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Nucleon-Antinucleon Mechanism for Pion-Pion Scattering Resonances

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The Chew-Mandelstam N/D equations for pion-pion scattering were modified to include contributions from the nucleon-antinucleon intermediate state, which was estimated in perturbation theory as well as by limitations imposed by unitarity alone. It was found that the Frazer resonance could not be obtained by such a simple mechanism, starting from S-wave dominant solutions.

An investigation was made as to whether or not the nucleon-antinucleon intermediate state might provide a simple mechanism for the Frazer $T=1$ P-wave pion-pion resonance proposed to explain the nucleon electromagnetic structure data.[1] Our philosophy is to proceed by iteration, starting with the subtraction constant λ as a first approximation, and retaining only S and P waves. In the elastic approximation of Chew and Mandelstam[2] this corresponds to starting from S-wave dominant solutions. We would expect that, if a mechanism is properly chosen, the P-wave dominant solution would appear instead.

The unitarity condition together with the dispersion relations imply a set of integral equations,

$$\tilde{M}_{Il}(\nu)=\tilde{S}_{Il}(\nu)+\int_0^\infty \frac{d\nu'}{\pi}\left(\frac{\nu'}{\nu'+1}\right)^{\frac{1}{2}}\frac{(\nu')^l|\tilde{M}_{Il}(\nu')|^2}{\nu'-\nu-i\epsilon},$$

where

$$\mathrm{Im}\tilde{S}_{Il}(\nu)=[\mathrm{Im}\tilde{S}]_{\pi\pi}{}^L+[\mathrm{Im}\tilde{S}]_{N\bar{N}}{}^L+[\mathrm{Im}\tilde{S}]_{N\bar{N}}{}^R,$$

$$[\mathrm{Im}\tilde{S}]_{\pi\pi}{}^L=\theta(-\nu-1)\sum_{I'l'}\int_0^{-\nu-1}\frac{d\nu'}{\pi}f_{Il,I'l'}(\nu,\nu')\times|\tilde{M}_{I'l'}(\nu')|^2.$$

We have also made use of crossing symmetry here. The solution of this equation is carried out by transforming to a set of N/D equations in the standard way.

The contribution from the nucleon-antinucleon channel was estimated first on the basis of a pole approximation for the $N\bar{N}\pi\pi$ amplitude which one encounters in the unitarity condition, symbolically,

$$\mathrm{Im}M_{\pi\pi,\pi\pi}=\int M_{\pi\pi,\pi\pi}{}^*M_{\pi\pi,\pi\pi}+\int M_{\pi\pi,N\bar{N}}{}^*M_{\pi\pi,N\bar{N}}.$$

This procedure leads to a resonance in π-π scattering, but the method is not justifiable because of unitarity limitations. The unitarity condition implies, on the right-hand cut,

$$[\mathrm{Im}M_{Il}(\nu)]_{N\bar{N}}\leqslant\tfrac{1}{4}[(\nu+1)/\nu]^{\frac{1}{2}},\quad(\nu>M^2-1).$$

This condition is so severely violated by the pole approximation that proposals to ignore the limitation on the left-hand cut, where the limitation does not rigorously apply, must be treated with some suspicion. What we have done is to look at the limitation imposed by unitarity. Thus we replace the inequality by an equality. We have furthermore approximated the left-hand cut by two poles to be interpreted as the pion-pion self-coupling, and the nucleon-antinucleon pair contribution on the left. The following refers to the $I=l=1$ state:

$$-[\mathrm{Im}\tilde{S}(\nu)]^L=-2\pi(5\lambda^2/9\pi)\delta(\nu+2)+\pi M^2\Gamma\delta(\nu+M^2).$$

The residues were fixed by conditions at threshold. Thus

$$\Gamma=[\tilde{S}(0)]_{N\bar{N}}{}^L.$$

Our numerical results are as follows. The $N\bar{N}$ contribution on the right is

$$[\tilde{S}(0)]_{N\bar{N}}{}^R\approx1/(4\pi M^2).$$

On the left we have many terms, one for each angular momentum. The contributions from the $I'=0$, S wave is

$$[\tilde{S}(0)]_{N\bar{N},I'=l'=0}{}^L\approx1/(144\pi M^4),$$

while the contribution from $I'=l'=1$ is

$$[\tilde{S}(0)]_{N\bar{N},I'=l'=1}{}^L\approx-1/(16\pi M^2).$$

The contributions are increasing, but within our approximations the left-hand contributions are numerically less important than the right-hand contributions. The left-hand contributions are represented

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[1] W. R. Frazer and J. R. Fulco, Phys. Rev. Letters **2**, 365 (1959).
[2] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467, 478 (1960).

by the constant Γ which we cannot accurately fix. We therefore allow a certain range, say

$$-1/300 \leqslant \Gamma < +1/300.$$

By using the N/D procedure, we calculated the scattering length and effective range for the P-wave pion-pion scattering channel. These parameters were defined by the equation,

$$\mathrm{Re}\frac{1}{\tilde{M}(\nu)}-\frac{2}{\pi}\nu\left(\frac{\nu}{\nu+1}\right)^{\frac{1}{2}}\tanh^{-1}\left[\left(\frac{\nu}{\nu+1}\right)^{\frac{1}{2}}\right]=-\frac{1}{a}+\frac{r}{2}\nu.$$

We found that

$$a\approx(5\lambda^2/9\pi)-\Gamma,$$

and

$$ra^2\approx-(5\lambda^2/9\pi)+(2\Gamma/M^2).$$

If we restrict λ to avoid bound states, say,

$$-0.3\leqslant\lambda\leqslant 0.3,$$

then this defines a region bounded by parabolas in the $(1/a, r/2)$ plane which lies well away from low-energy resonances. See Fig. 1.

A more exhaustive discussion of the above methods is given elsewhere.[3]

In conclusion, there is no simple explanation of the Frazer resonance as the effect of the nucleon-antinucleon intermediate state. It is the author's opinion that if a mechanism is to be found which can be used in conjunction with an iterative approach to obtain the P-wave dominant solutions, one must look to the four-pion intermediate state.

[3] J. G. Belinfante, Ph.D. thesis (unpublished).

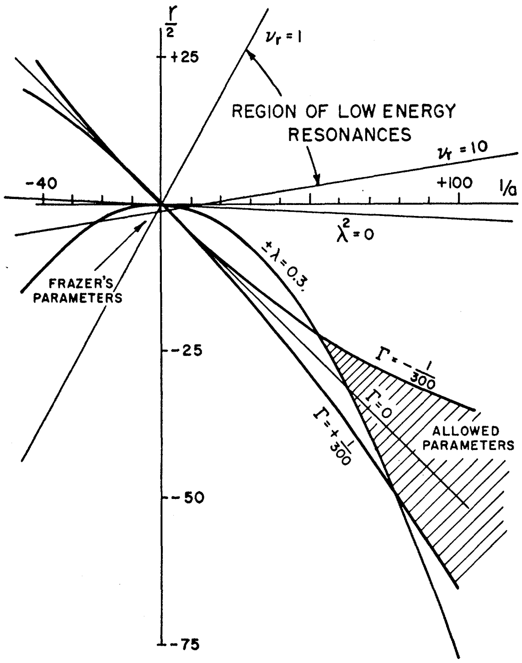

FIG. 1. Effect of the $N\bar{N}$ intermediate state.

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