Remanent State in One-Dimensional Micromagnetics

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A first integral is found for Brown's nonlinear equations in one dimension. When the external field is zero, another first integral can be found, which enables complete integration of the equations. For a unidirectional anisotropy with an easy direction perpendicular to the plane of the film, one of the integration constants is not determined by the boundary conditions, as if indicating a possibility of continuum of different remanence values for different histories. However, when a small field in the plane of the film is introduced as a perturbation, this degeneracy is removed, and the magnetization can change only in the plane defined by the field and the direction of anisotropy. There are still many discrete possible values for the remanence, each of which determines uniquely the susceptibility at that state.

I. INTRODUCTION

THE static equilibria of a ferromagnet are given by the Brown equations, which can be written in vector form,[1]

$$\mathbf{v}\times[C\nabla^2\mathbf{v}-(\partial w/\partial\mathbf{v})+J_s\mathbf{H}]=0. \tag{1}$$

Here $\mathbf{v}$ is a unit vector in the direction of magnetization; its Cartesian components are the direction cosines α,β,γ of the magnetization vector. The anisotropy energy per unit volume is denoted by w, while $\partial w/\partial\mathbf{v}$ represents a vector whose components are $\partial w/\partial\alpha$, $\partial w/\partial\beta$, $\partial w/\partial\gamma$; C is the exchange constant and J_s is the saturation magnetization. The field $\mathbf{H}$ is given by

$$\mathbf{H}=\mathbf{H}_0-\nabla V, \tag{2}$$

where $\mathbf{H}_0$ is the external applied field and V is the potential of volume and surface charges.

The boundary conditions for (1) are[1]

$$\mathbf{v}\times\partial\mathbf{v}/\partial n=0, \tag{3}$$

where n is the normal. Actually, since $\mathbf{v}$ is a unit vector, its derivative is perpendicular to it, so that (3) implies $\partial\mathbf{v}/\partial n=0$. However, Brown prefers the form (3) which seems better related to (1), and reveals the fact that (3), as well as (1), yields actually only two independent equations in terms of the components, and not 3.

In the two Eqs. (1), one of the components of $\mathbf{v}$ depends on the other two through the relation

$$\alpha^2+\beta^2+\gamma^2=1, \tag{4}$$

which complicates the handling of (1).

Solutions of Brown equations are known only when they are linearized by assuming small changes from saturation, i.e., $\alpha\ll1$ and $\beta\ll1$ and only first-order terms in α,β are considered.[2–6] Only for the case of an infinite cylinder, a solution of the nonlinear equations was tried,[4,7] under certain assumptions, but no solution was found in this case, and the hysteresis curve was taken as a rectangular loop. When an exchange anisotropy is added, one can find a solution of (1), for special one-dimensional cases.[8] In these cases, as well as in the study of imperfections,[9,10] one selects the appropriate solution by following it from nucleation, which proved efficient in all but one of the cases,[8] in which two such solutions were found. In the present treatment another approach is tried, namely to start from zero external field, for which fortunately there is an analytic solution of (1) in one dimension. It turns out that for zero external field there is a continuum of possible one-dimensional solutions, which one would expect if the Brown equations were to yield stable domain configurations. In this case, if one starts at a demagnetized state and applies a magnetic field which is large enough to reverse some of the domains, then removes it, the remanence state should depend on this previously applied field. Actually it is an experimental fact that one can get any remanence value between zero and its value on the curve which starts from saturation, the so-called limiting hysteresis curve. However, if a small field is added as a perturbation to one-dimensional case under study, most of these solutions disappear.

In Sec. II the general Eqs. (1) are written in another form, for the one dimensional case, and a first integral is found. In Sec. III another first integral is found for zero external field and the complete solution is carried out for the case of unidirectional anisotropy with an easy direction perpendicular to the film. In Sec. IV a small external field is added as a perturbation.

II. BROWN'S EQUATIONS IN ONE DIMENSION

A unidirectional anisotropy will be assumed for simplicity, with an easy or hard direction along one of the Cartesian axes (the case of a cubic symmetry is evidently treated similarly). In this case

$$w=K_1\alpha^2+K_2\beta^2. \tag{5}$$

[1] W. F. Brown, Jr., J. Appl. Phys. **30**, 62 S (1959).
[2] W. F. Brown, Jr., Phys. Rev. **105**, 1479 (1957).
[3] E. H. Frei, S. Shtrikman, and D. Treves, Phys. Rev. **106**, 446 (1957).
[4] A. Aharoni and S. Shtrikman, Phys. Rev., **109**, 1522 (1958).
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[9] A. Aharoni, Phys. Rev. **119**, 127 (1960).
[10] C. Abraham and A. Aharoni, Phys. Rev. **120**, 1576 (1960).

The relation (4) is fulfilled if one transforms α, β, γ into

$$\alpha = \sin\omega \cos\Omega, \tag{6a}$$

$$\beta = \sin\omega \sin\Omega, \tag{6b}$$

$$\gamma = \cos\omega, \tag{6c}$$

and there is no loss in generality, as long as ω and Ω are any functions of x, y, and z. It actually means using the angles between magnetization and coordinate axes, rather than the direction cosines.

The transformation (6) can be carried out for the general three-dimensional case. The results are given in Appendix I, for the sake of future study of more complicated cases, since they are easier to handle than (1) and (4), especially for numerical computations. In the present paper, however, we shall be interested only in one dimension, i.e., for ω, Ω (and V) assumed functions of x only, and the material is a slab infinite in the y and z directions, and extending from $-a$ to $+a$ in the x direction. In this one-dimensional case it is readily shown that the flux density, B, vanishes, which eliminates V of (2), so that the use of (5) and (6) in (1) implies

$$\frac{d^2\omega}{dx^2} - \tfrac{1}{2}\sin 2\omega \left(\frac{d\Omega}{dx}\right)^2 - \tfrac{1}{2}(g_1 \cos^2\Omega + g_2 \sin^2\Omega)\sin 2\omega - h\sin\omega = 0, \tag{7a}$$

$$\sin\omega \frac{d^2\Omega}{dx^2} + 2\cos\omega \frac{d\omega}{dx}\frac{d\Omega}{dx} + \tfrac{1}{2}(g_1 - g_2)\sin\omega \sin 2\Omega = 0. \tag{7b}$$

Here

$$g_1 = (2K_1 + 4\pi J_s)/C, \quad g_2 = 2K_2/C, \quad h = J_s H_0/C, \tag{8}$$

with the external field $\mathbf{H}_0$ assumed in the $+z$ direction. The boundary conditions (3) are transformed to

$$d\omega/dx = (\sin\omega)d\Omega/dx = 0. \tag{9}$$

Consider now the expression

$$A = \sin^2\omega \left(\frac{d\Omega}{dx}\right)^2 + \left(\frac{d\omega}{dx}\right)^2 - (g_1 \cos^2\Omega + g_2 \sin^2\Omega)\sin^2\omega + 2h\cos\omega. \tag{10}$$

By differentiating and substituting for the second derivatives from (7), one obtains $dA/dx = 0$; i.e., A is a first integral.

If $g_1 = g_2$ (the easy axis parallel to z), Eq. (7b) can be integrated once to yield

$$\sin^2\omega(d\Omega/dx) = \text{const}.$$

Since this expression is zero on the boundary, it is identically zero, i.e., $\Omega = \text{const}$. In this case (10) is an equation in ω only and the integration is straightforward. This case has been studied before[8,9] and will not be discussed here.

III. THE REMANENCE STATE

For $h=0$, it can be verified by differentiation that

$$B = (g_1 \sin^2\Omega + g_2 \cos^2\Omega)\left(\frac{d\omega}{dx}\right)^2 + \tfrac{1}{4}(g_1 \cos^2\Omega + g_2 \sin^2\Omega)\sin^2 2\omega \left(\frac{d\Omega}{dx}\right)^2 + \tfrac{1}{2}(g_1 - g_2)\sin 2\omega \sin 2\Omega \frac{d\omega}{dx}\frac{d\Omega}{dx} + g_1 g_2 \cos^2\omega \tag{11}$$

is another first integral.

In the following, only the case of easy axis in the x direction will be considered. In this case,

$$g_2 = 0, \tag{12a}$$

and for simplification of notations, we shall use

$$g = -g_1. \tag{12b}$$

Substituting (12) in (11), one obtains

$$B = -g\left(\sin\Omega \frac{d\omega}{dx} + \tfrac{1}{2}\cos\Omega \sin 2\omega \frac{d\Omega}{dx}\right)^2. \tag{13}$$

According to the boundary condition (9), $B=0$ and therefore (13) implies

$$\sin\Omega = b\cot\omega, \tag{14}$$

where b is a constant. By substituting (14) in (10) and using α of (6a), one obtains

$$4\alpha^2(1-\alpha^2)(A - g\alpha^2) = (d\alpha^2/dx)^2. \tag{15}$$

If the anisotropy in the x direction is large enough to overcome the demagnetizing field, $g>0$ and in this case the solution of (15) is

$$\alpha = k\,\mathrm{sn}(u,k), \quad u = g^{\frac{1}{2}}(x - x_0). \tag{16}$$

Here "sn" is the sine amplitude function,[11] x_0 is a constant, and

$$k^2 = A/g \tag{17}$$

may be regarded as an arbitrary constant, replacing A. The three constants x_0, k, b and the value zero already chosen for B should determine the general solution of (7). The case $g<0$ is of less interest, since in this case one would expect the magnetization to rotate in the yz plane. This is actually shown in Appendix II.

From (16) one can readily find that

$$\cos\omega = (1+b^2)^{-\frac{1}{2}}\,\mathrm{dn}u, \tag{18a}$$

$$\sin\Omega = b(b^2 + k^2\,\mathrm{sn}^2 u)^{-\frac{1}{2}}\,\mathrm{dn}u, \tag{18b}$$

and hence

$$d\omega/dx = k^2 g^{\frac{1}{2}}(b^2 + k^2\,\mathrm{sn}^2 u)^{-\frac{1}{2}}\,\mathrm{sn}u\,\mathrm{cn}u \tag{19a}$$

$$-\sin\omega(d\Omega/dx) = bkg^{\frac{1}{2}}(b^2 + k^2\,\mathrm{sn}^2 u)^{-\frac{1}{2}}\,\mathrm{cn}u. \tag{19b}$$

[11] P. F. Byrd and M. D. Friedman, *Handbook of Elliptic Integrals for Engineers and Physicists* (Springer-Verlag, Berlin, 1954).

To fulfill the boundary conditions (9) one should have

$$\mathrm{cn}u=0, \quad \text{for} \quad x=\pm a.$$

This implies

$$g^{\frac{1}{2}}(a-x_0)=(2n+1)K(k) \tag{20a}$$

$$-g^{\frac{1}{2}}(a+x_0)=(2m+1)K(k), \tag{20b}$$

where m and n are integers. They are practically arbitrary and their only limitation is that they should be different, since by subtracting (20b) from (20a)

$$g^{\frac{1}{2}}a=(n-m)K(k).$$

The left-hand side is a given constant of problem and is different from zero. Therefore

$$n\neq m. \tag{20c}$$

Equations (20) determine the constants x_0 and k, although not uniquely. All the boundary conditions are thus fulfilled, *with the constant b undetermined.*

The remanent magnetization in the direction of previously applied field is

$$j=\int_{-a}^{a}\cos\omega dx=(n-m)\pi g^{-\frac{1}{2}}(1+b^2)^{-\frac{1}{2}}. \tag{21}$$

This contains the quite arbitrary parameter b, permitting thus a continuum of possible remanence values. However, these are not stable and it will be shown in the next part that by applying a small magnetic field, b is uniquely determined.

IV. A SMALL PERTURBATION

Consider a field $|h|\ll 1$. One can evidently write

$$\omega=\omega_0+h\omega_1, \quad \Omega=\Omega_0+h\Omega_1, \tag{22}$$

where ω_0, Ω_0 are the functions calculated in the previous part, for $h=0$. To a first approximation in h, one can therefore take for example

$$\begin{aligned}\sin\omega&=\sin\omega_0+h\omega_1\cos\omega_0\\&=(1+b^2)^{-\frac{1}{2}}\{(9b^2+k^2\,\mathrm{sn}^2u)^{\frac{1}{2}}+h\omega_1\,\mathrm{dn}u\},\end{aligned}$$

and similarly for the other functions involved. Substituting (22) in (7a) and in (10), one obtains to a first approximation in h

$$\begin{aligned}&d^2\omega_1/du^2-2bk(1+b^2)^{-\frac{1}{2}}(b^2+k^2\,\mathrm{sn}^2u)^{-\frac{1}{2}}\\&\quad\times\mathrm{dn}u\{d(\Omega_1\,\mathrm{cn}u)/du\}+k^2(1-b^2-2k^2\,\mathrm{sn}^2u)\\&\quad\times(b^2+k^2\,\mathrm{sn}^2u)^{-2}\{k^2\,\mathrm{sn}^4u+b^2(\mathrm{sn}^2u-\mathrm{cn}^2u)\}\omega_1\\&\qquad=g^{-1}(b^2+k^2\,\mathrm{sn}^2u)^{\frac{1}{2}}(1+b^2)^{-\frac{1}{2}},\end{aligned} \tag{23}$$

and

$$\mathrm{cn}^2u\frac{d}{du}\left[\frac{k^2\,\mathrm{sn}u}{\mathrm{cn}u(b^2+k^2\,\mathrm{sn}^2u)^{\frac{1}{2}}}\omega_1+\frac{bk}{(1+b^2)^{\frac{1}{2}}}\frac{\Omega_1}{\mathrm{cn}u}\right]=A_1-\frac{\mathrm{dn}u}{g(1+b^2)^{\frac{1}{2}}}, \tag{24}$$

where

$$A_1=(A/g-k^2)/(2h)$$

is a constant replacing A.

Dividing (24) by cn^2u, integrating and multiplying by $\mathrm{cn}u$, one obtains a linear relation between ω_1 and Ω_1 involving known functions, the constant A_1 and another constant of integration, A_2. Differentiating this relation and using the boundary conditions $d\omega_1/du=d\Omega_1/du=0$, for $x=\pm a$, it can be shown that these are compatible with (20), only if $A_1=A_2=0$. This implies

$$bk\Omega_1+k^2(1+b^2)^{\frac{1}{2}}(b^2+k^2\,\mathrm{sn}^2u)^{-\frac{1}{2}}(\mathrm{sn}u)\omega_1+g^{-1}\,\mathrm{sn}u=0. \tag{25}$$

Using this to substitute for Ω_1 in (23), and using the transformation

$$\omega_1=g^{-1}(1+b^2)^{-\frac{1}{2}}(b^2+k^2\,\mathrm{sn}^2u)^{-\frac{1}{2}}v(u)\,\mathrm{dn}u, \tag{26}$$

one obtains finally an equation in v, whose first integral is

$$\frac{dv}{du}=\frac{b^2\,\mathrm{am}u-\mathrm{sn}u\,\mathrm{cn}u+\mathrm{const}}{\mathrm{dn}^2u}-\mathrm{sn}u\,\mathrm{cn}u. \tag{27}$$

Now from (26) it is seen that since $\mathrm{cn}u$ and $d\omega_1/du$ are zero on the boundary, so is dv/du. From (27), on the other hand, this condition is compatible with (20) only if both the integration constant *and* b are zero. The parameter b is thus uniquely determined, and its only possible value is zero. According to (14) and (6), this means that the magnetization does not have a component in the y direction. It can be shown now that for $b=0$

$$-kg\omega_1=\mathrm{sn}u, \tag{28}$$

and all the boundary conditions are actually fulfilled. The magnetization in the direction of previously applied field is, for small values of h,

$$j(h)=\frac{a\pi}{K(k)}+\frac{2ha}{gk^2}\left[1-\frac{E(k)}{K(k)}\right], \tag{29}$$

with k determined by (20). This contains the undetermined integers m and n allowing different discrete values for the remanence $j(0)$ and the susceptibility. The remanence and the susceptibility, however, determine each other uniquely by (29) and it would be interesting to try to verify this relation experimentally with thin films, for different remanence values.

The z component of magnetization in zero applied field is, according to (18a), a purely sinusoidal function of space and does not show any similarity to Bloch walls. The parameters m and n actually determine the number of cycles within the slab. This might be associated with the assumption of one dimension. The configuration obtained might also explain spin-wave excitation in thin films.

A small field *perpendicular* to the plane of the film can also be easily studied. This is best carried out by

assuming dependence on z rather than on x in the general equations of Appendix I and considering a boundary at $z=\pm a$. In this case it is readily shown that the only solution for $h=0$ is $\sin\omega=0$, i.e., a uniform magnetization.

V. CONCLUSION

The Brown equations in one dimension can yield nonuniform magnetization, when the anisotropy is perpendicular to the plane of the film, but not Bloch walls.[12] The magnetization lies in the plane determined by the direction of external field and that of the anisotropy. The remanence magnetization can assume arbitrary discrete values, but it uniquely determines the susceptibility.

APPENDIX I. THE GENERAL BROWN EQUATIONS

By substituting (2), (5), and (6) into (1) and using adequate linear combinations, one obtains finally, assuming $\mathbf{H}_0$ to be applied in the $+z$ direction,

$$C\left[\nabla^2\omega-\tfrac{1}{2}\sin2\omega\left\{\left(\frac{\partial\Omega}{\partial x}\right)^2+\left(\frac{\partial\Omega}{\partial y}\right)^2+\left(\frac{\partial\Omega}{\partial z}\right)^2\right\}\right]$$
$$-(K_2\sin^2\Omega+K_1\cos^2\Omega)\sin2\omega$$
$$-J_s\cos\omega\left(\sin\Omega\frac{\partial V}{\partial y}+\cos\Omega\frac{\partial V}{\partial x}\right)$$
$$-J_s\sin\omega\left(H_0-\frac{\partial V}{\partial z}\right)=0, \tag{30a}$$

$$C\left[\sin\omega\nabla^2\Omega+2\cos\omega\left(\frac{\partial\omega}{\partial x}\frac{\partial\Omega}{\partial x}+\frac{\partial\omega}{\partial y}\frac{\partial\Omega}{\partial y}+\frac{\partial\omega}{\partial z}\frac{\partial\Omega}{\partial z}\right)\right]$$
$$+(K_1-K_2)\sin\omega\sin2\Omega$$
$$+J_s\left(\sin\Omega\frac{\partial V}{\partial x}-\cos\Omega\frac{\partial V}{\partial y}\right)=0, \tag{30b}$$

where V inside the material should fulfill

$$\nabla^2V=4\pi J_s\left[\frac{\partial(\sin\omega\cos\Omega)}{\partial x}+\frac{\partial(\sin\omega\sin\Omega)}{\partial y}+\frac{\partial(\cos\omega)}{\partial z}\right]. \tag{30c}$$

Outside the material,

$$\nabla^2V_{\text{out}}=0, \tag{31}$$

and on the boundary

$$V_{\text{in}}=V_{\text{out}}, \tag{32a}$$

$$\partial V_{\text{in}}/\partial n-\partial V_{\text{out}}/\partial n=4\pi J_n. \tag{32b}$$

Also, by using (6) in (3), one obtains on the boundary

$$\partial\omega/\partial n=0, \tag{32c}$$

$$(\sin\omega)\partial\Omega/\partial n=0. \tag{32d}$$

In this form the Brown equations are much easier to handle, especially for numerical computations. A transformation similar to (6) can evidently be used for other coordinate systems. If one writes (1) in cylindrical coordinates and uses the following substitution for the components of $\mathbf{v}$,

$$\begin{aligned}\alpha_r&=\sin\omega^*\sin\Omega^*,\\ \alpha_\varphi&=\sin\omega^*\cos\Omega^*,\\ \alpha_z&=\cos\omega^*,\end{aligned} \tag{33}$$

and if one assumes a unidirectional anisotropy with an easy direction along the z axis, which is also the direction of the external magnetic field, the Brown equations are:

$$C\left[\nabla^2\omega^*-\tfrac{1}{2}\sin2\omega^*\left\{\left(\frac{\partial\Omega^*}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial\Omega^*}{\partial\varphi}-1\right)^2+\left(\frac{\partial\Omega^*}{\partial z}\right)^2\right\}\right]$$
$$-K\sin2\omega^*-J_s\cos\omega^*\left(\frac{\cos\Omega^*}{r}\frac{\partial V}{\partial\varphi}+\sin\Omega^*\frac{\partial V}{\partial r}\right)$$
$$-J_s\sin\omega^*\left(H_0-\frac{\partial V}{\partial z}\right)=0, \tag{34a}$$

$$C\left[\sin\omega^*\nabla^2\Omega^*+2\cos\omega^*\right.$$
$$\left.\times\left\{\frac{\partial\omega^*}{\partial r}\frac{\partial\Omega^*}{\partial r}+\frac{1}{r^2}\frac{\partial\omega^*}{\partial\varphi}\left(\frac{\partial\Omega^*}{\partial\varphi}-1\right)+\frac{\partial\omega^*}{\partial z}\frac{\partial\Omega^*}{\partial z}\right\}\right]$$
$$-J_s\left(\cos\Omega^*\frac{\partial V}{\partial r}-\frac{\sin\Omega^*}{r}\frac{\partial V}{\partial\varphi}\right)=0 \tag{34b}$$

$$\nabla^2V=4\pi J_s\left[\frac{\sin\omega^*\sin\Omega^*}{r}\left(1-\frac{\partial\Omega^*}{\partial\varphi}\right)\right.$$
$$+\frac{\cos\omega^*\cos\Omega^*}{r}\frac{\partial\omega^*}{\partial\varphi}+\cos\omega^*\sin\Omega^*\frac{\partial\omega^*}{\partial r}$$
$$\left.+\sin\omega^*\cos\Omega^*\frac{\partial\Omega^*}{\partial r}-\sin\omega^*\frac{\partial\omega^*}{\partial z}\right], \tag{34c}$$

with (31) and (32) the same as in the Cartesian coordinates.

APPENDIX II. THE CASE $g<0$

The solution of (15) in this case is

$$\alpha=\mathrm{dn}u,\quad u=(-g)^{\frac{1}{2}}(x-x_0),\quad k^2=1-g/A, \tag{35}$$

[12] A similar result was reported by M. W. Muller, Bull. Am. Phys. Soc. **6**, 125 (1961), and Phys. Rev. **122**, 1485 (1961). However, he used quite a different approach.

which implies

$$\cos\omega = k(1+b^2)^{-\frac{1}{2}}\,\mathrm{sn}u,\quad \sin\Omega = bk(b^2+\mathrm{dn}^2u)^{-\frac{1}{2}}\,\mathrm{sn}u, \tag{36}$$

$$d\omega/dx = -k(-g)^{\frac{1}{2}}(b^2+dn^2u)^{-\frac{1}{2}}\,\mathrm{cn}u\,\mathrm{dn}u, \tag{37}$$

$$\sin\omega(d\Omega/dx) = bk(-g)^{\frac{1}{2}}(b^2+\mathrm{dn}^2u)^{-\frac{1}{2}}\,\mathrm{cn}u, \tag{38}$$

$$V = 4\pi J_s(-g)^{-\frac{1}{2}}(\mathrm{am}u+C), \tag{39}$$

where C is a constant.

Using (37) and (38) in (9), one obtains that either $bk=0$, or $\mathrm{cn}u=0$, for $x=\pm a$. Suppose the latter is possible; then

$$u(a)=(2n+1)K(k),\quad u(-a)=(2m+1)K(k), \tag{40}$$

where m and n are integers. In this case

$$\mathrm{am}u(a)=(n+\tfrac{1}{2})\pi,\quad \mathrm{am}u(-a)=(m+\tfrac{1}{2})\pi.$$

Substituting in (39), V cannot fulfill (32a) unless $m=n$ and according to (40) and (35) this is impossible. It follows that the only possibility is $bk=0$. Now if $k=0$, one obtains from (36) that $\sin\Omega=0$ and according to (14)

$$b=0. \tag{41}$$

It can now be shown that the only solution of (7) which fulfills the boundary conditions is $\sin 2\omega=0$.

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Recovery of Electron Radiation Damage in n-Type InSb*

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The production and recovery of electron radiation damage in n-type InSb has been studied by means of Hall-coefficient and electrical-conductivity measurements. Irradiations were performed mainly at 80°K, since no recovery was observed between 4° and 80°K. The damage recovered in five well-defined stages with the recovery nearly complete at 320°K. Isochronal and isothermal recovery was monitored in each of the stages, allowing a determination of the activation energies for recovery and a study of the recovery kinetics. None of the recovery kinetics fit any simple models. There is evidence that the two lowest-temperature recovery stages involve the annihilation of close interstitial-vacancy pairs and that interactions of primary defects with impurities do not occur. However, the first-order kinetics expected for close-pair recovery is not explicitly observed. A possible explanation for the observed kinetics, involving the independent annihilation of two types of close-pair configurations in the same stage with an electrostatic interaction between the interstitial and vacancy, is proposed.

I. INTRODUCTION

THE recovery of electron radiation damage has been studied in some detail in germanium[1,2] and to a lesser extent in silicon.[1,3] Aukerman[4] has published some results on damage recovery in InSb; however, the present work probably represents the first extensive study of damage recovery in a compound semiconductor.

The production of damage in n-type InSb by electrons with energies as low as 0.24 Mev (corresponding to a maximum energy transfer to an indium atom of 5.7 ev) has been reported earlier.[5] The present paper reports a detailed study of recovery of electron radiation damage produced principally at 0.4, 0.7, and 1.0 Mev. The results are mainly concerned with recovery in samples with initial electron concentrations of about 1.5×10^{16} cm^{-3}, and, unless specified otherwise, the results quoted are for such samples. Some differences which occur when purer samples are used are also mentioned, and these effects are the subject of further investigations.

InSb crystallizes in the zincblende structure. In this structure, each indium atom is tetrahedrally surrounded by four antimony atoms, and each antimony is similarly surrounded by four indium atoms. The indium atoms and antimony atoms each lie on a fcc sublattice, and there are two more sublattices of vacant sites. One of these has tetrahedrally arranged indium atoms for nearest neighbors, and the other has nearest neighbor antimony atoms in the same arrangement. This leads to four possible interstitial configurations, i.e., an indium interstitial with either indium or antimony nearest neighbors and similar configurations for an antimony interstitial. There are also two possible kinds of vacancies and two kinds of replacements in which the originally displaced atom strikes an atom of the other type displacing it and remaining in its lattice position.

One might think that this multiplicity of possible point defects would lead to complications in the radiation damage studies in InSb in comparison to elemental

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