Yield Ratios for the Isomeric Pair $Sc^{44m,44}$ Formed in $(\alpha,\alpha n)$ and (α,n) Reactions*†

SYLVIA M. BAILEY‡
University of California, Lawrence Radiation Laboratory, Berkeley, California
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The yield ratio of Sc^{44m}/Sc^{44} was measured in $Sc^{45}(\alpha,\alpha n)Sc^{44}$ reactions with helium ions of energies between 20 and 43 Mev and at 320 Mev. The measured ratio was nearly constant at a value of 1.5 between 20 and 43 Mev and was 0.62 at 320 Mev. The Sc^{44m}/Sc^{44} ratio was measured in $K^{41}(\alpha,n)Sc^{44}$ reactions at 10 and 43 Mev with values of 0.3 and 0.9, respectively. The isomer ratio was calculated for the K^{41}(10-Mev α,n)Sc^{44} reaction by means of a compound-nucleus model, and for the Sc^{45}(320-Mev $\alpha,\alpha n$)Sc^{44} reaction by means of a classical knock-on model. The calculated ratios were 0.32 and 0.51, respectively.

INTRODUCTION

NUCLEAR reaction theory, whether for compound-nucleus formation[1] at low energies or direct interactions[2] at high energies, must concern itself with angular momentum conservation. Since an isomeric pair provides two discernable states of the same nucleus differing considerably in spin, it would seem that some information on the angular-momentum transfer could be obtained by studying relative yields.

In the case of thermal neutron capture, the analysis should be relatively uncomplicated. The excited compound state can only differ in spin from the target nucleus by $\frac{1}{2}$ unit and since the γ-ray cascade would proceed by lowest possible multipolarity, the isomer favored should be that closest in spin to the target nucleus. Data in the literature[3–5] bear this out qualitatively.

Studies have also been made of reactions in which the incoming particle can deposit varying amounts of angular momentum.[6–9] Segrè and Helmholz[4] suggested that at sufficiently high excitation energies the statistical weights of the isomeric states should determine the isomer-yield ratio. However, Levy[10] analyzed the problem in greater detail and pointed out that the statistical weights do not present a limiting value.

Meadows and Diamond[11] measured isomer-yield ratios for three different (p,pn) reactions covering a proton energy range from near threshold to 100 Mev. They attempted to analyze their results in terms of reaction mechanisms, and concluded that compound-nucleus formation was the principal mechanism up to excitation energies of about 20 Mev and knock-on processes were dominant at the higher energies.

EXPERIMENTAL

Bombardment Procedures

Spectroscopically pure scandium oxide powder was used as a target for alpha particles on the 60-in. cyclotron and the 184-in. synchrocyclotron. Reagent-grade tribasic potassium phosphate powder was used as a target for alpha particles in the 60-in. cyclotron. About 10 mg of a paste made by mixing Sc_2O_3 powder and Duco cement was spread in a 10-mil platinum "hat." This platinum hat was covered with a 1-mil platinum cover foil and mounted in a microtarget assembly described by Ritsema[12] for bombardment on the 60-in. cyclotron with helium ions. About 15 mg of potassium phosphate (K_3PO_4) powder was similarly mounted as a target on the 60-in. cyclotron and bombarded with helium ions. The platinum cover foil was weighed in each bombardment. Weighed aluminum foils were used to degrade the energy of the helium ions from the 60-in. cyclotron as described by Thomas.[13] The energy of the helium ions was obtained from the range-energy curves of Aron, Hoffman, and Williams.[14]

For bombardments on the 184-in. syncrhocyclotron with 320-Mev helium ions, a paste of scandium oxide powder and Duco cement was wrapped in aluminum foil about 2 mils thick and clamped in a copper target holder.

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† The material in this article was taken from a thesis which was submitted in partial fulfillment of the requirements for the degree of doctor of philosophy.
‡ Present address: National Bureau of Standards, Washington 25, D. C.
[1] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952).
[2] R. Serber, Phys. Rev. **72**, 1114 (1947).
[3] A. W. Fairhall, Massachusetts Institute of Technology, Laboratory for Nuclear Science and Engineering, Progr. Rept. MIT-NSE-PR(5-31-52), May 31, 1952 (unpublished).
[4] E. Segrè and A. C. Helmholz, Revs. Modern Phys. **21**, 271 (1949).
[5] L. Seren, H. N. Friedlander, and S. H. Turkel, Phys. Rev. **72**, 888 (1947).
[6] L. Katz, L. Pease, and H. Moody, Can. J. Phys. **30**, 476 (1952); L. Katz, R. G. Baker, and R. Montalbetti, Can. J. Phys. **31**, 250 (1953); J. Goldemberg and L. Katz, Phys. Rev. **90**, 308 (1953).
[7] R. Sagane, Phys. Rev. **85**, 926 (1952).
[8] F. Boehm, P. Marmier, and P. Preiswerk, Helv. Phys. Acta **25**, 599 (1952).
[9] Alexis C. Pappas and Rodman A. Sharp, J. Inorg. & Nuclear Chem. (to be published).
[10] Harris B. Levy, thesis, University of California Radiation Laboratory, 1953 (unpublished).
[11] J. W. Meadows, R. M. Diamond, and R. A. Sharp, Phys. Rev. **102**, 190 (1956).
[12] Susanne E. Ritsema, Master's thesis, University of California Radiation Laboratory, 1956 (unpublished).
[13] Thomas Darrah Thomas, thesis, University of California Radiation Laboratory, 1957 (unpublished).
[14] W. A. Aron, B. G. Hoffman, and F. C. Williams, Atomic Energy Commission Report AECU-663, May, 1951 (unpublished).

Chemical Procedures

In the procedure for the Sc_2O_3 targets from the 60-in. cyclotron, the scandium oxide was dissolved in HCl solution, and three scandium hydroxide and two scandium fluoride precipitations and passage through a Dowex A-2 anion-exchange column were used in purification. Scandium hydroxide was mounted for counting. The procedure for Sc_2O_3 targets from the 184-in. synchrocyclotron was similar except that the use of the Dowex A-2 anion-exchange column was omitted. In the procedure for the K_3PO_4 targets from the 60-in. cyclotron, the purification steps were the same as for the Sc_2O_3 targets from the 60-in. cyclotron except that the fluoride precipitations were omitted.

Counting Procedures

The scandium activity was counted on a 100-channel pulse-height analyzer in which the detecting unit was a sodium iodide (thallium-activated) scintillation crystal. The 1.16-Mev gamma ray of 3.9-hr Sc^{44} and the 270-Mev gamma ray of 59-hr Sc^{44m} were seen.

The decay schemes of Sc^{44m} and Sc^{44}, from the Table of Isotopes by Strominger, Hollander, and Seaborg,[15] are shown in Fig. 1. The length of the bombardments was from $\frac{1}{4}$–$\frac{1}{2}$ hr, and the procedures for chemical separation took 4 or 5 hrs. The independent-yield ratio of Sc^{44m} to Sc^{44} was obtained by following the decay of the 1.16-Mev gamma ray of 3.9-hr Sc^{44}. From the 59-hr decay curve of the 1.16-Mev gamma peak of Sc^{44} in transient equilibrium with its parent, the growth curve for the 3.9-hr Sc^{44} by isomeric transition was constructed. This Sc^{44} growth curve was subtracted from the experimental decay curve in order to obtain the 3.9-hr decay curve of Sc^{44} formed from the nuclear reaction.

RESULTS AND DISCUSSION

Table I gives the Sc^{44m}/Sc^{44} cross-section ratio when Sc^{45} (spin $\frac{7}{2}$) was bombarded with helium ions at energies of 20.4–320 Mev.

Table II gives the Sc^{44m}/Sc^{44} cross-section ratio when K^{41} of spin $\frac{3}{2}$ was bombarded with helium ions at 10-Mev and 43-Mev energies

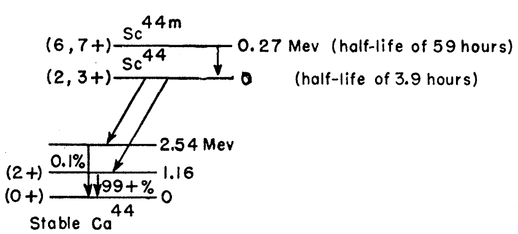

FIG. 1. Decay schemes of Sc^{44m} and Sc^{44}.

[15] D. Strominger, J. M. Hollander, and G. T. Seaborg, Revs. Modern Phys. **30**, 585 (1958).

Matsuo and Sugihara[16] measured the cross section and the Sc^{44m}/Sc^{44} ratio for the $K^{41}(\alpha,n)Sc^{44}$ reaction in the energy range 6–30 Mev. The plot of the cross section vs projectile energy gave a symmetrical curve with a maximum at 18 Mev; therefore, a compound-nucleus mechanism for the reaction at 10 Mev is indicated to be valid. Also, the increase of the isomer ratio from 0.30 at 6 Mev to 2.00 at 25 Mev is in accord with the expectation that, if the reaction proceeds by a compound-nucleus mechanism, the isomer ratio should increase with higher projectile energies.

The isomer ratio Sc^{44m}/Sc^{44} from the K^{41}(10-Mev α,n)Sc^{44} reaction was calculated in the following way. The alpha-particle energy of 10 Mev in the laboratory system gives an entrance-channel energy E_α of 9.1 Mev in the center-of-mass system. Since the target nucleus K^{41} has a spin of $\frac{3}{2}+$ and the alpha particle has 0 spin, the entrance-channel spin S_α is $\frac{3}{2}+$. In the entrance channel, the cross section for the formation of the compound nucleus with a particular angular momentum is given by the formula

$$\sigma_{cl}(\alpha)=(2l+1)\pi\lambda\!\!\!^{-2}T_l(\alpha),$$

TABLE I. $Sc^{45}(\alpha,\alpha n)Sc^{44}$ results.

Bombarding energy (Mev)	20.4	25.2	33.7	43	320
Sc^{44m}/Sc^{44}	1.7	1.3	1.5	1.4	0.62

with $\lambda\!\!\!^{-}$ being the de Broglie wavelength divided by 2π, $T_l(\alpha)$ the transmission coefficient for each l, and l the quantum number for the orbital angular momentum according to which the square of the angular momentum equals $l(l+1)\hbar^2$. The transmission coefficient $T_l(\alpha)$ was obtained from Feshbach, Shapiro, and Weisskopf.[17] The percent of each l value which contributes inside the nucleus is calculated. The channel spin S_α and l combine to give J_c:

$$J_c=|l-S_\alpha|,\ \cdots,\ l+S_\alpha.$$

For each S_α and l combination, the percentage of J_c is determined by the statistical weight, $2J_c+1$. Cameron[18] concluded that, to a good approximation, the nuclear-level spacing was inversely proportional to $(2J+1)$.

The nuclear temperature θ was assumed to be 1.4 Mev. The average energy of the emitted neutron is twice the nuclear temperature or 2.8 Mev. Transmission coefficients $T_l(\alpha)$ for neutrons are taken from Feld, Feshbach, Goldberger, Goldstein, and Weisskopf.[19] The

[16] T. Matsuo and T. T. Sugihara, Clark University, Department of Chemistry, Ann. Progr. Rept. NYO-7759, 1959 (unpublished).

[17] Herman Feshbach, M. M. Shapiro, and V. F. Weisskopf, Clark University Ann. Progr. Rept. NYO-3077; NDA-15B-5, June, 1953 (unpublished).

[18] A. G. W. Cameron, Chalk River Report, PD-292, October, 1957 (unpublished).

[19] B. T. Feld, H. Feshbach, M. L. Goldberger, H. Goldstein, and V. F. Weisskopf, Final Report of the Fast Neutron Data Project, NYO-636, January, 1951 (unpublished) p. 115–116.

formula

$$\sigma_{cl}=(2l+1)\pi\lambda\!\!\!^{-2}T_l,$$

was used to calculate the cross section for emission of neutrons of differenct l values.

From the decay scheme of Blue and Bleuler,[20] it is assumed that Sc^{44m} and Sc^{44} have even parity. The parity of K^{41} is even, and the intrinsic parities of helium ions, neutrons, and protons are even. Since the shell model of the nucleus shows that states having between twenty and forty particles have odd parity, the assumption is made that for several Mev above the ground state, the parity of the odd-odd nucleus Sc^{44} is even. In order to conserve parity, the l values of the neutron must be odd if the l value of the helium ion was odd, and the l value of the neutron must be even if the l value of the helium ion was even.

The spin states in a nuclear reaction, $A(a,c)C$, are given as follows:

$$(A+a)\rightarrow B\rightarrow (C+c),$$
$$(\mathbf{I}_1+\mathbf{i}_1)+\mathbf{l}_i=\mathbf{J}_c=\mathbf{l}_f+(\mathbf{I}_2+\mathbf{i}_2),$$
$$\mathbf{S}_\alpha+\mathbf{l}_i=\mathbf{J}_c=\mathbf{l}_f+\mathbf{S}_\beta.$$

Here B is the compound nucleus; l_i and l_f are l values of a and c, respectively; I_1 and I_2 are spins of A and B, respectively; i_1 and i_2 are intrinsic spins of a and c, respectively; S_α is entrance-channel spin; and S_β is exit-channel spin. The S_β values which result from this combination of l_f and J_c were calculated, and the percentages of S_β were determined by their statistical weights, $2S_\beta+1$.

The exit-channel spin S_β is a combination of the intrinsic neutron spin of $\frac{1}{2}$ and the angular momentum of the residual nucleus I_2. The percent of I_2 was determined by its statistical weight $(2I_2+1)$.

By means of gamma cascades, the residual nucleus goes to the final products, Sc^{44m} ($I=7$ or 6) and Sc^{44} ($I=3$ or 2). It is assumed that all the I_2 values greater than 7 or 6 decay to 7 or 6 and that all I_2 values less than 3 or 2 decay to 3 or 2. I_2 values within one unit of an isomer spin are assumed to go to that isomer. The spin state midway between the two isomers is divided between the isomers on the basis of their statistical weights.

With the assumption that the spin of Sc^{44m} is 7 and the spin of Sc^{44} is 3, one finds that the ratio of the cross section for the metastable state σ_m to the cross section for the ground state σ_g is $\sigma_m/\sigma_g=0.32$. With the assumption that the spin of Sc^{44m} is 6 and the spin of Sc^{44} is 2, one finds that the cross-section ratio is $\sigma_m/\sigma_g=0.77$. The experimental yield ratio Sc^{44m}/Sc^{44} for this reaction is 0.3.

The near constancy of the Sc^{44m}/Sc^{44} ratios from the $Sc^{45}(\alpha,\alpha n)Sc^{44}$ reaction in the 20- to 43-Mev energy range is consistent with a direct-interaction mechanism or with a mixture of compound-nucleus and direct-interaction mechanisms if the changes in the amount and the ratio of the compound-nucleus mechanism are counterbalanced by changes in the amount or ratio of the direct-interaction mechanism.

[20] J. W. Blue and E. Bleuler, Phys. Rev. **100**, 1324 (1955).

TABLE II. $K^{41}(\alpha,n)Sc^{44}$ results.

Energy (Mev)	10	43
Sc^{44m}/Sc^{44}	0.3 ± 0.1	0.9 ± 0.1

A calculation is now made for a $Sc^{45}(p,pn)Sc^{44}$ or $Sc^{45}(\alpha,\alpha n)Sc^{44}$ knock-on reaction in which the charged particle strikes a neutron and both particles go out. This is a classical calculation. In order for the wavelength of the projectile to be small enough to enable the projectile to interact classically with only one nucleon, the energy of the projectile must be high. Benioff[21] has used a quantum-mechanical method of calculating (p,pn) cross sections based on the optical and shell models.

The binding energy of the least-bound particle in the nucleus of Sc^{44} is the 6.87-Mev binding energy of a proton. The Coulomb barrier for the proton is 3.92 Mev.[22] The addition of the proton binding energy and the Coulomb barrier gives a total of 10.8-Mev excitation needed to eject a proton from the Sc^{44} nucleus, but the binding energy of a neutron in Sc^{44} is only 9.86 Mev. Therefore, the energy level of all neutrons knocked out of the nucleus of Sc^{45} in a knock-on reaction must be not lower than 9.86 Mev from the top level in Sc^{44} which has particles in it. The energy-level scheme in the potential well of the nucleus was taken from Ross, Mark, and Lawson.[23] The neutrons in the $1f_{7/2}$, $1d_{3/2}$, and $2s_{1/2}$ levels are certainly available to be knocked out in a knock-on reaction, and the availability of neutrons in the $1d_{5/2}$ level is questionable.

A calculation for a knock-on reaction, $Sc^{45}(p,pn)Sc^{44}$ or $Sc^{45}(\alpha,\alpha n)Sc^{44}$, is now made with the assumption that the neutrons in the $1f_{7/2}$, $1d_{3/2}$, and $2s_{1/2}$ levels have equal probabilities of being knocked out. The spin of the knocked-out neutron adds vectorially with the $\frac{7}{2}$ spin of the Sc^{45} target to give the spin I_2 of the residual nucleus, which is assumed to gamma cascade to the isomer products of Sc^{44m} (spin of 7 or 6) and Sc^{44} (spin of 3 or 2). The I_2 spin values are formed in proportion to their statistical weights, $2I_2+1$.

A similar calculation for the same knock-on reaction is made with the additional assumption that the six neutrons in the $1d_{5/2}$ level are also equally available for a knock-on reaction.

The results of these two calculations for the knock-on

[21] Paul A. Benioff, Phys. Rev. **119**, 316, 324 (1960).
[22] K. J. LeCouteur, Proc. Phys. Soc. (London) **A63**, 259 (1950).
[23] A. A. Ross, Hans Mark, and R. D. Lawson, Phys. Rev. **102**, 1613 (1956).

reactions, $Sc^{45}(\alpha,\alpha n)Sc^{44}$ or $Sc^{45}(p,pn)Sc^{44}$, are summarized below:

Neutron levels	$1f_{7/2}$, $1d_{3/2}$, $2s_{1/2}$	$1f_{7/2}$, $1d_{3/2}$, $2s_{1/2}$, $1d_{5/2}$
Sc^{44m}/Sc^{44}		
for spins 7, 3	0.46	0.56
for spins 6, 2	1.4	1.5

It is observed that the calculated isomer ratio, 0.46 or 0.56, from the knock-on calculation agrees fairly well with the experimental isomer ratio 0.62 for 320-Mev helium ions. The agreement between the calculated and experimental yield ratios of Sc^{44m}/Sc^{44} for the K^{41}(10-Mev α,n)Sc^{44} reaction, which was assumed to proceed by a compound-nucleus mechanism, and for the Sc^{45}(320-Mev α, αn)Sc^{44} reaction, which was assumed to occur by a knock-on mechanism, indicate the usefulness of these methods for calculating isomer ratios from nuclear reactions. Katz[6] used a compound-nucleus model to calculate isomer ratios from nuclear reactions and obtained agreement between calculated and experimentally measured ratios, and Rudstam[24] used the Serber[25] model and made Monte Carlo cascade calculations to calculate an isomer ratio in general agreement with experiment.

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I wish to express my gratitude to Professor Isadore Perlman, under whose guidance this work was done.

[24] Gösta Rudstam, thesis, University of Uppsala, 1956 (unpublished).
[25] R. Serber, Phys. Rev. 72, 1114 (1947).

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Angular Distribution and Ranges of N^{13} Particles from N^{14} on N^{14}

K. S. Toth
Oak Ridge National Laboratory, Oak Ridge, Tennessee*
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The distribution of N^{13} particles from the neutron transfer reaction $N^{14}(N^{14},N^{13})N^{15}$ was investigated from 3.5–32° in the laboratory. Range curves for N^{13} particles were obtained from 16–32° (laboratory system). Transfers leaving both residual nuclei in their ground states were distinguished from those in which the products are left in excited states. It is found that (1) the ground-state transfer cross section decreases as the bombarding energy is lowered; (2) of the N^{15} excited states, only the first and/or second contribute significantly to the transfer cross section; (3) for a given bombarding energy the excited-state transfer distribution peaks at an angle larger than that at which the ground-state transfer distribution reaches its maximum; (4) from the ground-state transfer distribution an r_0 of 2.2 f can be determined, while the excited-state transfer distribution yields an r_0 of about 1.65 f; and (5) the excited-state distributions are more consistent with the tunneling mechanism of Breit and Ebel than are the ground-state distributions.

INTRODUCTION

THE reaction $N^{14}(N^{14},N^{13})N^{15}$ has been investigated previously by Reynolds and Zucker.[1] Excitation functions were obtained using thick AgCN targets and, with the use of nitrogen gas targets, a rough angular distribution of N^{13} particles was determined. Reynolds and Zucker interpreted the reaction as proceeding by means of a neutron transferring from one N^{14} nucleus to the other. The present work was undertaken to improve and extend the N^{13} angular distribution data by using thin targets of a solid nitrogen-bearing compound.

At the outset, it was noted that the reaction was quite distinctive in that: (1) N^{13} excited states are unstable with respect to proton emission to C^{12}, so that N^{13} nuclei detected are necessarily found in their ground states, and (2) the first excited state in N^{15} occurs at 5.28 Mev. Thus, N^{13} nuclei resulting from transfers to the N^{15} ground state (ground-state transfers) are approximately 5 Mev more energetic than those leaving N^{15} recoils in excited states (excited-state transfers). The large energy gap made it possible to distinguish the two sets of N^{13} particles.

An earlier brief communication[2] dealt with the change in the relative amounts of ground- and excited-state transfers at various angles as the incident N^{14} energy was varied. The data reported at that time indicated that with decreasing bombarding energy, excited-state transfers become more numerous than ground-state transfers. The observation is confirmed by the present experimental information obtained with improved resolution at the larger angles.

EXPERIMENTAL METHOD

The N^{14} beam consisted of N^{3+} ions accelerated to about 28 Mev in the Oak Ridge 63-in. cyclotron. The experimental arrangement is shown in Fig. 1. A circular

* Operated by the U. S. Atomic Energy Commission by Union Carbide Corporation.
[1] H. L. Reynolds and A. Zucker, Phys. Rev. 101, 166 (1956).
[2] K. S. Toth, Phys. Rev. 121, 1190 (1961).