few electrons in the ⟨111⟩ valleys contribute to μ_i^* which have a thermal energy ϵ_{th} which obeys the equation $h\nu+\epsilon_{th}\approx\Delta E$. This energy is much higher than the minimum energy of these valleys while the final energy is nearly equal to the minimum energy of the ⟨100⟩ valleys. Now the term proportional to $(\mathbf{k}-\mathbf{k}_0)^2$ in (4) is predominant, yielding the factor $\bar{\alpha}$ in (9b).

(c) If $h\nu\approx\Delta E$, both terms in the squared matrix element of (4) are of equal importance, yielding the factor $\frac{1}{2}(\bar{\alpha}+\bar{\beta})$ of (8c).

Furthermore, it can be seen from (5) that μ_i^* becomes very small for sufficiently low temperatures and sufficiently long wavelengths such that $[\Delta E-(h\nu+\hbar\omega_i^*)]/kT\gg1$. With $\Delta E/k=2030°\mathrm{K}$, $\hbar\omega_i^*/k=316°\mathrm{K}$[7], and $T=300°\mathrm{K}$, we therefore find that the effect of nonequivalent intervalley scattering on the absorption constant becomes rapidly negligible at room temperature if the wavelength λ becomes larger than about $14\,\mu$.

[7] This corresponds, rather arbitrarily, to a value found by Weinreich and quoted by Brockhouse, J. Phys. Chem. Solids **8**, 400 (1959).

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Single-Crystal and Polycrystal Resistivity Relationships for Yttrium*

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Different proposals for calculating polycrystal resistivities from single-crystal values are applied to yttrium metal. It is shown that a simple average yielding $\rho_{poly}=\frac{1}{3}(2\rho_\perp+\rho_{||})$ gives the best fit to experimental data.

INTRODUCTION

IT is reasonable to expect that a general relation exists between the principal resistivities of a single crystal and the isotropic resistivity of a polycrystalline sample of the same substance. There is, however, little agreement in the literature as to the form of this relation; several methods of calculating the polycrystal resistivity have been proposed. Voigt[1] suggests

$$\sigma_{poly}=\tfrac{1}{3}(2\sigma_\perp+\sigma_{||}),$$

which is the same as

$$\rho_{poly}=3\rho_\perp\rho_{||}/2\rho_{||}+\rho_\perp, \tag{1}$$

where $\rho_{||}$ and $\rho_\perp$ are the principal resistivities of a hexagonal crystal measured parallel and perpendicular to the axis of symmetry, and where $\rho_\perp=1/\sigma_\perp$, $\rho_{||}=1/\sigma_{||}$, $\rho_{poly}=1/\sigma_{poly}$. Andrade and Chalmers[2] obtain

$$\rho_{poly}=\frac{[\rho_\perp(\rho_{||}-\rho_\perp)]^{\frac{1}{2}}}{\tan^{-1}\{[(\rho_{||}-\rho_\perp)/\rho_\perp]^{\frac{1}{2}}\}}$$

for the case $\rho_{||}>\rho_\perp$. This is equivalent to

$$\rho_{poly}=\frac{[\rho_\perp(\rho_\perp-\rho_{||})]^{\frac{1}{2}}}{\tanh^{-1}\{[(\rho_\perp-\rho_{||})/\rho_\perp]^{\frac{1}{2}}\}} \tag{2}$$

for $\rho_\perp>\rho_{||}$. Bruggeman[3] proposes

$$\rho_{poly}=\tfrac{1}{4}\{\sigma_\perp+[\sigma_\perp(8\sigma_{||}+\sigma_\perp)]^{\frac{1}{2}}\},$$

or

$$\rho_{poly}=4\Big/\left\{\frac{1}{\rho_\perp}+\left[\frac{1}{\rho_\perp}\left(\frac{8}{\rho_{||}}+\frac{1}{\rho_\perp}\right)\right]^{\frac{1}{2}}\right\}. \tag{3}$$

Another method,[4] which has been proposed more recently, yields

$$\rho_{poly}=\tfrac{1}{3}(2\rho_\perp+\rho_{||}). \tag{4}$$

This relation is the result of an average over the total solid angle of the well-known expression,

$$\rho(\theta)=\rho_{||}\cos^2\theta+\rho_\perp\sin^2\theta,$$

for the resistivity along an arbitrary direction of current flow in a hexagonal crystal.[5]

Several attempts[2,3] to resolve this question experimentally have been indecisive; if the anisotropy ratio, $\rho_\perp/\rho_{||}$, is very near unity, the small difference between values given by any of Eqs. (1)–(4) is masked by the probable error of the experiment. Recently Hall *et al.*[6] have reported the principal single-crystal resistivities of yttrium (see Table I). Their values at 300°K lead to the exceptionally high anisotropy ratio of

$$\rho_\perp/\rho_{||}=2.07,$$

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[1] W. Voigt, *Lehrbuch der Kristallphysik* (B. G. Teubner, Leipzig, 1928), p. 959.
[2] E. N. da C. Andrade and B. Chalmers, Proc. Roy. Soc. (London) **A139**, 413 (1932).
[3] D. A. G. Bruggeman, Ann. Physik **25**, 645 (1936).
[4] J. L. Nichols, J. Appl. Phys. **26**, 470 (1955).
[5] W. Boas and J. K. Mackenzie, *Progress in Metal Physics*, edited by B. Chalmers (Interscience Publishers, Inc., New York, 1950), Vol. II, p. 90.
[6] P. M. Hall, S. Legvold, and F. H. Spedding, Phys. Rev. **116**, 1446 (1959).

TABLE I. Comparison of experimental and calculated results.[a]

	Residual resistivity (10^{-6} ohm-cm)	Resistivity at 300°K (Residual subtracted) (10^{-6} ohm-cm)
Basal plane[b]	2.6	71.6
c axis[b]	1.5	34.6
Y-I[c]	9.6	60.0
Y-II[c]	5.2	59.5
Y-III[c]	3.2	59.6
Eq. (1)[d]	...	52.8
Eq. (2)[d]	...	56.8
Eq. (3)[d]	...	55.2
Eq. (4)[d]	...	59.3

[a] The probable errors are estimated to be ±1% for the experimental values and ±2% for the calculated values.
[b] Hall *et al.*[6]
[c] Present experimental results.
[d] Values obtained from substitution of Hall's results into Eqs. (1)–(4).

which suggests that a fair test of the various calculated ρ_{poly} might by made through a comparison with measurements on polycrystalline samples of this metal.

EXPERIMENTAL PROCEDURE

The polycrystalline yttrium data were obtained from three separate arc-melted samples produced in this Laboratory, The finished samples were cylinders 2 in. long and $\frac{3}{16}$ in. in diameter. They were annealed at 900°C for two hours, cold swaged with a 10% reduction in diameter, and reannealed at 800°C for one hour. These operations eliminated grain orientation and reduced the size of the grains. The samples were examined and found to have an average grain diameter of 0.14 mm. Analysis by back-reflection x-ray methods showed that the grains were randomly oriented. Table II lists the impurities found by spectrographic and vacuum fusion analyses on one of the samples, Y-II. Except for the silicon content which is unusually high, these values are representative of the other samples.

The resistivities of the three samples were measured using the method and apparatus described by Colvin *et al.*[7]

TABLE II. Impurity analysis of sample Y-II. (Values in %).

Er≤0.005; Ho≤0.05; Dy≤0.005; Tb≤0.04; Gd≤0.01; Sm≤0.05; Ca<0.02; Fe, 0.05; Mg<0.01; Si, 0.05; Ta<0.1; Cr, 0.1; O_2, 0.033; Mn, Ti, Cu, Ni, Ca, Al, Cr, Mg, Si, Be, trace present.

[7] R. V. Colvin, S. Legvold, and F. H. Spedding, Phys. Rev. **120**, 741 (1960).

RESULTS AND CONCLUSIONS

The electrical resistivity of a representative sample (Y-II) in the temperature range 1.3° to 300°K is shown in Fig. 1. Table I shows the residual resistivities together with the resistivities at 300°K (less residuals) of the three polycrystalline samples and the single crystals. The calculated results of substituting the measured principal single-crystal resistivities into Eqs. (1)–(4) are also included. The estimated probable errors are taken to be ±1% for the experimental values and

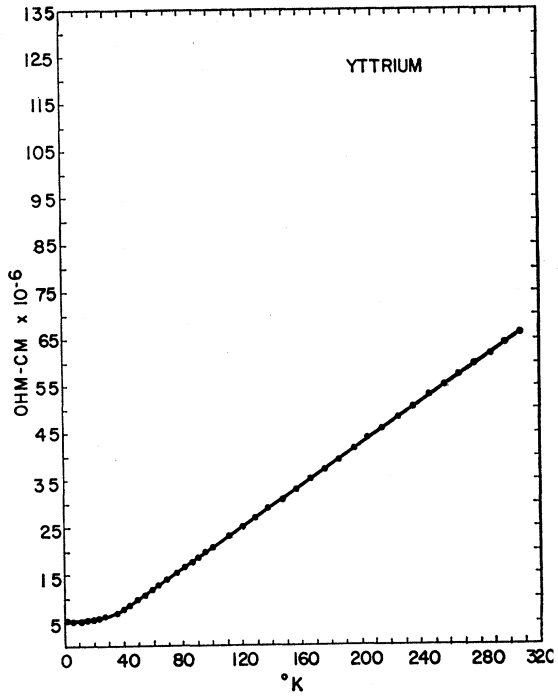

FIG. 1. The electrical resistivity of yttrium (sample Y-II) as a function of temperature.

±2% for the calculated values. It is clear that the best fit to the experimental data is obtained from Eq. (4). It therefore seems appropriate to use this relation for hexagonal metals.

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