however, suggests that the strong  $\gamma$  ray is a transition from the previously unknown 1.14 level to ground. The fact that it is populated by an allowed electron capture transition from the  $\frac{3}{2}^+$  ground state of  $\text{Nd}^{141}$  indicates that its spin and parity are  $\frac{1}{2}^+$ ,  $\frac{3}{2}^+$ , or  $\frac{5}{2}^+$ . The apparent absence in the  $\text{Nd}^{141}$  decay scheme of a transition to the  $\frac{7}{2}^+$  state at 0.14 in competition with the transition to the  $\frac{5}{2}^+$  ground state suggests that the 1.14 level is not  $\frac{5}{2}^+$  and probably not  $\frac{3}{2}^+$  (if the  $M1$  and  $E2$  matrix elements are not untypical). It is therefore very probably  $\frac{1}{2}^+$ , as expected from shell model. The 1.30 level is then very probably  $\frac{3}{2}^+$  as expected from shell model. Its assignment<sup>15</sup> as  $\frac{5}{2}^+$  was based on the strong 1.14-Mev  $\gamma$  ray to the  $\frac{7}{2}^+$  level which we now explain differently. All of the higher energy peaks in the Pr data of Fig. 5 must also be assigned as levels in  $\text{Pr}^{141}$ , the only isotope of that element.

The levels in Nd up to 1.34 are all known previously; the high-energy region has not been investigated previously. Er has a well-known rotational spectrum; in the two even isotopes, 0.26 and 0.55 are the  $4^+$  and  $6^+$  second and third rotational levels, respectively, so that it is perhaps somewhat surprising that they are so strongly excited in these reactions. The 0.82 level is

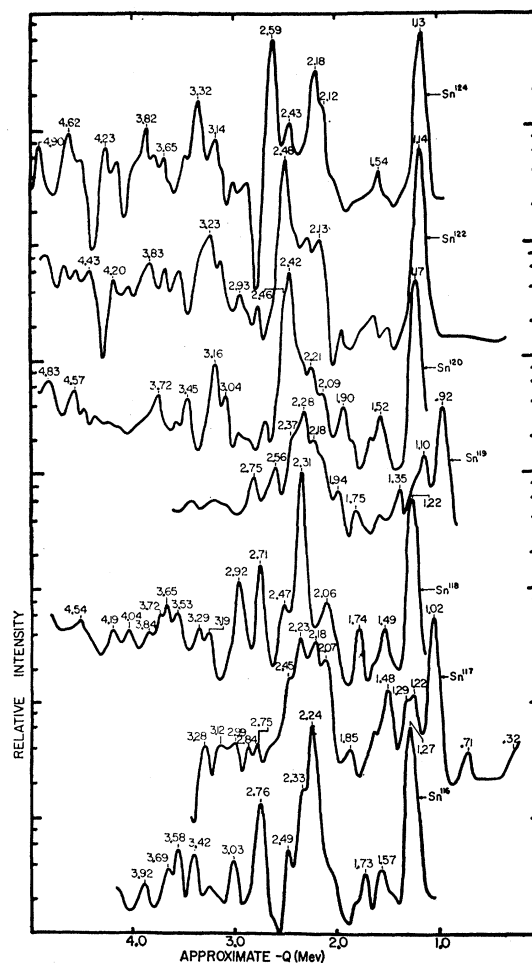


Fig. 6. Energy spectra of deuterons inelastically scattered from various isotopes of Sr. See caption for Fig. 3.

the ground state of a  $K=2$  excited rotational band, so that its strong excitation is also somewhat unexpected. In Ta, the other distorted nucleus studied, higher members of the ground-state rotational band are again excited: the 0.30, 0.50, and 0.71 levels are thought<sup>15</sup> to be the  $11/2^+$ ,  $13/2^+$ , and  $15/2^+$  members of the ground-state ( $\frac{7}{2}^+$ ) rotational band. Here, however, the intensity ratios among the three lines is quite large. All higher energy peaks must be due to previously unknown levels in  $\text{Ta}^{181}$ , as Ta is monoisotopic. In Pt, the low-energy levels are known from decay scheme work; the high-energy region again shows a rather large amount of structure for a heavy element with three major isotopes.

The data for the Sn isotopes are shown in Fig. 6. The isotopic purity of the targets is sufficient to assure that all labeled peaks represent levels in the target isotope. There is some possibility of slight oxygen and carbon contamination; elastic peaks from these would give apparent peaks at about 1.52 and 2.10 Mev, respectively, with extra width, due to angular resolution, of about 100 kev.

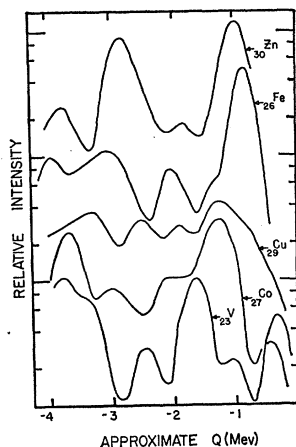


FIG. 7. Low-resolution energy spectra of inelastically scattered deuterons from lighter nuclei. Resolution has been reduced by calculation so as to have Gaussian shape with 450-kev full width at half maximum; elastic peaks are removed before smearing resolution.

In  $\text{Sn}^{116}$ , the only previously known levels observed here are the  $1.27-2^+$  and  $2.76-4^+$ . In  $\text{Sn}^{117}$ , only the levels up to 1.02 Mev were known. In  $\text{Sn}^{118}$ , the  $1.22-2^+$  was known previously; three higher levels known from  $\text{Sb}^{118}$  decay are not excited here. In  $\text{Sn}^{119}$ , only the 0.92 level was known. In  $\text{Sn}^{120}$ , the 1.18 is a well-known  $2^+$  level; levels at 2.21 and 2.42 are known from the decay of  $\text{Sb}^{120m}$ , where they are assigned as  $4^+$  and  $6^+$ , respectively, because  $\gamma$ -ray transitions connecting them and connecting 2.21 to 1.18 are pure  $E2$  from internal conversion data. However, evidence will be presented below that 2.42 is of odd parity, and very probably  $3^-$ . It is, of course, possible that there are two different levels at about the same energy. In both  $\text{Sn}^{122}$  and  $\text{Sn}^{124}$ , only the first excited ( $2^+$ ) states were known previously.

### B. Gross Structure of Energy Distributions

In order to aid in viewing the gross structure of the energy distributions the data of Figs. 3-6 was mathematically "smeared-out" to a Gaussian resolution function with 0.45-Mev full width at half maximum; the results are shown in Figs. 7-10. They are equivalent to what would be measured with the best scintillation resolution, except that the elastic peak is completely removed; this eliminates a very major source of difficulty in measurements with scintillators.

The most interesting gross structure effects are those in Fig. 8 which covers atomic numbers 30-56. The lowest energy peaks are due to the well-known one-phonon vibrational states. Their energy variation is chiefly featured by increases near closed shells for 50 neutrons (Sr, Y, Zr) and 50 protons (Sn). The two-phonon vibrational states are not strongly excited. The anomalous peak,<sup>8</sup> between 1.8 and 3.2 Mev, is very strong throughout the region. In most cases, it appears to be double with the higher energy portion having generally lower intensity. Further evidence on this is obtainable from the data for the Sn isotopes, shown in Fig. 10. There seems to be a fairly regular structure in the 3-4 Mev region; it is not even difficult to imagine

regularities in this region from the high-resolution Sn data of Fig. 6. Considering the even isotopes in order from 116 to 124, it is impossible to ignore the regularity in the 2.24, 2.31, 2.42, 2.48, and 2.59 levels, all of which are the most strongly excited levels in their respective spectra; further evidence on this regularity from angular distributions is presented below (Sec. C). There is strong temptation then to imagine a regularity in the 2.76,  $2.71+2.92$ , 3.16, 3.23, and 3.32 peaks, and another in the  $3.58+3.42$ , 3.65, 3.72, 3.83, and 3.82 peaks. Several other regularities at lower excitation energy are also easily imagined from Fig. 6.

In the lighter elements whose gross structure is shown

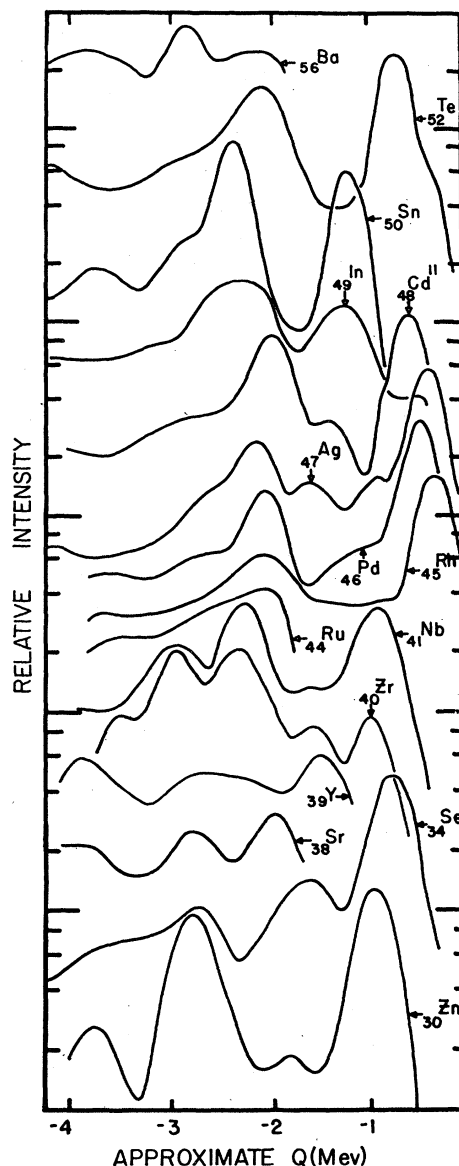


FIG. 8. Low-resolution energy spectra of inelastically scattered deuterons from medium weight nuclei. See caption for Fig. 7.

in Fig. 7, the regularities seem to be strongest among the even  $Z$  elements as a group and among the odd  $Z$  elements as a group. This was also found in the  $(p, p')$  data.<sup>8</sup> However, it is difficult not to derive the impression from Fig. 7 that the  $1.62+1.82$  in V, the  $1.19+1.47$  in Co, and the  $0.97+1.33$  in Cu are in some way the analog of the  $2^+$  first excited states in the even nuclei (0.85 in Fe and 0.99 in Zn). In addition, it is difficult not to believe that the broad peaks in V at 3.8, in Co at 3.8, in Cu at 3.3, and in Fe at 3.1 are the analog of the  $2.98+2.78+2.72$  peaks in Zn which have been identified<sup>16</sup> as the anomalous peak in the three principal Zn isotopes.

The gross structure effects in the heavy elements are difficult to surmise from Figs. 5 and 9. The variations in the low-energy peaks are all well understood. The sharp difference between Pr and Nd is due to the fact that the only isotope of Pr has 82 neutrons so that its first vibrational state lies at  $\sim 1.2$  Mev. The 82 neutron isotope of Nd ( $Nd^{142}$ ) has its first vibrational state at 1.57 Mev, but Nd has several other isotopes whose neutron shells are not closed so that their vibrational states lie lower (0.70 in  $Nd^{144}$ , 0.46 in  $Nd^{146}$ , and 0.30 in  $Nd^{148}$ ). The low-lying states of Er are not shown in Fig. 9 as they are not resolved from the ground states.

Perhaps the most striking feature of Fig. 9 as compared to Fig. 10 is the lack of a sharp anomalous peak. One can imagine the 2.83 peak in Ba, the 2.09 peak in Nd, and perhaps the 1.39 peak in Ta as being associated with it, but this is far from definite, and the evidence for it is essentially nonexistent in Pr and Pt. Furthermore, the energy does not seem to be consistent with its known location at 2.6 Mev in the Pb isotopes.<sup>8</sup>

The fact that the anomalous peak is missing in this region was surmised on the basis of much poorer data in reference 16. It was pointed out there that the nuclei involved here are either distorted or easily distortable. It may be that this causes the anomalous group to be greatly broadened, so that it is not easily recognizable.

### C. Angular Distribution Studies

It has been widely speculated that the "anomalous peaks" found in inelastic scattering with  $-Q \approx 2-3$  Mev are due to the  $T=0$ ,  $3^-$  collective oscillation mode.<sup>7</sup> However, there is essentially no direct experimental confirmation for this. An opportunity to test this hypothesis is presented by the recent work of Blair<sup>10</sup> who showed that angular distributions of inelastically scattered particles for even-parity levels are "out of phase" with those for odd-parity levels. Indeed this method has been used<sup>9,10</sup> to establish the parity of levels in lighter nuclei that may well be associated with the anomalous peaks. However, the most striking evidence for the anomalous peak is in the  $Z=40-52$  region (see Fig. 8), and this region has not been investigated previously with this technique because of resolution difficulties.

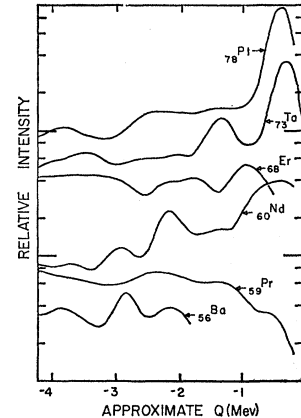


FIG. 9. Low-resolution energy spectra of inelastically scattered deuterons from heavy nuclei. See caption for Fig. 7.

The method consists essentially of comparing angular distributions of inelastically scattered deuterons which excite the well-known  $2^+$  vibrational level (first excited state in even-even nuclei) with those which excite prominent levels in the anomalous region. If the present interpretation of the theory is correct, they should have opposite parities so that the respective angular distributions should be out of phase. It is readily shown that the difference in energies is of essentially no consequence.

Some results are shown in Fig. 11 for Zr, and in Figs. 12, 13, and 14 for the Sn isotopes. In Zr, the four principal peaks in the anomalous region are approximately out of phase with the  $0.93-2^+$  level, although they are not perfectly in phase with each other. This latter fact may be explained by the presence of other weak levels which are not perfectly resolved, so that it seems quite reasonable to conclude that the anomalous peaks in Zr have negative parity; each is probably due to a different isotope, although this remains to be established.

In the Sn isotopes, the most strongly excited level in the anomalous region does seem to have negative parity, but there are relatively strongly excited levels in this region which apparently have positive parity (e.g., 2.18

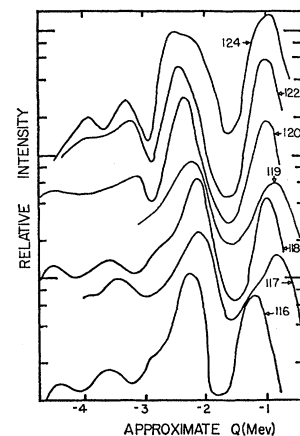


FIG. 10. Low-resolution energy spectra of inelastically scattered deuterons from various isotopes of Sn. See caption for Fig. 7.

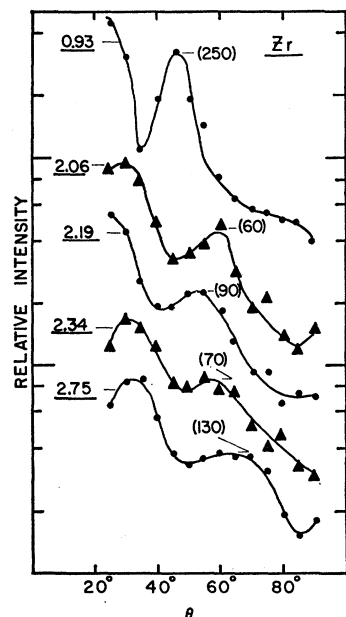


FIG. 11. Angular distributions of various peaks from  $Zr(d,d')$  reactions. Underlined numbers are values of  $-Q$  (see Fig. 4) in Mev, and numbers in parentheses are relative intensities at the levels shown.

in  $Sn^{124}$ ), or which give angular distributions which are not simple to analyze in this way. The latter cases may be explained as mixtures of unresolved levels, some with each parity.

In general, the evidence here shows that the principal contribution to the anomalous peaks have opposite parity from the ground states, which supports the contention that the anomalous peaks are indeed the  $3^-$  collective excitation. With regard to the other levels

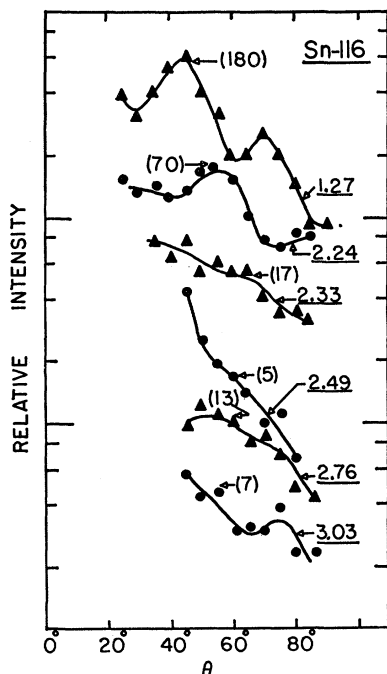


FIG. 12. Angular distributions of various peaks from  $Sn^{116}(d,d')$  reactions. See caption for Fig. 11.

present, it should be pointed out that the energy used in these experiments is somewhat below that for maximum excitation of the anomalous peaks<sup>16</sup>; presumably at higher bombarding energies, the other levels would be still less important.

Since we have now identified the  $2^+$  and  $3^-$  levels in the various even  $Sn$  isotopes, it is interesting to note the dependence of their energy on mass number. This is shown in Fig. 15; the values for the odd isotopes are taken as the center of the broad peaks which are clearly associated with the collective states. It is readily apparent that the energy variations with mass are very regular and are in opposite directions for the  $2^+$  and  $3^-$  state. The energies are lower for the odd isotopes than for the even isotopes, but by a far larger amount for the

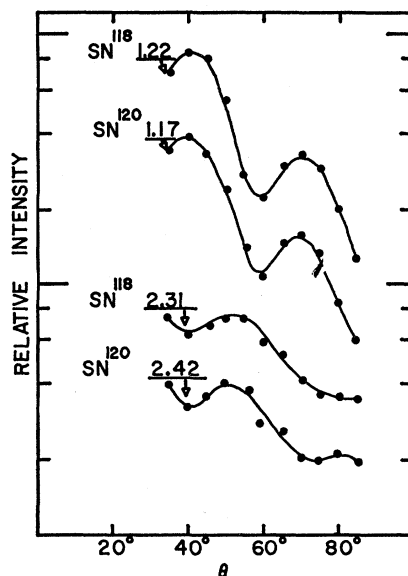


FIG. 13. Angular distribution of various peaks from  $Sn^{118}$  and  $Sn^{120}(d,d')$  reactions. See caption for Fig. 11.

$2^+$  than for the  $3^-$ . There is no very clear explanation for these behaviors. They are at least in some ways inconsistent with what is known about the behavior of these levels in  $Zn$  and  $Zr$ ; this indicates that it is very probably not a symmetry energy effect, but rather a straightforward mass effect.

#### D. Correlation with $(d,p)$ and $(d,t)$ Reactions

In view of the fact that  $(d,p)$  and  $(d,t)$  reactions excite single-particle states (actually single quasi-particle states) while  $(d,d')$  reactions excite collective states, one might expect intuitively that there should be a negative correlation between the strengths with which given levels are excited by the two types of reaction. However, there is good evidence that this is much too simple a picture. In a study of the excitation of first-excited states of even-even nuclei (the well-known



collective  $2^+$  states) by  $(d,p)$  and  $(d,t)$  reactions, it was found<sup>18</sup> that these were somewhat less strongly excited than the ground states (well-known single-particle states), but there were exceptions, and in general the difference was not large. Furthermore, one must consider mixing of collective and single-particle character between levels at nearly the same energy with the same spin and parity. Another factor to be considered is that there should be a negative correlation between the cross section for exciting given levels by  $(d,p)$  and  $(d,t)$  reactions,<sup>19</sup> so that this must complicate the negative correlation between each of these and  $(d,d')$ .

In order to study these matters, cross sections for exciting various levels in the Sn isotopes by  $(d,d')$  reactions are plotted in Figs. 16–21 against the cross section for exciting these same levels by  $(d,p)$  or by  $(d,t)$

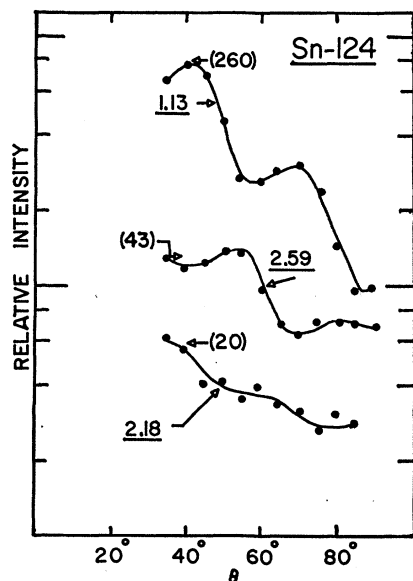


FIG. 14. Angular distributions of various peaks from  $\text{Sn}^{124}(d,d')$  reactions. See caption for Fig. 11.

reactions in the appropriate isotopes.<sup>19</sup> A positive or negative correlation would be evidenced if the points grouped about lines with slopes of  $+45^\circ$  or  $-45^\circ$ , respectively. In some cases, the energies are not known with sufficient certainty to be sure that the levels excited in the two reactions actually are the same levels. In all doubtful cases, it is assumed that they are the same; this biases the data so as to make the correlation more positive. A similar effect results from plotting unobserved levels at their upper limits; such data are shown in the figures by arrows pointing in the directions where the true values probably lie.

The general impression obtained from Figs. 16–21 is that the correlation is zero or slightly negative. In view of the aforementioned biases, it seems fairly cer-

<sup>18</sup> B. L. Cohen and R. E. Price, Phys. Rev. **118**, 1582 (1960).

<sup>19</sup> B. L. Cohen and R. E. Price, Phys. Rev. **121**, 1441 (1961).

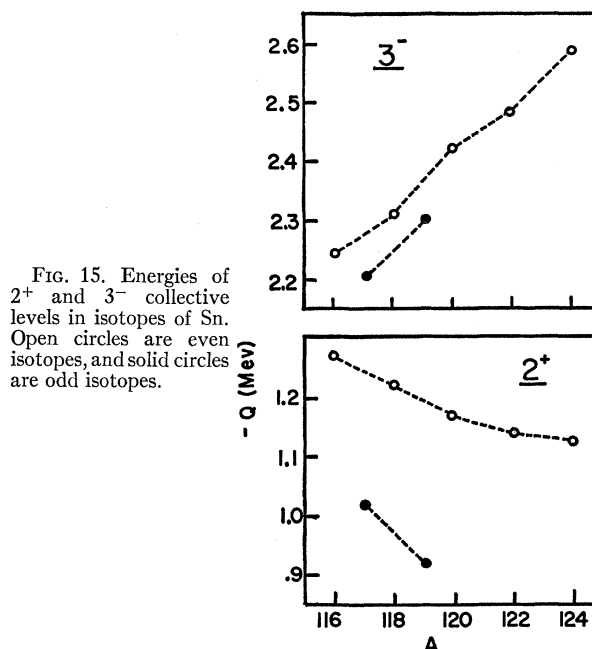


FIG. 15. Energies of  $2^+$  and  $3^-$  collective levels in isotopes of Sn. Open circles are even isotopes, and solid circles are odd isotopes.

tain that there is at least something of a negative correlation in agreement with the simple theory. However, there are several notable exceptions to this general trend. In the even isotopes (Figs. 16–19), first excited ( $2^+$ ) states are relatively strongly excited by  $(d,t)$  re-

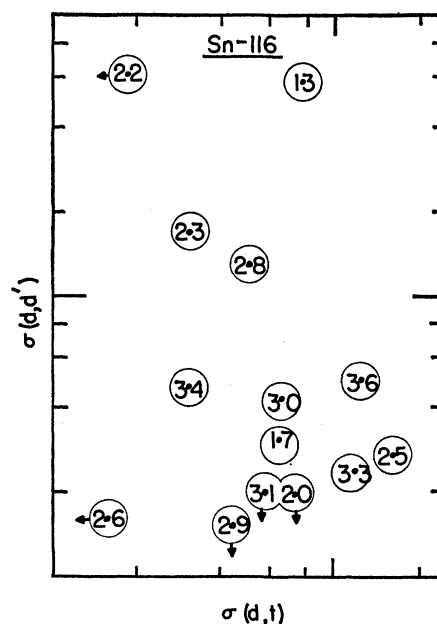


FIG. 16. Correlation between  $\sigma(d,d')$  in  $\text{Sn}^{116}$  and  $\sigma(d,t)$  in  $\text{Sn}^{117}$  for exciting given levels of  $\text{Sn}^{116}$ . Numbers give excitation energies of levels in Mev; decimal point should be between the two digits. Arrows indicate upper limits.  $\sigma(d,d')$  is taken proportional to differential cross section at  $60^\circ$ , and  $\sigma(d,p)$  is taken proportional to differential cross section at  $30^\circ$ . Cross sections are in arbitrary units. The levels at 1.3 and 2.2 Mev are the  $2^+$  and  $3^-$  collective levels, respectively.

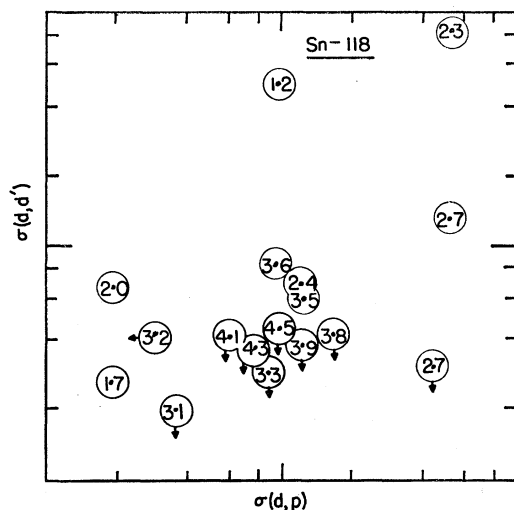


FIG. 17. Correlation between  $\sigma(d,d')$  in  $\text{Sn}^{118}$  and  $\sigma(d,p)$  in  $\text{Sn}^{117}$  for exciting given levels of  $\text{Sn}^{118}$ . See caption for Fig. 16. The levels at 1.2 and 2.3 Mev are the  $2^+$  and  $3^-$  collective levels, respectively.

actions as well as by  $(d,d')$ , in agreement with reference 18; it should be noted, however, that absolute cross sections for exciting any single level in even-even nuclei are rather small because of low fractional parentage coefficients<sup>19</sup> and unfavorable statistical factors. The  $3^-$  collective states in the even isotopes are rather strongly excited by  $(d,p)$  reactions as well as by  $(d,d')$ , which can perhaps be explained as a mixing of these states with the single-particle level of configuration  $(s_{1/2}f_{7/2})_3$  which is expected to lie in the same energy region. In Fig. 18, it appears as though the  $3^-$  state is also fairly strongly excited by  $(d,t)$ ; this would correspond to a configuration  $(s_{1/2}f_{5/2}^{-1})$  which should be at a much higher excitation energy.<sup>20</sup> This is a region of high level density, so that the levels excited in the two experiments may be different; no such effect is observed in  $\text{Sn}^{116}$  (Fig. 16).

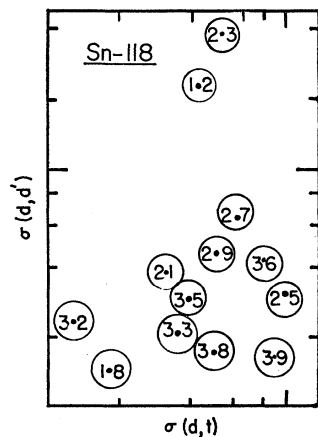


FIG. 18. Correlation between  $\sigma(d,d')$  in  $\text{Sn}^{118}$  and  $\sigma(d,t)$  in  $\text{Sn}^{119}$  for exciting given levels of  $\text{Sn}^{118}$ . See caption for Fig. 16. The levels at 1.2 and 2.3 Mev are the  $2^+$  and  $3^-$  collective levels, respectively.

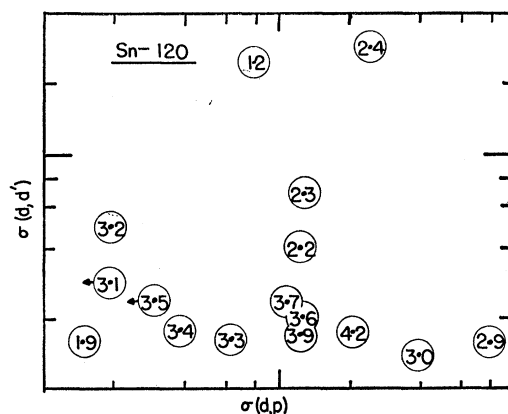


FIG. 19. Correlation between  $\sigma(d,d')$  in  $\text{Sn}^{120}$  and  $\sigma(d,p)$  in  $\text{Sn}^{119}$  for exciting given levels of  $\text{Sn}^{118}$ . See caption for Fig. 16. The levels at 1.2 and 2.4 Mev are the  $2^+$  and  $3^-$  collective levels, respectively.

In the odd isotopes of Sn (Figs. 20 and 21) perhaps the most notable feature is the mixing between the  $d_{5/2}$  single quasi-particle state, and the  $2^+$  vibrational state based on the  $\frac{1}{2}^+$  ground state. The positions of these states, listed in Table II, are known from the fact that they give  $l_n=2$  angular distributions in  $(d,p)$  reactions.<sup>19</sup>

The relative cross sections with which each of these levels is excited in  $(d,p)$ ,  $(d,d')$ , and  $(d,t)$  reactions are listed in Table II. The situation is quite different in  $\text{Sn}^{117}$  than in  $\text{Sn}^{119}$ . In the former, the same nuclear level (1.02) is most strongly excited in each of the three reactions. In order to ascertain that there are not two very close-lying levels, the energy was accurately measured for the three cases; it was identical within a possible error of 10 kev. Furthermore, two levels of the same spin and parity cannot lie close together without very strong mixing which would result in separating

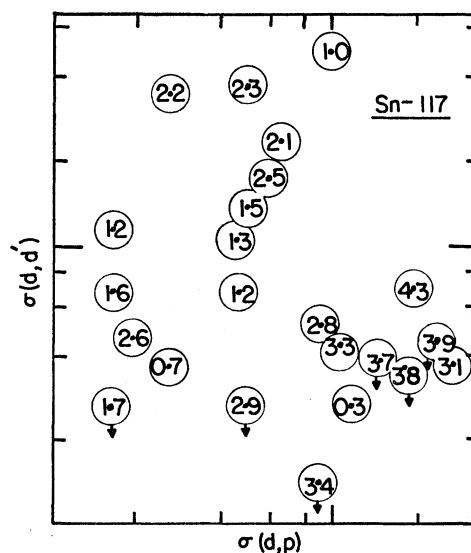


FIG. 20. Correlation between  $\sigma(d,d')$  in  $\text{Sn}^{117}$  and  $\sigma(d,p)$  in  $\text{Sn}^{116}$  for exciting given levels of  $\text{Sn}^{117}$ . See caption for Fig. 16.

<sup>20</sup> B. L. Cohen and R. E. Price, Nuclear Phys. 17, 129 (1960).

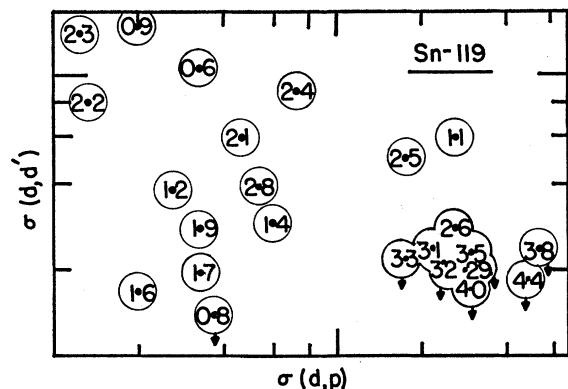


FIG. 21. Correlation between  $\sigma(d, d')$  in  $\text{Sn}^{119}$  and  $\sigma(d, p)$  in  $\text{Sn}^{118}$  for exciting given levels of  $\text{Sn}^{119}$ . See caption for Fig. 16.

them drastically, so that the possibility that 1.02 in  $\text{Sn}^{117}$  is a double level is very remote. Thus the same nuclear level is both the principal  $d_{5/2}$  single quasi-particle level and the principal  $2^+$  vibrational level. Yoshida<sup>21</sup> has shown that this is indeed possible with fortuitous parameters for the unperturbed levels. In  $\text{Sn}^{119}$ , on the other hand, the situation is more in line with expectations; the principal  $d_{5/2}$  single quasi-particle level is at 1.09 whereas the principal  $2^+$  vibrational level is at 0.92. There is still, however, a considerable amount of mixing.

### E. Comparison with Coulomb Excitation

It is tempting to hypothesize that the cross section for direct interaction inelastic scattering to a given level depends on the degree to which that level is col-

TABLE II. Excitation cross sections at  $60^\circ$  for  $\frac{5}{2}^+$  levels in odd Sn isotopes by  $(d, p)$ ,  $(d, d')$ , and  $(d, t)$  reactions.

$E$ (Mev)	$\text{Sn}^{117}$ $d\sigma/d\Omega$ (relative)			$E$ (Mev)	$\text{Sn}^{119}$ $d\sigma/d\Omega$ (relative)		
	$(d, p)$	$(d, d')$	$(d, t)$		$(d, p)$	$(d, d')$	$(d, t)$
1.02	48	69	61	0.92	7	64	9
1.18	25	$\leq 14$	26	1.09	79	24	70
1.50	27	17	$\leq 12$	1.35	15	12	21

lective. The best available quantitative index for the latter quantity is the value of  $B(E2)$  obtained from Coulomb excitation experiments. Our hypothesis can therefore be checked by studying the correlation between  $B(E2)$  and the cross section for inelastic scattering at an angle sufficiently large that actual Coulomb excitation should not be important. This was done in reference 16 for  $(p, p')$  reactions, and a reasonably good correlation was found. However, because of the improved resolution, a much more extensive study may be made with the data from this work.

Figure 22 shows a plot of cross sections for  $(d, d')$  reactions observed in the present work vs  $B(E2)$  from Coulomb excitation experiments.<sup>22, 23</sup> The numbers in that figure are atomic numbers of the nuclei. There is clearly a strong positive correlation, but the data seem to lie along different lines for light ( $Z=23-34$ ) than for heavy ( $Z=45-78$ ) nuclei, with the exceptions of the Nd isotopes and the low-abundance isotope  $\text{Te}^{125}$ . The latter might be explained away if one of the abundant isotopes of Te has a level at the same energy, but the Nd results are apparently true exceptions. The difference between light and heavy nuclei was also observed in reference 16.

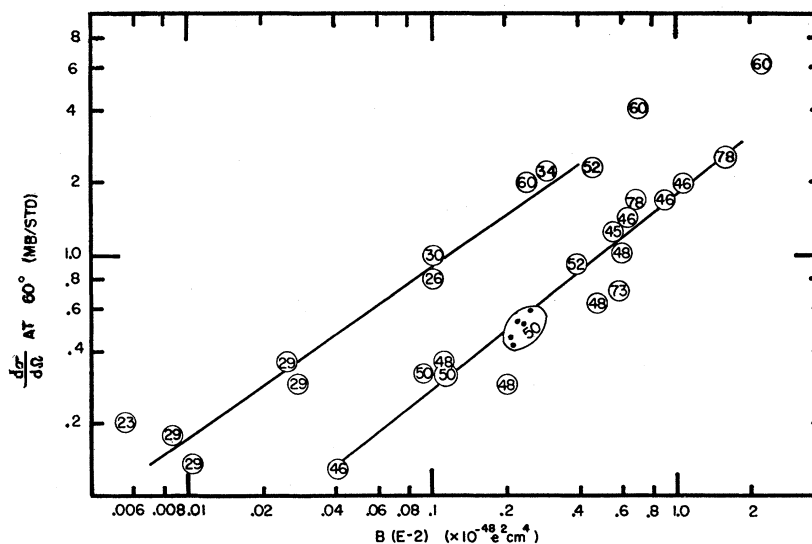


FIG. 22.  $d\sigma/d\Omega$  at  $60^\circ$  for exciting levels by  $(d, d')$  reactions vs  $B(E-2)$  for exciting these same levels by Coulomb excitation.

<sup>21</sup> S. Yoshida (private communication).

<sup>22</sup> K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Revs. Modern Phys. 28, 432 (1956).

<sup>23</sup> P. H. Stelson and F. K. McGowan, Phys. Rev. 110, 489 (1958). D. G. Alkhazov, D. S. Anreev, K. I. Erokhina, and I. Kh. Lemberg, J. Exptl. Theoret. Phys. 33, 1347 (1957) [translation: Soviet Phys.-JETP 6, 1036 (1958)].

The choice of  $B(E2)$  as an index of the degree to which a level is collective is not completely clear. Arguments might be made for using the ratio  $B(E2)/B(E2)_{SP}$ , the denominator representing the single-particle value,<sup>22</sup> or  $d\sigma_{CE}/d\Omega$ , the cross section for Coulomb excitation as calculated from the standard formulas.<sup>22</sup> A plot of observed cross sections vs the latter quantity is shown in Fig. 23; it is obtained from Fig. 22 by multiplying the abscissas by  $C/Z^2$ , where  $C = (2ME/\hbar^2) \times f(60^\circ, \xi)_{\xi=0} \times 10^{-48} \text{ cm}^2$ , a constant for all data in the present experiment.<sup>22</sup> The clear-cut distinction between light and heavy nuclei is now largely removed, but there is a tendency for data from each mass region to cluster together. It is very important to point out that the abscissa is not the actual Coulomb excitation cross section, since the standard formulas<sup>22</sup> ignore penetration of the nuclear surface. The latter occurs for angles larger than  $30^\circ$  for even the heaviest nuclei<sup>24</sup>; at  $60^\circ$  the standard formulas give much too large a cross section since the classical orbit for  $60^\circ$  deflection penetrates the nucleus quite substantially, and the actual wave function inside the nucleus is strongly attenuated by the imaginary potential (i.e., other reactions take place). It is therefore clear from Fig. 23 that direct-interaction inelastic scattering is the predominant process in these experiments.

If  $B(E2)/B(E2)_{SP}$  had been chosen as the abscissa, a plot intermediate between Figs. 22 and 23 would be obtained, as the abscissas in such a plot are obtained by dividing the abscissas of Fig. 22 by  $A^{1/2}$ . The important result of this section remains, however, that there is a very strong correlation between cross sections for direct interaction inelastic scattering and the degree to which a level is collective.

#### F. Search for 5-Mev Peaks of Yntema and Zeidman

In a low-resolution survey of inelastic deuteron scattering at 21.6 Mev,<sup>3</sup> large peaks were reported at

<sup>24</sup> Massachusetts Institute of Technology Laboratory Nuclear Science Progress Report, May 1951, NP 3151 (unpublished); also February, 1951, NP 3019 (unpublished).

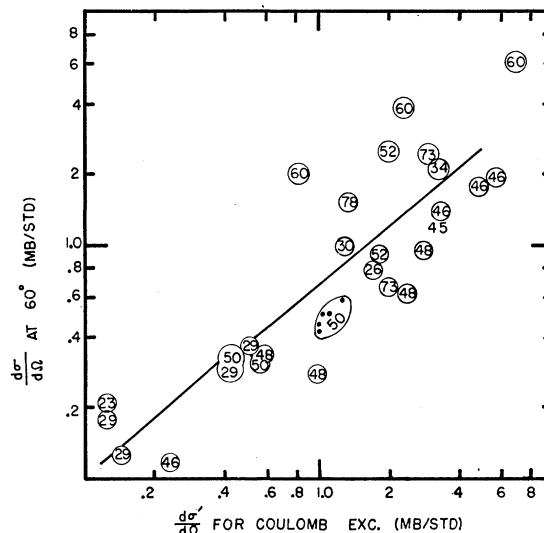


FIG. 23.  $d\sigma/d\Omega$  at  $60^\circ$  for exciting levels by  $(d,d')$  reactions vs  $d\sigma'/d\Omega$ , the cross section for exciting these same levels by Coulomb excitation according to standard formulas.<sup>22</sup>

about 4–5-Mev excitation energy in the spectra from Rh, Ag, and Sn. Since photographic plate spectra are obscured in this region by protons from  $(d,p)$  reactions, a search for these peaks was made using a scintillation detector at the focus of the particle-analyzing magnet; the protons and deuterons were then easily separated by pulse-height analysis. No trace of these peaks was found; the spectrum continues to decrease monotonically with increasing excitation energy. The large peaks at about 3-Mev excitation in Ta and Pt reported in reference 3 are also not seen here (see Fig. 9).

It is perhaps worth noting that all of these peaks are at approximately the energy where large peaks of tritons from  $(d,t)$  reactions are expected. The experimental method of reference 3 would give some difficulty in distinguishing these from inelastically scattered deuterons, so that these peaks may be due to tritons.<sup>25</sup>

<sup>25</sup> Note added in proof. The authors of reference 3 have kindly informed me that they have found the peaks in question to be due to tritons.