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Electric-Field-Induced Anisotropies in an Inhomogeneous Plasma*

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It is shown on the basis of the Boltzmann equation that in the absence of a dc magnetic field, an electric field alone can produce anisotropies in electron diffusion and conductivity in a weak inhomogeneous plasma. These anisotropies have their origin in the ac component of the electron density resulting from the interaction between the electric field and electron density gradients. Electron diffusion is reduced in the direction of the field, while the induced ac current acquires an additional component in the direction of the gradient of the ac electron density. This component will in general lie in a direction different from that of the exciting field. Although these effects are usually small, they may become significant under suitable circumstances.

I. INTRODUCTION

THE effect of a steady magnetic field on electron flow in a weak plasma is well known, the general result being the introduction of characteristic anisotropy terms into the flow parameters. In particular, both the diffusion and conductivity coefficients become tensors.[1] The purpose of this paper is to show that in the absence of a magnetic field, an alternating electric field alone can alter the properties of an inhomogeneous plasma in a somewhat similar way.

We will consider a weak, inhomogeneous plasma imbedded in the gas from which it is created by partial ionization. With this model, it may be assumed that while the ion densities are functions of position, the electron collision frequency is not. This implies that electron–neutral particle collisions dominate, and thus gives a measure of the required diluteness of the plasma. The effects to be considered here will involve the interaction between an alternating electric field and stationary electron concentration gradients. Accordingly, we will restrict our attention to the steady-state case in which the concentrations are maintained by continuous ionization; thus we might have in mind a microwave-induced gas discharge in a cavity, or the region behind a hypersonic shock front.

The behavior of the electrons will be described by the electron distribution function, approximated on the basis of the familiar decomposition of the Boltzmann equation which results from the expansion of this function in spherical harmonics in velocity space and in a Fourier series in harmonics of the frequency of the applied field. For electrons in the presence of an ac electric field of sufficiently high frequency, the rapid convergence of this expansion permits us to describe our model by means of the first-order equations given by Allis.[1] The ac electric field will be written $\mathbf{E}=\mathbf{E}_a e^{j\omega t}$, where $\mathbf{E}_a$ is constant in magnitude and direction over the region of interest. There will be no magnetic field. Assuming an applied field of this form to permeate the plasma, the first-order equations take the form,

$$\tfrac{1}{3}v\nabla\cdot\mathbf{f}_0{}^1-\frac{e}{6mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}_a\cdot\mathbf{f}_{1r}{}^1)=B_{0,\mathrm{in}}+\frac{m}{M}\frac{1}{v^2}\frac{\partial}{\partial v}\left[\nu_c v^3\left(f_0{}^0+\frac{kT_g}{mv}\frac{\partial f_0{}^0}{\partial v}\right)\right],\qquad(1)$$

$$j\omega f_1{}^0+\tfrac{1}{3}v\nabla\cdot\mathbf{f}_1{}^1-\frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}_a\cdot\mathbf{f}_0{}^1)=0,\qquad(2)$$

$$\nu_c\mathbf{f}_0{}^1+v\nabla f_0{}^0-\frac{1}{2}\frac{e}{m}\mathbf{E}_a\frac{\partial f_{1r}{}^0}{\partial v}=0,\qquad(3)$$

$$(j\omega+\nu_c)\mathbf{f}_1{}^1+v\nabla f_1{}^0-\frac{e}{m}\mathbf{E}_a\frac{\partial f_0{}^0}{\partial v}=0.\qquad(4)$$

The subscripts 0 and 1 designate the zeroth and first Fourier amplitudes of the symmetric, f^0, and asymmetric, $\mathbf{f}^1$, components of the electron distribution function, thus the above set represents eight partial differential equations for the two functions $f_0{}^0$ and $f_1{}^0$ and the six scalar components of $\mathbf{f}_1{}^1$ and $\mathbf{f}_0{}^1$. Since some of these may be complex, the additional subscript r is used to indicate that the real part only is to be taken. $B_{0,\mathrm{in}}$ is the inelastic collision term, e is the charge on the electron, m and M are the masses of the electron and gas molecule, respectively, ν_c is the collision frequency for momentum transfer, and T_g is the temperature of the neutral gas. The components of the distribution function depend on $\mathbf{r}$ and v only, the integration over angles in velocity space having already been performed. The equations above are incomplete, since certain terms have been ignored. The elastic collision term $B_1{}^0$, which would appear on the right of (2), has been neglected under the assumption that $\omega\gg\nu_c(m/M)$. Imposing the further requirement that the energy exchanged per collision between the electron and the field be small compared with the random energy of the electron, the second-harmonic components $\mathbf{f}_2{}^1$ and $f_2{}^0$ normally appearing in the last terms of (2) and (4), respectively, have also been neglected.

The macroscopic quantities associated with the various components of the distribution function will

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[1] W. P. Allis, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. 21, p. 404 ff.

be defined as follows:

the zeroth-order (stationary) electron density,

$$n_0(\mathbf{r})=\int_0^\infty f_0{}^0(\mathbf{r},v)4\pi v^2dv; \tag{5}$$

the first-order ac component of the electron density,

$$n_1(\mathbf{r})=\int_0^\infty f_1{}^0(\mathbf{r},v)4\pi v^2dv; \tag{6}$$

the stationary flow vector,

$$\mathbf{\Gamma}_0(\mathbf{r})=\int_0^\infty \mathbf{f}_0{}^1(\mathbf{r},v)(4\pi/3)v^3dv; \tag{7}$$

the first-order ac electric current,

$$\mathbf{j}_1(\mathbf{r})=-e\int_0^\infty \mathbf{f}_1{}^1(\mathbf{r},v)(4\pi/3)v^3dv. \tag{8}$$

In order to find these quantities, Eqs. (1)–(4) should be solved for the two scalar and two vector components of the distribution function. For many purposes, however, it is unnecessary to go this far. By obvious manipulations, Eqs. (2)–(4) can be transformed into three new equations, each relating one of the components $f_1{}^0$, $\mathbf{f}_0{}^1$, or $\mathbf{f}_1{}^1$ to $f_0{}^0$ alone. If these new equations can be solved, then the integrands of (6)–(8) will involve only $f_0{}^0$. Moreover, the assumption that the electron collision frequency is independent of position permits us to write $f_0{}^0(\mathbf{r},v)=n_0(\mathbf{r})F_0(v)$, where the velocity distribution $F_0(v)$ is normalized by (5). In this way, the integrals in (6)–(8) are reduced to products of the stationary density $n_0(\mathbf{r})$ and its derivatives, and sums of terms of the form

$$\int_0^\infty g(v)F_0(v)4\pi v^2dv=\langle g\rangle, \tag{9}$$

where $g(v)$ is some function of v only, and $\langle g\rangle$ is its mean value relative to the velocity distribution F_0. Even with this simplification, the velocity integrals can be quite complicated when ν_c is a function of velocity, as it usually is. Therefore, in order to display the basic features of the results as clearly as possible, it will be assumed in what follows that ν_c is independent of velocity. Thus the results to be given could also be obtained, apart from numerical factors, from a continuum formulation. (See the reference in footnote 2 for a particular example.)

II. ELECTRON DENSITY COMPONENT $n_1(\mathbf{r})$

If an alternating electric field is applied over a region in which the plasma is inhomogeneous, Eq. (4) may be solved for $\mathbf{f}_1{}^1$ in terms of $f_1{}^0$ and $f_0{}^0$. Substituting this into (2), one obtains

$$j\omega\left(f_1{}^0-\frac{v^2}{3j\omega(j\omega+\nu_c)}\nabla^2f_1{}^0\right)=-\frac{(e/m)v}{3(j\omega+\nu_c)}\nabla\cdot\left(\mathbf{E}_a\frac{\partial f_0{}^0}{\partial v}\right)+\frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}_a\cdot\mathbf{f}_0{}^1). \tag{10}$$

If the second term on the left side of this equation is ignored, and the resulting solution for $f_1{}^0$ inserted into (6), the integral of the second term on the right will vanish at the limits, and the remaining term gives an approximation for $n_1(\mathbf{r})$ in the form

$$n_1(\mathbf{r})\approx-\frac{(e/m)}{3j\omega}\nabla\cdot\left[\mathbf{E}_an_0(\mathbf{r})\int_0^\infty\frac{v^3}{(j\omega+\nu_c)}\frac{\partial F_0}{\partial v}4\pi dv\right]. \tag{11}$$

Doing the integral by parts (for constant ν_c) and recalling that $\mathbf{E}_a$ is independent of position, we obtain the simple expression

$$n_1(\mathbf{r})\approx-\frac{(e/m)}{j\omega(j\omega+\nu_c)}\mathbf{E}_a\cdot\nabla n_0(\mathbf{r}). \tag{12}$$

The alternating component of the electron density therefore arises as a result of the interaction between the applied ac electric field and the steady-state flow associated with electron density gradients. This observation has, in one form or another, been made before.[1–3] It is of particular interest because the dependence of n_1 on the applied field provides a nonlinear mechanism through which first order mixing of two electromagnetic waves may occur.[2,3]

The validity of the approximation depends upon the conditions under which the second term on the left of (10) may be neglected. If we estimate v^2 by its mean value $\langle v^2\rangle$, and write $\lambda=(2\pi c/\omega)$ as the nominal wavelength of the applied field, then we must require that

$$\left|\frac{\langle v^2\rangle}{3j\omega(j\omega+\nu_c)}\frac{\nabla^2f_1{}^0}{f_1{}^0}\right|\approx10^{-2}\left|\frac{\langle v^2\rangle}{c^2}\frac{\lambda^2}{\Lambda^2}\right|\ll1. \tag{13}$$

Since the spatial dependence of $f_1{}^0$ enters through $f_0{}^0$ only, the ratio $\nabla^2f_1{}^0/f_1{}^0$ has been approximated by $1/\Lambda^2$, where Λ is a diffusion length characteristic of the given concentration. The term in question may therefore be ignored in a nonrelativistic plasma, provided the ratio λ/Λ is not too large. This condition must be examined with particular care in certain situations, as in the case of ionization by a hypersonic shock wave, where the electron temperature is high and the electron concentration changes rapidly over a few mean free paths.

[2] L. Wetzel, J. Appl. Phys. **32**, 327 (1961).
[3] V. L. Ginsburg, Soviet Phys.—JETP **8**, 1100 (1959).

III. PERTURBATION OF THE FLOW VECTOR $\mathbf{\Gamma}_0(\mathbf{r})$

The electric field influences the stationary diffusion current through the last term of Eq. (3), where $\mathbf{E}_a$ appears with the real part of f_1^0. This represents the dc component of the second-order interaction between the applied field and the ac perturbation of f^0. (The ac component of this interaction appears in $\mathbf{f}_2^1$, which was left out of the original equations.) To determine the effect of this term on $\mathbf{\Gamma}_0$, let us assume that the condition (13) is satisfied, so that (10) can be solved for f_1^0 in the form

$$f_1^0 \approx -\frac{(e/m)v}{3j\omega(j\omega+\nu_c)}\nabla\cdot\left(\mathbf{E}_a\frac{\partial f_0^0}{\partial v}\right) - \frac{je}{3\omega m v^2}\frac{\partial}{\partial v}(v^2\mathbf{E}_a\cdot\mathbf{f}_0^1). \quad (14)$$

Since $\mathbf{f}_0^1$ is real, the last term of (14) is pure imaginary, so the real part of f_1^0 is just the real part of the first term. Thus (3) yields $\mathbf{f}_0^1$ in the following form:

$$\mathbf{f}_0^1 \approx -\frac{v}{\nu_c}\nabla f_0^0 + \frac{(e/m)^2}{6}\mathbf{E}_a\mathbf{E}_a \cdot\nabla\left\{\frac{1}{\nu_c}\frac{\partial}{\partial v}\left[\frac{v}{(\omega^2+\nu_c^2)}\frac{\partial f_0^0}{\partial v}\right]\right\}. \quad (15)$$

The flow vector $\mathbf{\Gamma}_0$, obtained by putting (15) into (7), consists of two parts. The integral of the first term on the right of (15) gives

$$\mathbf{\Gamma}_0^{(0)}(\mathbf{r}) = -\frac{\langle v^2\rangle}{3\nu_c}\nabla n_0(\mathbf{r}) = -D_0\nabla n_0(\mathbf{r}), \quad (16)$$

where D_0 denotes the unperturbed diffusion coefficient for the electrons. The second term may be integrated by parts to give the additional contribution,

$$\mathbf{\Gamma}_0^{(1)}(\mathbf{r}) = \frac{(e/m)^2}{2\nu_c(\omega^2+\nu_c^2)}\mathbf{E}_a\mathbf{E}_a\cdot\nabla n_0(\mathbf{r}). \quad (17)$$

Since these two vector components do not, in general, lie in the same direction, their sum can be written in tensor (dyadic) notation in the form

$$\mathbf{\Gamma}_0(\mathbf{r}) \approx \mathbf{\Gamma}_0^{(0)}(\mathbf{r}) + \mathbf{\Gamma}_0^{(1)}(\mathbf{r}) = -\mathfrak{D}\cdot\nabla n_0(\mathbf{r}), \quad (18)$$

with

$$\mathfrak{D} = D_0\left[\mathbf{1} - \frac{3}{2}\frac{(e/m)^2E_a^2}{\langle v^2\rangle(\omega^2+\nu_c^2)}\hat{k}\hat{k}\right], \quad (19)$$

where $\mathbf{1}$ is the unit tensor and $\mathbf{E}_a$ has been written as $E_a\hat{k}$, $\hat{k}$ being a unit vector in the direction of the applied field. This interesting result shows that an ac electric field alone can cause a plasma to behave anisotropically. Since the coefficient of $\hat{k}\hat{k}$ in (19) is negative, the effect of the electric field will be to reduce diffusion in the direction of the field, leaving the transverse diffusion unaltered. A recent calculation of body forces in a plasma concentration suggests a similar conclusion.[4]

The magnitude of the anisotropy term may be simply expressed in terms of two energy parameters. Let $U_A = \frac{1}{2}m\langle v^2\rangle$ be the random energy of the electrons, and $U_C = e^2E_a^2/2m(\omega^2+\nu_c^2)$ the average energy gained by an electron from the electric field between collisions. Using these parameters, (19) can be written

$$\mathfrak{D} = D_0[\mathbf{1} - \tfrac{3}{2}(U_C/U_A)\hat{k}\hat{k}]. \quad (20)$$

The magnitude of the anisotropy is therefore measured by the ratio of the average energy gained from the field per collision to the random energy of the electron.

In general the random energy U_A is a function of U_C which depends upon the various ways in which the electron can lose the energy it gains from the field. For example, if the electron undergoes only elastic collisions with the gas molecules, then U_A is of the order of U_C/δ, where δ is the energy loss parameter $2m/M$. In this case the anisotropy will be of order 10^{-4}, a negligible effect. However, if the electrons are allowed to excite or ionize the gas molecules with which they collide, fast electrons are replaced by one or more slow electrons, thereby changing the energy distribution and causing U_A to increase less rapidly with U_C. This situation has been considered by Reder and Brown,[5] who calculated the average electron energy in helium as a function of E_e/p (E_e is the effective electric field defined by $E_e^2 = (E_a^2\nu_c^2)/2(\omega^2+\nu_c^2)$, and p is the gas pressure). The nature of the dependence of U_A on U_C strongly affects the magnitude of the diffusion anisotropy, as can be illustrated in the following example. Consider helium at a pressure of 2 mm Hg in an ac electric field of rms amplitude $E_a/\sqrt{2} = 300$ v/cm at a frequency of 3000 Mc/sec. If we were to assume elastic collisions only, then $U_C/U_A \approx \delta \approx 3\times10^{-4}$, so the size of the anisotropy term in this case would be only about 4×10^{-4}. Now, using the value $\nu_c = 5.1\times10^9$/sec for He at 2 mm Hg, we find that $E_e = 78$ v/cm and $U_C = 0.4$ ev. Referring to Fig. 6 of reference 5 for the random energy when inelastic collisions are taken into account, U_A is found to be 11 ev for $E_e/p = 39$, and the anisotropy term becomes 5×10^{-2}, or two orders of magnitude greater than when such collisions are ignored. Thus in an actively ionized gas, where E_e/p is sufficiently high that the electron energy loss is mainly through ionization and excitation, one might expect the ac electric field to produce diffusion anisotropies of at least a few percent. In the usual microwave discharge, effects of this size would be difficult to verify by the indirect methods used by Lax, Allis, and Brown[6] in studying

[4] L. M. Kovrizhnykh, Soviet Phys.—JETP 6, 54 (1958).
[5] F. H. Reder and S. C. Brown, Phys. Rev. 95, 885 (1954).
[6] B. Lax, W. P. Allis, and S. C. Brown, J. Appl. Phys. 21, 1297 (1950).

the analogous behavior in a dc magnetic field. The electric field effects would probably require a more direct experimental confirmation.

The stationary electron density may be determined from the diffusion equation. That is, multiplying (1) by $4\pi v^2$, integrating over v, and introducing $\mathbf{\Gamma}_0$ from the definition (7), one obtains[6]

$$\nabla\cdot\mathbf{\Gamma}_0(\mathbf{r})=\int_0^\infty B_{0,\mathrm{in}}4\pi v^2dv=\nu_i n_0(\mathbf{r}), \tag{21}$$

where ν_i is the effective collision frequency for ionization. Using (20) for $\mathbf{\Gamma}_0$, the resulting equation in $n_0(\mathbf{r})$ may be solved subject to the boundary conditions appropriate to the given enclosure. The effect of the anisotropy is to produce an effective dilation of the enclosure in the direction of the electric field. (See the discussion in reference 6 for the analogous case of a magnetic field induced anisotropy.)

IV. PERTURBATION OF THE AC ELECTRIC CURRENT $j_1(\mathbf{r})$

The ac electric current is given in (8) in terms of $\mathbf{f}_1^1$, which can be found by eliminating $\mathbf{f}_0^1$ and f_1^0 from (2)–(4). Let us first substitute $\mathbf{f}_0^1$ from (3) into (2), whence

$$f_1^0=-\frac{v}{3j\omega}\nabla\cdot\mathbf{f}_1^1-\frac{(e/m)}{3j\omega v^2}\frac{\partial}{\partial v}\left(\frac{v^3}{\nu_c}\mathbf{E}_a\cdot\nabla f_0^0\right)+\frac{(e/m)^2}{6j\omega v^2}\frac{\partial}{\partial v}\left(\frac{v^2}{\nu_c}E_a^2\frac{\partial f_{1r}^0}{\partial v}\right). \tag{22}$$

The contribution of the last term to the perturbation of $\mathbf{j}_1$ can be shown to be of order U_C/U_A smaller than that of either of the other terms.[7] Since U_C/U_A is generally small, we are justified in ignoring this term in what follows. Substituting (22) into (4), we may write the latter in the form

$$\left(1-\frac{v^2}{3j\omega(j\omega+\nu_c)}\nabla\nabla\right)\cdot\mathbf{f}_1^1=\frac{(e/m)}{(j\omega+\nu_c)}\mathbf{E}_a\frac{\partial f_0^0}{\partial v}+\frac{(e/m)}{3j\omega v(j\omega+\nu_c)}\frac{\partial}{\partial v}\left[\frac{v^3}{\nu_c}\nabla(\mathbf{E}_a\cdot\nabla f_0^0)\right], \tag{23}$$

where tensor notation has been used on the left. It might be expected that the second term in the tensor could be ignored on the basis of (13). If this were done, however, it would be found that the perturbation of $\mathbf{j}_1$ contributed by the last term of (23) would have the magnitude of the neglected term. Therefore we cannot neglect terms of this size, although we will assume that we may ignore their squares. With this mind, we perform a scalar multiplication of (23) on the left by the tensor $[1+v^2\nabla\nabla/3j\omega(j\omega+\nu_c)]$, ignore terms of order higher than the first, and obtain the following approximate solution for $\mathbf{f}_1^1$:

$$\mathbf{f}_1^1\approx\frac{(e/m)}{(j\omega+\nu_c)}\mathbf{E}_a\frac{\partial f_0^0}{\partial v}+\frac{v^2}{3j\omega(j\omega+\nu_c)}\nabla\nabla\cdot\left[\frac{(e/m)}{(j\omega+\nu_c)}\mathbf{E}_a\frac{\partial f_0^0}{\partial v}\right]+\frac{(e/m)}{3j\omega v(j\omega+\nu_c)}\frac{\partial}{\partial v}\left[\frac{v^3}{\nu_c}\nabla(\mathbf{E}_a\cdot\nabla f_0^0)\right]. \tag{24}$$

It has been assumed that $f_0^0=n_0(\mathbf{r})F_0(v)$, and since $\mathbf{E}_a$ is a constant vector and $\nabla n_0(\mathbf{r})$ is irrotational, the space-dependent factors of the last two terms of (24) are both equal to $\mathbf{E}_a\cdot\nabla\nabla n_0(\mathbf{r})$. Substitution of (24) into (8) leaves three velocity integrals which can be reduced to the form of (9). After a few algebraic manipulations, the approximation for the ac electric current becomes

$$\mathbf{j}_1(\mathbf{r})\approx\frac{(e^2/m)}{(j\omega+\nu_c)}\mathbf{E}_a\cdot\left[1+\mathfrak{F}(\tau)\frac{\langle v^2\rangle}{\nu_c\omega}\nabla\nabla\right]n_0(\mathbf{r}), \tag{25}$$

where

$$\mathfrak{F}(\tau)=[5\tau+j(7\tau^2+2)]/9(\tau^2+1),\quad \tau=\nu_c/\omega. \tag{26}$$

The tensor in (25) is symmetric, so the current can be written

$$\mathbf{j}_1(\mathbf{r})\approx\mathfrak{C}\cdot\mathbf{E}_a, \tag{27}$$

with the conductivity tensor defined as

$$\mathfrak{C}=\frac{(e^2/m)}{(j\omega+\nu_c)}\left[1+\mathfrak{F}(\tau)\frac{\langle v^2\rangle}{\nu_c\omega}\nabla\nabla\right]n_0(\mathbf{r}). \tag{28}$$

For the reasons given above, the spatial dependence of the perturbation term in (25) may be written $\nabla(\mathbf{E}_a\cdot\nabla n_0)\propto\nabla n_1$, showing that the ac current has acquired a component in the direction of the gradient of the ac electron density. This implies the possibility of induced currents perpendicular to the direction of the applied field. It is clear from (13) that in most cases these perturbation currents will be quite small. However, under suitable conditions of temperature, frequency, and inhomogeneity, this term should become large enough to predict a detectable transverse current distribution.

V. CONCLUSION

The anisotropies appearing in the perturbations of the steady state flow vector and the ac electric current are seen to have their origin in the distribution function component f_1^0, (or, if one prefers, in the ac electron density component n_1 defined by it), which exists only in the presence of plasma inhomogeneities and is generally neglected. Although these effects are small,

[7] L. Wetzel, Brown University Scientific Report No. AF4561/9 (1960), (unpublished).

they are not beyond detection. For example, it was mentioned earlier that n_1 should give rise to nonlinear interaction between two electromagnetic waves. Experimental verification of this interaction appears to be feasible in the sharp electron density gradient existing behind a hypersonic shock front, and such an experiment is being planned.[8]

The effect of a dc magnetic field on the perturbations produced by the ac electric field is examined in the Appendix for a few simple cases. It is shown that when the magnetic and electric fields are parallel, both n_1 and the ac field induced perturbation of $\mathbf{\Gamma}_0$ are unaffected. Thus in this case the transverse and longitudinal flow retardations, produced, respectively, by the magnetic and electric fields, are simply superimposed on one another.

APPENDIX

In the presence of a dc magnetic field $\mathbf{B}_0$, the only change in Eqs. (1)–(4) is the addition of the term $-\boldsymbol{\omega}_b\times\mathbf{f}_0{}^1$ to (3), and the term $-\boldsymbol{\omega}_b\times\mathbf{f}_1{}^1$ to (4), where $\boldsymbol{\omega}_b=(e/m)\mathbf{B}_0$. Using the abbreviation $(j\omega+\nu_c)=\nu$, the modified form of Eq. (4) becomes

$$(\nu-\boldsymbol{\omega}_b\times)\mathbf{f}_1{}^1=-v\nabla f_1{}^0+(e/m)\mathbf{E}_a(\partial f_0{}^0/\partial v). \quad \text{(A.1)}$$

The inverse of the vector operator multiplying $\mathbf{f}_1{}^1$ has the form[1,6]

$$\mathbf{O}=\frac{(\nu^2+\boldsymbol{\omega}_b\boldsymbol{\omega}_b\cdot+\nu\boldsymbol{\omega}_b\times)}{\nu(\nu^2+\omega_b{}^2)}. \quad \text{(A.2)}$$

Therefore, we may write

$$\mathbf{f}_1{}^1=-v\mathbf{O}\nabla f_1{}^0+(e/m)\mathbf{O}\mathbf{E}_a(\partial f_0{}^0/\partial v), \quad \text{(A.3)}$$

and substitute into (2). Since the operator $\mathbf{O}$ is of order $(j\omega+\nu_c)^{-1}$, we will again ignore the term in $[\langle v^2\rangle\nabla^2/\omega^2]$ on the left of (10), and write

$$f_1{}^0\approx-\frac{v}{3j\omega}\frac{e}{m}\nabla\cdot\mathbf{O}\mathbf{E}_a\frac{\partial f_0{}^0}{\partial v}+\frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}_a\cdot\mathbf{f}_0{}^1). \quad \text{(A.4)}$$

Putting this into (6), assuming that ν_c is constant, and observing that the last term vanishes upon integration, we obtained, finally,

$$n_1(\mathbf{r})\approx\frac{(e/m)}{j\omega}\nabla\cdot\left[\frac{\nu^2+\boldsymbol{\omega}_b\boldsymbol{\omega}_b\cdot\ +\nu\boldsymbol{\omega}_b\times}{\nu(\nu^2+\omega_b{}^2)}\right]\mathbf{E}_a n_0(\mathbf{r}). \quad \text{(A.5)}$$

[8] *Note added in proof.* In recent experiments at Los Alamos, H. Dreicer (private communication) has observed interactions between two ac electric fields which indicate the existence of the electron density oscillations given by n_1. I would like to thank Dr. Dreicer for communicating these results, which will be described at the IAEA Conference on Plasma Physics, Salzburg, September, 1961.

For convenience consider the cases in which $\mathbf{E}_a$ and $\boldsymbol{\omega}_b$ are either parallel or perpendicular.

$\mathbf{E}_a$ *parallel to* $\boldsymbol{\omega}_b$. Here $\boldsymbol{\omega}_b\times\mathbf{E}_a=0$ and $\boldsymbol{\omega}_b\boldsymbol{\omega}_b\cdot\mathbf{E}_a=\omega_b{}^2\mathbf{E}_a$, so (A.5) becomes

$$n_1(\mathbf{r})\approx\frac{(e/m)}{j\omega}\nabla\cdot\left[\frac{(\nu^2+\omega_b{}^2)}{\nu(\nu^2+\omega_b{}^2)}\right]\mathbf{E}_a n_0(\mathbf{r})$$

$$=\frac{(e/m)}{j\omega(j\omega+\nu_c)}\mathbf{E}_a\cdot\nabla n_0(\mathbf{r}), \quad \text{(A.6)}$$

which is the same as (12).

$\mathbf{E}_a$ *perpendicular to* $\boldsymbol{\omega}_b$. Here $\boldsymbol{\omega}_b\cdot\mathbf{E}_a=0$, and if we let $\boldsymbol{\omega}_b=\omega_b\hat{e}_z$ and $\mathbf{E}_a=E_a\hat{e}_y$, then $\boldsymbol{\omega}_b\times\mathbf{E}_a=-\omega_bE_a\hat{e}_x$. In this case (A.5) becomes

$$n_1(\mathbf{r})\approx\frac{(e/m)}{j\omega}\nabla\cdot\left[\frac{\nu E_a\hat{e}_y-\omega_bE_a\hat{e}_x}{(\nu^2+\omega_b{}^2)}\right]n_0(\mathbf{r})$$

or

$$n_1(\mathbf{r})\approx\frac{(e/m)}{j\omega}\frac{E_a}{(\nu^2+\omega_b{}^2)}\left[\nu\frac{\partial n_0}{\partial y}-\omega_b\frac{\partial n_0}{\partial x}\right]. \quad \text{(A.7)}$$

This reduces to (A.6) when $\omega_b=0$, but when $\omega_b\neq0$ the dependence of n_1 on spatial variations of n_0 becomes considerably more complicated, and magnetic resonances can occur.

In the case of parallel $\mathbf{E}_a$ and $\mathbf{B}_0$, (A.4) reduces to (14), and the equation for $\mathbf{f}_0{}^1$ corresponding to (15) may be written

$$\mathbf{f}_0{}^1=-v\mathbf{O}_c\nabla f_0{}^0+\frac{(e/m)^2}{6}\mathbf{O}_c\mathbf{E}_a\mathbf{E}_a$$

$$\cdot\left\{\frac{\partial}{\partial v}\left[\frac{v}{(\omega^2+\nu_c{}^2)}\frac{\partial}{\partial v}\nabla f_0{}^0\right]\right\}, \quad \text{(A.8)}$$

where $\mathbf{O}_c$ is defined by (A.2) with ν replaced by ν_c. Just as in (A.6), $\mathbf{O}_c\mathbf{E}_a=\mathbf{E}_a/\nu_c$, so the second term on the right of (A.8) is identical to the second term on the right of (20). This means that the diffusion anisotropy generated by the electric field is unaltered by the application of a parallel magnetic field. The first term on the right of (A.8) contains the magnetically induced anisotropy found in reference 6, [see their Eq. (25)]. This term results in a retardation of diffusion in directions *normal* to the magnetic, hence also electric, field, so we conclude that for parallel fields, these two anisotropies are uncoupled and add independently.