

# THE PHYSICAL REVIEW

*A journal of experimental and theoretical physics established by E. L. Nichols in 1893*

SECOND SERIES, VOL. 123, NO. 2

JULY 15, 1961

## Comparison of Two Theoretical Approaches to Electron Behavior in Ar-CO<sub>2</sub>, Ar-N<sub>2</sub>, Ar-H<sub>2</sub>, and Ar-CO Gas Mixtures\*

MARTIN A. UMAN

*The University of Arizona, Tucson, Arizona*

(Received March 7, 1961)

The electron drift velocity and electron average energy for low-energy electrons in binary gas mixtures of Ar-CO<sub>2</sub>, Ar-N<sub>2</sub>, Ar-H<sub>2</sub>, and Ar-CO are determined using two theoretical methods of approach: (1) a "distribution function," or "Boltzmann equation" approach; and (2) an "average electron" approach. The results of the two theoretical methods of approach are compared and discussed.

### I. INTRODUCTION

THE study of the behavior of low-energy electrons in gas mixtures under the influence of a dc electric field is a quite recent one. By and large, the research conducted in this field has been of an experimental nature. The scanty amount of theoretical work which has appeared in the literature makes use of the "average electron" model.<sup>1</sup> In a recent article the author has presented a method of describing electron behavior in gas mixtures based on a solution of the Boltzmann equation.<sup>2</sup> In that article, theoretically derived drift velocity vs  $E/p$  curves for electrons in argon and in Ar-CO<sub>2</sub> and Ar-N<sub>2</sub> mixtures were compared to corresponding experimental data. The results of this comparison appear to confirm the validity of the theoretical approach. In the present paper the results of the Boltzmann equation theory, or distribution function theory, are compared to those of the average electron theory for electron behavior in gas mixtures of Ar-CO<sub>2</sub>, Ar-N<sub>2</sub>, Ar-H<sub>2</sub>, and Ar-CO. The electron drift velocity and the electron average energy as functions of  $E/p$  are plotted for these gases. The two theoretical methods of approach yield results which are in good general agreement. Discrepancies between the two theories can be attributed to the assumptions made in deriving the average electron theory.

### II. "AVERAGE ELECTRON" THEORY

The most expedient method of analysis of the behavior of low-energy electrons in gas mixtures is the so-called "average electron" method. As the name implies, this model assumes that all electrons are endowed with the properties of one suitably chosen "average" electron. The effects of the electron velocity distribution are ignored, with the exception that average values of speed can be related to mean squared speed by making use of an assumed distribution function.

Townsend<sup>3</sup> has shown that, assuming a Maxwell-Boltzmann distribution, the equations describing the behavior of an average electron in a *single* gas are

$$v^2 = 0.407Fc^2, \quad (1)$$

$$vc = 0.815Ee/Nm\sigma, \quad (2)$$

where  $v$  is the drift velocity,  $F$  the fractional electron energy loss per collision,  $c$  the mean speed,  $E$  the electric field,  $N$  the gas particle density,  $e$  the electronic charge,  $m$  the electron mass, and  $\sigma$  the gas cross section. The cross section is related to the mean free path by  $\lambda = 1/N\sigma$ .

The extension of Eq. (1) and Eq. (2) to the binary gas case involves the determination of an effective  $F$  and an effective  $N$  for the binary gas mixture. The average fractional electron energy loss per collision for the mixture,  $F_M$ , is the weighted average of the fractional energy losses to each of the two constituent gases of the gas mixture. The probability of an electron

\* This work was conducted in part at Princeton University, Princeton, New Jersey, and supported in part by RCA Laboratories, Princeton, New Jersey.

<sup>1</sup> For example, W. H. English and G. C. Hanna, *Can. J. Phys.* **31**, 768 (1953).

<sup>2</sup> M. A. Uman and G. Warfield, *Phys. Rev.* **120**, 1542 (1960).

<sup>3</sup> J. S. Townsend, *Electrons in Gases* (Hutchinson and Company, Ltd., London, 1947), p. 50.

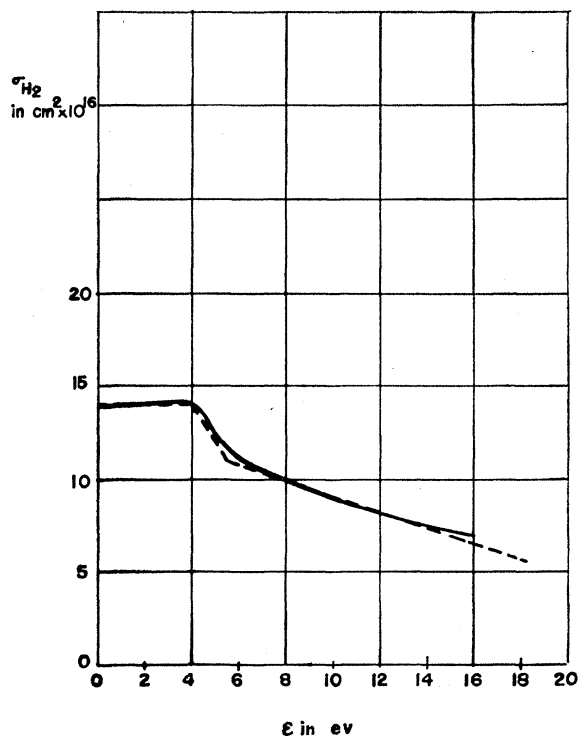


FIG. 1. Molecular hydrogen cross-section vs energy.

colliding with one of the constituent gases, say gas  $A$ , is proportional to the product of the density of gas  $A$ ,  $N_A$ , and the collision cross section of gas  $A$ ,  $\sigma_A$ . For a binary mixture, then, the average fractional energy loss per collision is

$$F_M = (N_A \sigma_A F_A + N_B \sigma_B F_B) / (N_A \sigma_A + N_B \sigma_B). \quad (3)$$

Similarly, the total collision cross section becomes

$$(N\sigma)_M = N_A \sigma_A + N_B \sigma_B. \quad (4)$$

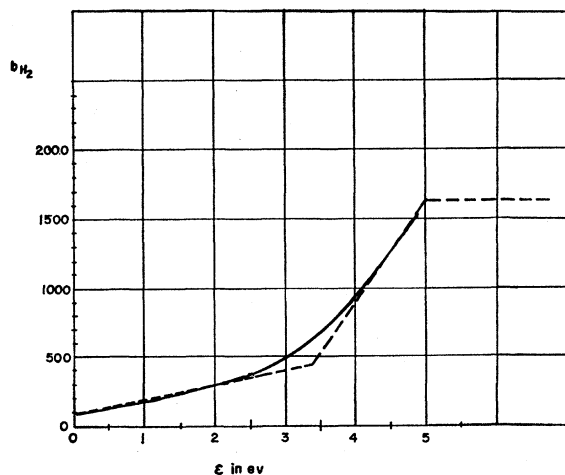


FIG. 2. Molecular hydrogen energy absorption coefficient vs energy.

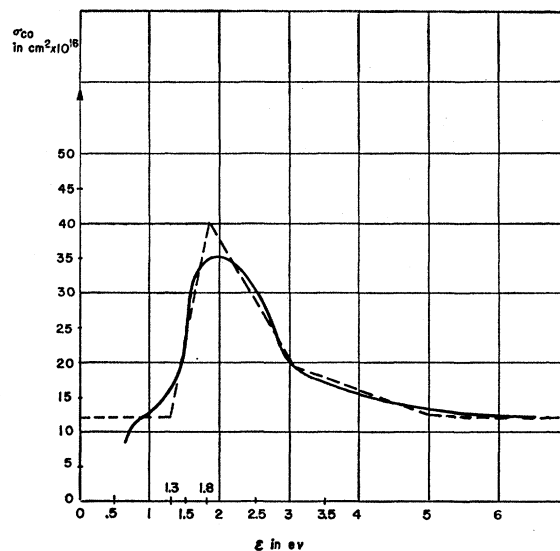


FIG. 3. Carbon monoxide cross section vs energy.

After substitution of Eq. (3) and Eq. (4) into Eq. (1) and Eq. (2), respectively, the basic equations for the determination of the behavior of electrons in gas mixtures become

$$v^2 = 0.407 \left[ \frac{N_A \sigma_A F_A + N_B \sigma_B F_B}{N_A \sigma_A + N_B \sigma_B} \right] c^2, \quad (5)$$

$$vc = 0.815 \frac{eE}{m} \left( \frac{1}{N_A \sigma_A + N_B \sigma_B} \right). \quad (6)$$

To obtain the relationship between  $v$ ,  $\epsilon$  (electron energy), and  $E/p$ , Eq. (5) and Eq. (6) must be solved simultaneously using  $c$  as a parameter.

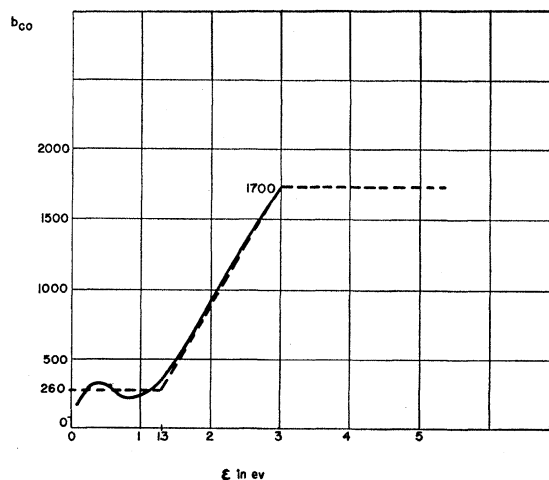
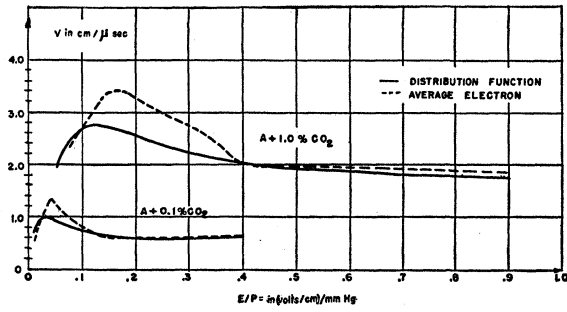


FIG. 4. Carbon monoxide energy absorption coefficient vs energy.

FIG. 5. Drift velocity vs  $E/p$  in Ar-CO<sub>2</sub>.

### III. "BOLTZMANN EQUATION" THEORY

The "average electron" model is very useful for obtaining a qualitative picture of the behavior of electrons in binary gas mixtures. It is inaccurate in that it cannot adequately take account of the effects of the electron energy distribution. In a previous paper by the author<sup>2</sup> a more sophisticated mathematical theory, based on the existence of the electron distribution, was developed. The derivation involves a solution of the Boltzmann equation for an electron phase space distribution function of the form

$$f(\mathbf{c}) = f_0(c) + [c_x/c]f_1(c), \quad (7)$$

from which the electron drift velocity and electron average energy are calculated:

$$v_x = \frac{1}{3} \left( \frac{2}{m} \right)^{1/2} \left[ \int_0^\infty \epsilon f_1 d\epsilon / \int_0^\infty \epsilon^{3/2} f_0 d\epsilon \right], \quad (8)$$

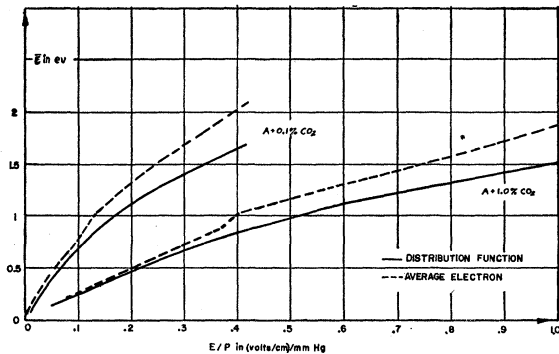
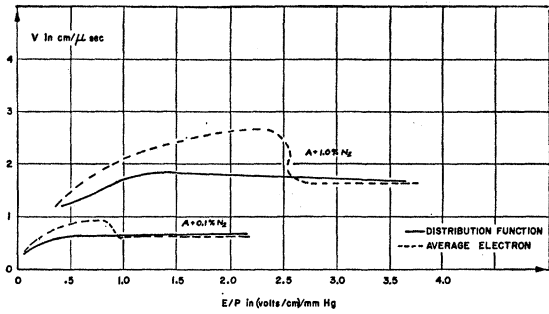
$$\bar{\epsilon} = \int_0^\infty \epsilon^{3/2} f_0 d\epsilon / \int_0^\infty \epsilon^{1/2} f_0 d\epsilon. \quad (9)$$

It is found that

$$f_0 = G \exp[-(6m/M)(N/E)^2 \times (A\epsilon^5 + B\epsilon^4 + C\epsilon^3 + D\epsilon^2)], \quad (10)$$

$$f_1 = (6m/M)(N/E)[H\epsilon^3 + L\epsilon^2 + J\epsilon]f_0, \quad (11)$$

where  $A, B, C, D, H, L,$  and  $J$  are functions of the gas

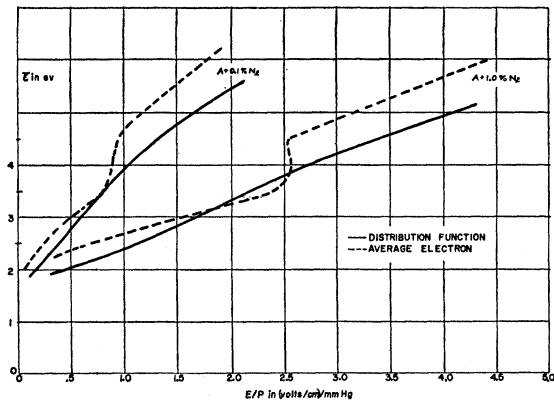
FIG. 6. Average energy vs  $E/p$  in Ar-CO<sub>2</sub>.FIG. 7. Drift velocity vs  $E/p$  in Ar-N<sub>2</sub>.

parameters and percentage mixture,  $G$  is the normalization constant, and  $N$  is the gas density of the majority constituent.<sup>2</sup>

### IV. RESULTS AND DISCUSSION

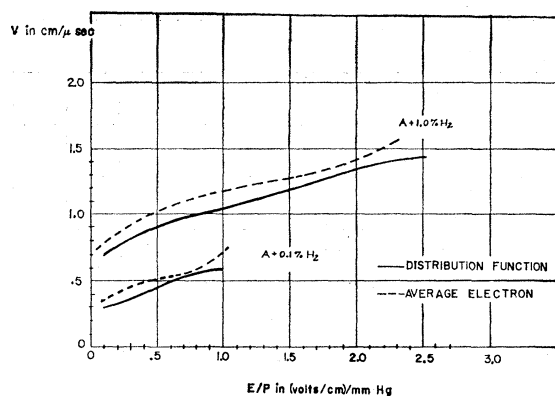
In this section the behavior of low-energy electrons in gas mixtures of Ar-CO<sub>2</sub>, Ar-N<sub>2</sub>, Ar-H<sub>2</sub>, and Ar-CO is investigated using the two theoretical methods of approach. The electron drift velocity and the electron average energy as predicted by the "average electron" theory and by the "Boltzmann equation" theory are compared. All theoretical computations were performed on an IBM 650 digital computer using straight-line approximations to the experimental cross section and to the experimental energy absorption coefficient,  $b = F_B/F_A$ .

The cross sections for argon, carbon dioxide, and molecular nitrogen and the energy absorption coefficients for carbon dioxide and molecular nitrogen have been presented previously.<sup>2</sup> The straight line approximations to the molecular hydrogen cross section and energy absorption coefficient plotted in Figs. 1 and 2 are based on the data of Brown<sup>4</sup> and of Healey and Reed,<sup>5</sup> respectively. The straight-line approximations

FIG. 8. Average energy vs  $E/p$  in Ar-N<sub>2</sub>.

<sup>4</sup> S. C. Brown, *Basic Data of Plasma Physics* (Technology Press, Cambridge, Massachusetts, 1959), p. 5.

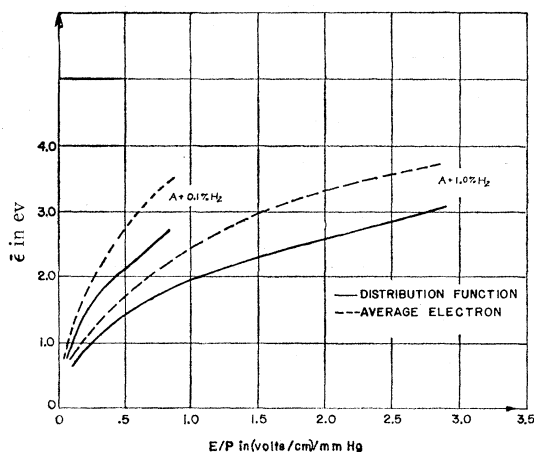
<sup>5</sup> R. H. Healey and J. W. Reed, *The Behavior of Slow Electrons in Gases* (Amalgamated Wireless, Ltd., Sydney, Australia, 1941).

FIG. 9. Drift velocity vs  $E/p$  in Ar- $H_2$ .

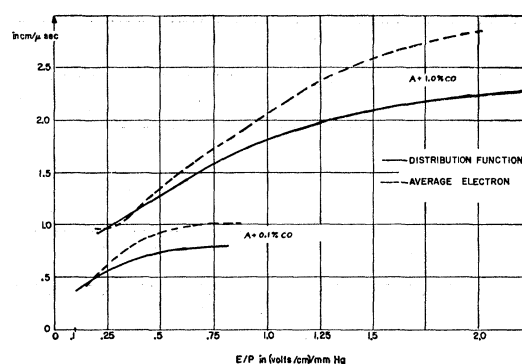
to the carbon monoxide cross section and energy absorption coefficient plotted in Figs. 3 and 4 are based on the data of Normand<sup>6</sup> and of Healey and Reed,<sup>5</sup> respectively. It should be noted that the argon atoms suffer only elastic collisions with the electrons for the range of electron energy under study.

The drift velocity curves for electrons in Ar- $CO_2$ , Ar- $N_2$ , Ar- $H_2$ , and Ar- $CO$  are shown in Figs. 5, 7, 9, and 11, respectively. The average energy curves for electrons in the above gas mixtures are shown in Figs. 6, 8, 10, and 12, respectively.

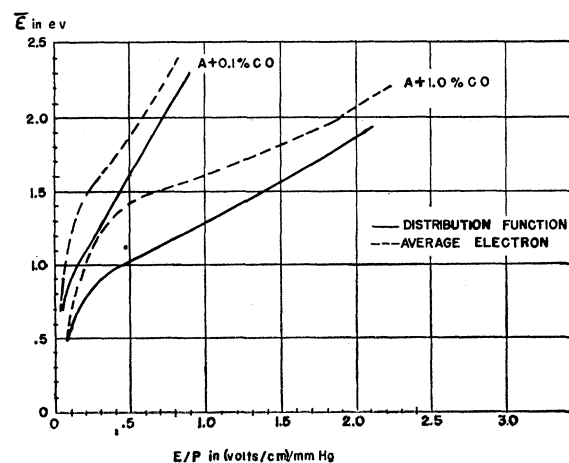
In comparing the results of the distribution function theory with the average electron theory, two general discrepancies are apparent: (1) For a given  $E/p$ , the average electron theory in almost all cases predicts a larger value of electron drift velocity and average energy than does the distribution function theory. (2) The average electron theory predicts sharper peaks and in general more drastic variation in the drift velocity and average energy curves than does the distribution function theory.

FIG. 10. Average energy vs  $E/p$  in Ar- $H_2$ .

<sup>6</sup> C. E. Normand, Phys. Rev. **35**, 1217 (1930).

FIG. 11. Drift velocity vs  $E/p$  in Ar- $CO$ .

The reasons for these discrepancies can be attributed to the simplifying assumptions used in deriving the average electron theory: (1) The average electron theory assumes that the electrons are distributed in a Maxwellian fashion,  $\exp[-(\epsilon/kT)]$ , about the mean energy. This assumption is used in relating the mean speed to the mean square speed to compute the constants in Eq. (1) and Eq. (2). The distribution function theory shows that the actual distribution function decreases much more rapidly than  $\exp[-(\epsilon/kT)]$ ; in fact, as rapidly as  $\exp[-(\epsilon/kT)^5]$ . Thus the electrons are forced to remain at a lower average energy for a given  $E/p$  than would be the case were the distribution Maxwellian. It should be noted that, according to the distribution function theory, the electrons are forced to remain at a lower average energy for a given  $E/p$  than would be the case were the distribution any function of the form  $\exp[-(\epsilon/kT)^n]$ , where  $n < 4$ , the minimum power [Eq. (10),  $A=0$ ] of the distribution function theory. The average electron theory-distribution function theory comparison will give similar results no matter what distribution is used to determine the constants in Eq. (1) and Eq. (2) as long as  $n < 4$ . Physically, the rapid distribution function dropoff with

FIG. 12. Average energy vs  $E/p$  in Ar- $CO$ .

increasing energy is caused in part by the increase in the argon cross section with energy. The larger the cross section, the smaller is the number of electrons which can be sustained at that cross section if a steady state is to exist. The distribution function dropoff is also influenced by the increase in the energy absorption coefficients with energy, causing higher-energy electrons to be scattered back to lower energies. (2) The average electron theory assumes that all electrons behave like one suitably chosen "average" electron. Thus the effect of an abrupt change in the cross-section or energy absorption coefficient with energy is to change abruptly the behavior of all electrons as they increase in energy through the change region. Since, in the distribution function theory, the electrons are spread out in energy, only a fraction of them experiences any abrupt change in cross section or energy absorption as  $E/p$  is increased. Therefore, the distribution function theory tends to smooth out the  $v-E/p$  and  $\bar{\epsilon}-E/p$  curves from any abrupt variation which the average electron theory may predict.

The distribution function theory and the average electron theory yield results which are in good general agreement. In particular, there is good correspondence between the magnitudes and the curve forms derived from the two theoretical approaches. If a distribution function which decreases more rapidly with energy than the Maxwellian is used in the derivation of the average electron theory, the two theoretical methods of approach will be in even closer agreement, although the peaking effects of the average electron theory will still exist. The peaking effects can be diminished slightly by increasing the accuracy of the cross-section and energy absorption coefficient approximations; but the improvement is minimal.

#### ACKNOWLEDGMENTS

The author would like to express his appreciation to Professor George Warfield of Princeton University for his valuable counsel during the course of this work and to Dr. Edwin Langberg of Elcon Laboratory, Incorporated, who provided the impetus for this study.