Angular Distribution of Sputtered Potassium Atoms*

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The angular distributions of potassium particles issuing from a potassium surface under bombardment by noble gas ions were observed under moderately good vacuum conditions. Sputtered potassium atoms were detected for incident ion energies above approximately 15 ev and useful observations of angular distributions were obtained for incident ion energies in the range 50 to 450 ev for all available values of the incident angle. A means was discovered for the discrimination between the total sputtered flux and that fraction of it possessing particle energies above a certain threshold. The apparatus and experimental procedures are described and the observed distribution patterns and a two-collision sputtering mechanism are discussed, along with related observations.

I. INTRODUCTION

RECENTLY several mechanistic theories of sputtering have been advanced by Henschke,[1] Langberg,[2] Silsbee,[3] Harrison,[4] and others, in which particle ejection is evisioned as the terminal event in a succession of collision processes. These theories have been supported by the valuable experimental contributions of Wehner,[5–7] Moore,[8] Bradley,[9] Thompson,[10] Bader,[11] and others. In spite of the general success of these studies, certain details of the sputtering mechanism have not yet been established, and it was thought that observations of the spatial distribution of sputtered particles performed during the surface bombardment would provide valuable information.

The method adopted was a more extensive application of the experimental procedure developed by Bradley[9] for his studies of the sputtering of potassium by argon. In this adaptation, freshly formed potassium surfaces were bombarded at various angles of incidence by noble gas ions. The flux per solid angle of sputtered potassium was measured with a surface ionization detector which could be swung in the plane of incidence about the target. The energies of the incident ions were varied over the range of 0 to 450 ev although consistent data were obtained at energies above 50 ev.

It was discovered in the course of the experiment that some fraction of the incident atom flux at the detector filament becomes ionized and detected with the filament unheated.[12] Preliminary study indicates that the signals obtained under cold filament conditions originate from potassium particles whose energies after rebound from the platinum surface are sufficient to overcome the surface adsorptive forces. A means was thus made available for the discrimination between these relatively energetic particles and the total sputtered flux. (See Sec. IV.)

The observed experimental distribution curves were in agreement with the hypothesis that at incident ion energies of less than one hundred times the sublimation energy (approximately) the dominant sputtering mechanism is a two-collision chain. This chain is of the type proposed by Langberg[2] in the case of argon, krypton, or xenon where the incident ion mass is equal to or greater than the potassium mass. The chain was at least partially of the rebound type proposed by Henschke[1] and suggested as likely by Wehner[13] for the case of helium or neon ion bombardment, where the ion mass is less than that of the surface particle. A three-collision mechanism was in evidence in the case of incident energies greater than one hundred times the sublimation energy particularly at near normal incidence.

The experiment was complicated, and the results clouded in some measure by the presence of small quantities of contaminant residual gases. The potassium surface became partially coated during the experiments, so that a minimum bombarding flux was required to maintain static conditions, a requirement which precluded measurements at the lower bombarding energies. For this reason the sputtering yield (sputtered atoms per incident ion) is dependent upon the bombarding flux and upon the apparatus vacuum history. Despite this difficulty, it was possible by suitable techniques and procedures to obtain distributions in flux exhibiting the same principal features on different days for the same conditions of incident energy and angle.

II. EXPERIMENTAL APPARATUS

The capillary-arc ion source shown in the experimental apparatus schematic diagram, Fig. 1, is similar to one

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[1] E. B. Henschke, Phys. Rev. **106**, 4 (1957).
[2] E. L. Langberg, Phys. Rev. **111**, 91 (1958).
[3] R. H. Silsbee, J. Appl. Phys. **28**, 1246 (1957).
[4] D. E. Harrison, Phys. Rev. **102**, 1473 (1956).
[5] G. K. Wehner, *Advances in Electronics and Electron Physics* (Academic Press, Inc., New York, 1955), Vol. 7.
[6] G. K. Wehner, Phys. Rev. **108**, 35 (1957).
[7] G. K. Wehner, Phys. Rev. **112**, 1120 (1958).
[8] W. J. Moore, Am. Scientist **48**, 2 (1960).
[9] R. C. Bradley, Phys. Rev. **93**, 719 (1954).
[10] M. W. Thompson, Phil. Mag. **4**, 37, 139 (1959).
[11] M. Bader *et al.*, Proceedings of the Atomic and Molecular Beams Conference, University of Denver, Denver, Colorado, June, 1960 (unpublished), p. 167.
[12] F. C. Hurlbut, Proceedings of the Atomic and Molecular Beams Conference, University of Denver, Denver, Colorado, June, 1960 (unpublished), p. 152.
[13] G. K. Wehner (private communication).

described by Kosch and Bendt-Nielson.[14] Tank helium and argon of standard purity were employed, and neon, krypton, and xenon of somewhat greater purity. The helium and argon gases were passed through a dry-ice–acetone trap and sometimes, although its effect was not apparent, passed over heated potassium before admission to the arc source. The maximum impurity in the argon was 0.06%, and the xenon contained 0.03% krypton and 0.005% argon.

The portion of the ion beam passing through the entrance hole was focused on the target by means of the $\frac{1}{2}$-in.-diam lenses shown in Fig. 1. The target could be rotated about an axis normal to the incident plane by a shaft passing through an "O"-ring seal. The target surface itself was formed by slicing a potassium billet with a knife blade which was restricted to move in the plane passing through the axis of rotation. Fresh surfaces of potassium could be formed as required by "feeding" the billet forward with a threaded screw, and as many as 10 fresh surfaces could be obtained without disturbing the vacuum. The actuating mechanism was an O-ring sealed push-rod mounted on a sylphon.

The detector consisted of a $\frac{1}{16}$in.$\times\frac{1}{4}$in.$\times$0.5-mil platinum strip whose supporting stems were mounted on a Mycalex base. In Fig. 1 the filament has been rotated 90° from its normal position for the sake of illustration. The ion collector was a rectangular copper strip enclosing the filament. The detector was maintained at 20 v negative with respect to the ground filament. A screening grid consisting of a single loop of copper wire was maintained at 85 or more volts positive potential with respect to the filament. This screen, located between the entrance pupil and the filament, served to prevent scattered ions and secondary electrons from striking the filament or collector electrode.

The solid angle subtended by the detector was 0.002 sr as determined by the filament area and its distance from the target. The detector block was made of aluminum and was maintained at -100°C or less by conduction of the filament energy through copper braids to a liquid-nitrogen-cooled trap. As indicated in Fig. 1, the detector could be swung freely around the target in the plane of incidence. The range of the angular measurements was set by the geometrical configuration such that the detector could not be brought closer than 20° to the incident ion beam.

The pressure in the experimental chamber was maintained at about 10^{-6} mm Hg with source flow as measured with B.A. ion gauge. Liquid nitrogen trapping was employed. The pressure of the chamber without source flow was around 10^{-7} mm Hg. This pressure may have been lower in the area of the target, since liquid nitrogen cooled surfaces subtended an appreciable part of the solid angle visible to the potassium surface. Conditions permitting useful observations in the case of the 50-v-argon run were achieved by use of a liquid helium trap.

[14] J. Kosch and B. Bendt-Nielson, Kgl. Danske Videnskab. Selskab. Mat.-fys. Medd. **21**, No. 8 (1944).

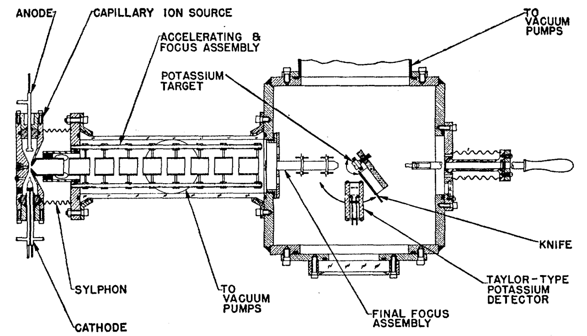

FIG. 1. Sputtering apparatus schematic diagram.

The incident ion beam flux density at the potassium surface was not known accurately, since the beam diameter at the target was not measured for each run. However, this diameter was measured once in the case or argon ions by means of a probe and was shown in this instance to be 2 mm.

The total ion beam current striking the target varied from 10^{-8} to 0.25×10^{-6} amp, depending upon the energy and composition of the beam. Electrostatic analysis of this beam showed an energy width of 7 ev. Multiply charged species were determined to be below 5% of the incident flux with a strong-focusing mass spectrometer.[15,16]

A zero-beam detector current was observed to be a function of detector position. In order to permit the correction of the data in respect to the zero-beam signal the ion beam was "chopped" electrostatically. The total measurement cycle was 20 sec in length, one-third being the beam-off period. Both detector and target current were continuously monitored.

Changes in surface conditions during observation were made apparent by observation at alternate points on successive sweeps. This procedure was employed for observation under both hot and cold filament conditions. The time to take a single distribution curve at a given angle was generally less than 10 min.

The time interval between the formation of new surfaces was usually less than an hour, depending in some degree on vacuum and beam current conditions.

III. DISCUSSION OF THE RESULTS

The data have been presented in the form of polar plots, Figs. 2–4, on which the heavy lines indicate the direction of the incident ion beam relative to the potassium surface. The short lines are drawn between experimental points obtained sequentially at the same detector angle, as an indication of the internal consistency of the data. Data obtained both with detector filament hot and with filament cold are presented and suitably identified.

[15] W. Paul, H. P. Renhard, and V. von Zahix, Z. Physik **152**, 143 (1958).

[16] M. Hertzberg, D. McKibbin, and D. Briglia, Rev. Sci. Instr. (to be published).

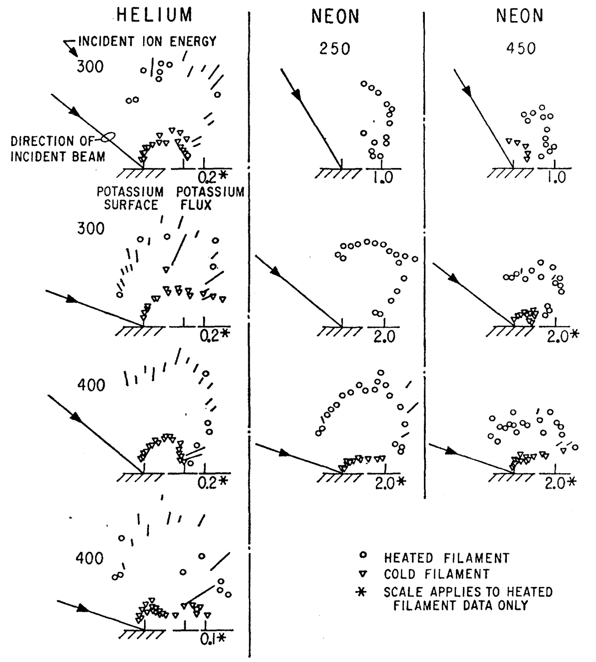

FIG. 2. Polar plots of sputtered potassium flux for helium and neon ions incident on potassium.

Representative argon data at various angles of incidence for 50-, 100-, and 240-ev ions are shown in Fig. 2. These data exhibit, as do the other data of these studies, a predominance of sputtering in forward directions and a general lobal configuration of the distribution. Certain limiting angles are ordinarily seen at the extremes of the patterns.

Data for helium, neon, krypton, and xenon are presented in Figs. 3 and 4. It should be noted that the neon hot filament curves show unusually large sputtered potassium flux by comparison with the other gases. In some instances the limiting angles at the right-hand extremes of the cold filament patterns are not defined; for example, xenon at 70°, 250 ev. It has not been determined what fraction of signals in these regions is due to sputtered potassium, since signals were also obtained with the platinum filament removed from the detector. These signals may be attributable to secondary electrons emitted from the collector upon incidence of neutral energetic atoms of the noble gases and of potassium.

An inspection of the typical distribution curves herein presented shows that sputtering is not an evaporative phenomenon in this energy range, and that a connection exists between the direction of incident momentum and the direction of ejection of sputtered particles. As the energy of the incident beam is increased, however, the curves do show a tendency toward the cosine distribution of the evaporative process, a trend also shown in recent observations of Wehner and Rosenberg.[17]

[17] G. K. Wehner and D. Rosenberg, J. Appl. Phys. 31, 177 (1960).

The observed flux distributions differ in magnitude from run to run, but exhibit characteristic configurations. The total sputtered yields are estimated to vary from three or four at a maximum to 0.05 at a minimum, depending upon surface conditions and the energy of the ion beam. However, these yield estimates suffer from numerous sources of inaccuracy, among which is a lack of precise information relative to distributions out of the plane of incidence. In general, there is a substantial difference between the data obtained with the hot and with the cold detector filament, the chief difference lying in the strength of signal, the ratio of hot to cold signals varying from 3 to 100, and decreasing for the higher energies of impact. The magnitude of this ratio is an indication of the fraction of atoms having energies sufficient for rebound (3 or more volts are suggested in the last section). The shapes of the cold filament and hot filament patterns also differ somewhat, notably at the largest angles from the incident beam. In these regions the cold filament pattern is reduced in amplitude, an effect which can be understood in consideration of the energy which can be delivered to particles at large angles from the initial direction. (See Figs. 6 and 7 and the remarks which follow.) This effect is more apparent at the lower incident ion energies and for the heavier gases. Data were obtained in the case of neon and argon at sufficiently low energies so that the large-angle energy requirement would influence the distribution of the heated filament patterns as well.

These observations are consistent with a sputtering model in which a series of two-body collisions takes place. The series is terminated by a favorable collision which directs the particle away from the surface with sufficient energy to overcome the surface binding forces.

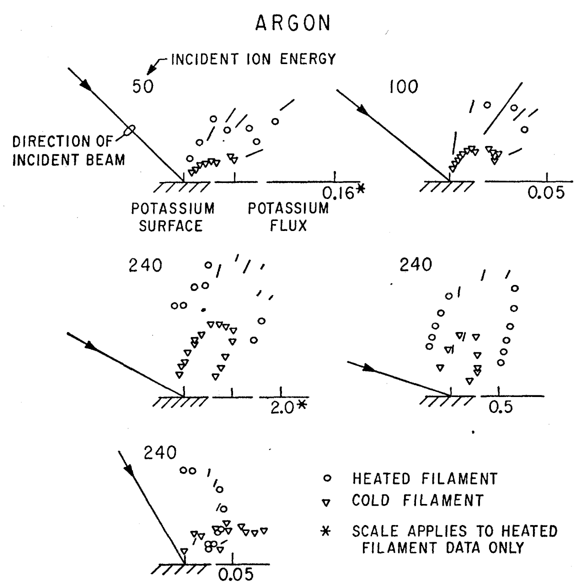

FIG. 3. Polar plots of sputtered potassium flux for argon ions incident on potassium.

The total sputtering process would hence be a composite of sputtering chains of one or more collisions.

One can interpret the experimental distributions in the sputtered flux as a measure of the probability of the occurrence of sputtering. A realistic computation of these probabilities, even for the billiard ball or hard elastic sphere model, is quite difficult in view of the roughness, both macroscopic and microscopic, of the surface, and in view of surface modification under continued ion bombardment. However, one may gain insight from computation of the probability distribution of particles after two billiard-ball collisions, where equal particle masses (argon-potassium) and random locations of the potassium atoms are assumed. Such a calculation was carried out, the results of which are shown in Fig. 5. A comparison of this curve with the observations reveals that in some cases, particularly at 30° incidence, the experimental values are larger than calculated. In this event a three-or-more-collision model is necessary to explain the large yields. At lower energies the requirement of multi-collision models is not imposed by the experimental observations, so that two-or-more-collision mechanisms could be dominant throughout. Although the collisional picture is not in question, the present distribution curves at the higher energies do at times resemble those of an evaporative process in regions at large angles from the normal in the sense that a cosine distribution may be fitted without difficulty.[17] Apparently the various randomizing factors, surface roughness, for example, are of particular importance here.

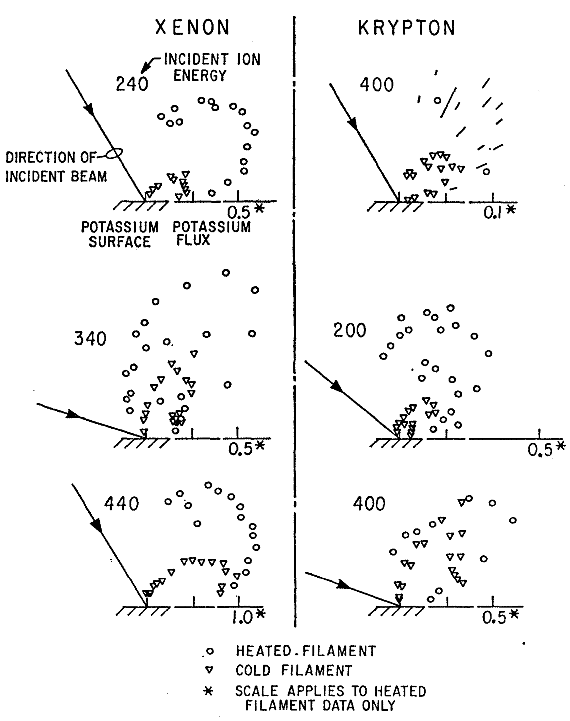

FIG. 4. Polar plots of sputtered potassium flux for xenon and krypton ions incident on potassium.

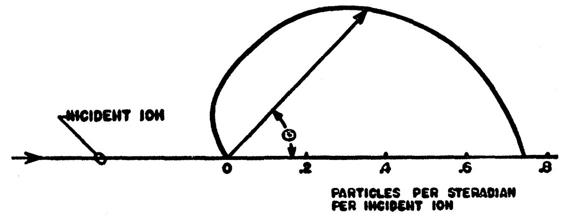

FIG. 5. Polar plot of the angular probability distribution of struck atom after two collisions, assuming hard elastic spheres of equal mass.

One may also calculate quite simply maximum energies of sputtered particles for a two-collision mechanism, assuming no dissipative losses. The maximum energy is found as a function of the angle θ which the ejected particles make with the incident direction. The results of such a computation are presented in Figs. 6 and 7. Here the ratio of the maximum energy of the sputtered particle to that of the incident ion energy is plotted in terms of the angle θ as indicated above. The curves of Fig. 6 correspond to the chain wherein the noble gas particle rebounds from a potassium particle and ejects another potassium particle on the subsequent collision in the manner postulated by Henschke.[1] Figure 7 illustrates the chain wherein an initially struck potassium particle subsequently collides and ejects either itself or another potassium particle in the manner advanced by Langberg.[2] The curves are nearly identical for argon because of the similarity of the masses of potassium and argon. Where the incident ion mass is greater than the surface particle mass, the energy ratio approaches zero with increasing θ in accordance with the expression

$$E/E_0 = B\cos^4(\theta/2), \qquad (1)$$

where B is the maximum energy fraction which can be delivered with a head-on collision. Thus, no zero-energy points are available in the cases of helium or neon. In these calculations we have assumed that the energy of ionization of the incident ion can be neglected. This energy is assumed to be given to a conduction electron in the manner proposed by Hagstrum[18] from studies of secondary electron production at clean surfaces by low-energy positive-ion bombardment.

On the basis of these computations one may find maximum ejection angles if one assumes the energy of release from the surface to be approximately that of the sublimation energy, 0.75 ev. The values of θ so computed lie beyond 145°, the approximate limit of experimental observation, except for the cases of argon at 50 and 100 v, respectively. For these cases the angles (139° and 145°) were not in disagreement with the observed limiting values.

If one requires the sputtered potassium particles to have a minimum energy of 3.75 ev, as an example, for detection at the cold filament, the total energy delivered to the sputtered particle before ejection must be at least

[18] H. D. Hagstrum, Phys. Rev. 96, 336 (1954).

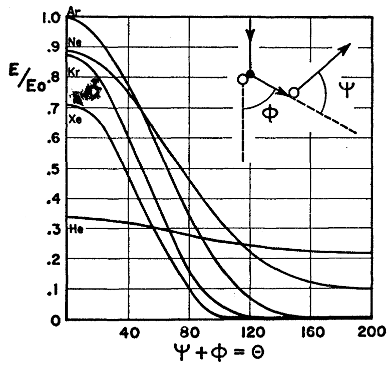

FIG. 6. Ratio of maximum terminal energy to incident energy as a function of total deflection angle θ.

4.5 v. Maximum angles for ejection in the various experimental situations have been found on this basis and are listed in Table I. Again the observed cutoff angles and the tabulated values are not in disagreement for mass ratios greater than or equal to 1. Similar agreement was obtained in the case of a distribution previously reported for 82-v argon ions incident at 60° upon a potassium surface.[19] Experimental limiting values were also observed with either hot or cold filament in the cases of helium and neon (30° and 50°) which are not in agreement with limiting values from process 1, Fig. 6. It may be concluded that process 2, Fig. 7, has influenced the results. It should be noted that the fourth-root dependency of $\cos(\theta/2)$ makes the angle insensitive to small variations of incident energy or energy requirements for detection.

The experimental observations reported herein and the preceding conclusions are not inconsistent with the observed preference of single cyrstals to sputter along directions corresponding to the nearest neighbor axis, as described by Wehner.[5] The potassium surfaces of the present experiment consist of many crystals possessing nearest neighbor axes at various orientations so that

TABLE I. Values of maximum angle for the observation of potassium sputtering using the two-collision model.

Ion	Bombarding energy (ev)	Minimum energy (ev)	Angle Calculated	Angle Measured
Argon	50	4.5	113°	98°
	100		125°	148°
	240		136°	163°
Krypton	200	4.5	133°	140°
	400		140°	143°
Xenon	240	4.5	132°	122°
	340		137°	130°
	440		140°	140°
Helium	300 } 30° incident	4.5	123°	146°
	400 } angle		130°	155°
	300 } 50° incident	4.5	123°	137°
	400 } angle		130°	148°
	300 } 70° incident	4.5	123°	140°
	400 } angle		130°	144°
Neon	75	4.5	119°	Insufficiently
	150		129°	precise
	250		136°	data
	450		142°	
Argon	50	0.75	139°	126°
	100		145°	123°
Neon	75	0.75	142°	
	150		148°	

structure in the distributions due to preferred directions would be obscured. It has also been pointed out by Silsbee[3] that potassium crystals which have much wider lattice spacings than most metals might be expected in consequence to exhibit directional effects less strongly.

IV. OTHER OBSERVATIONS

In the calculations above and in the interpretations relating to them, the specific arrangements of the atoms participating in the two-collision process have been ignored. Realistic orientations in two-collision chains will be relatively unimportant in view of the random orientations of the microcrystals, but will be important to a complete understanding of sputtering where three-or-more-collision processes are dominant.

The surface may be considered to have been in quasi-equilibrium throughout the measurements. Fluctuations of ion intensity were accompanied by an immediate proportional change in the observed sputtering rate, followed by slower changes as new levels of surface coverage became established. For example, in the case of xenon and krypton it was noted that an increase in bombarding current increased the yield, the time constants for this process again being of the order of a minute. If sufficiently high signal levels were not maintained the sputtering yield would decrease to zero in times of the order of minutes. Even with the best vacuums and the highest bombardment fluxes, a decay of signal after several hours was observed for all gases. The flux of incident ions was of the same order of magnitude as the flux of background noble gas under the best conditions, and ten times larger than the unknown im-

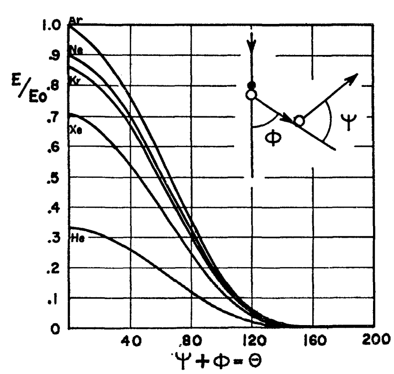

FIG. 7. Ratio of maximum terminal energy to incident energy as a function of total deflection angle θ.

[19] R. P. Stein, *First Symposium on Surface Effects on Space Craft Materials,* edited by F. Clauss (John Wiley & Sons, Inc., New York, 1960).

purity. One easily understands the variation of yield in this situation as noted by Wehner[5] and more recently by Yonts and Harrison.[20] An independent means of observing the surface coverage was not at hand. Observations of sputtering at the lowest incident energies were obtained immediately after formation of a clean surface (argon on potassium at 15 ev), but flux distributions could not be obtained at this energy because of the rapid deterioration of the signal. The very low threshold values observed by Bradley,[9] approximately 2 ev, were not observed, in part, quite possibly, because of insufficient observation time. The sputtering yield was observed to be greatest at angles of incidence in the neighborhood of 45° corroborating the observations of other experimenters. The sputtering yield in the case of neon was observed to be as great as that in the case of argon in spite of the mass inequality and resultant limitation on energy transfer, a possible consequence of the more efficient rebound mechanism available to neon. Sputtering in the case of helium was reduced, indicating that here the mass inequality effect was dominant.

V. ENERGY REQUIREMENTS FOR COLD FILAMENT DETECTION

Operation of the Taylor surface ionization detector is governed by the Saha-Langmuir equation where thermodynamic equilibrium is presumed to apply at the detector surface. Prediction is followed quite closely by experiment in the case of potassium and other alkali metal atoms on tungsten filaments, as indicated for example in the work of Werning.[21] However, careful observations by Datz and Taylor[22] of potassium ionization at a platinum surface require the assumption of reflection without either ionization or adsorption for some fraction of the incident atoms. The Saha-Langmuir equation predicts that the efficiency of ionization, which is ordinarily a substantial fraction of unity in the present case, increases as the ionizing surface temperature decreases, although it is ordinarily observed that the platinum filament must be maintained at a certain elevated temperature before any ionization is observed at all. This is attributable to the requirement that the surface be clean, and that the elevated temperature be sufficient to evaporate from the surface all incident potassium atoms, as well as the atoms of residual contaminant gases. In a very clean vacuum system the platinum filament remains clean after an initial flashing for some period of time. If, now, the incident potassium atom has sufficient kinetic energy to overcome the attractive forces, it might well rebound from a cold filament as an ion. It is known from the present experimental results that this effect does in fact occur. However, since equilibrium theory cannot be expected to apply, and since independent observations of the energies of sputtered atoms of this system have not yet been made, many questions are presented which remain unanswered.

One may make a crude estimate of the minimum energy requirement for the escape of potassium atoms by setting the energy of the recoiling particle equal to the energy of adsorption of potassium on the surface. If the energy of adsorption is taken to be 2.5 ev (the value for potassium on tungsten), the minimum incident energy becomes 5.7 ev for a head-on collision, and 3.75 ev for a collision in which the potassium ion rotates its momentum vector by 90°. Such a process is credible if the adsorption energy is principally due to Coulomb attraction since the incoming neutral particle will not have its kinetic energy augmented. If a portion of the adsorption energy can be represented in terms of a potential well common to both incident and leaving particles, a correspondingly lower figure is obtained for the single two-body collisions. Accurate prediction is complicated also by the likelihood that the effective threshold is in fact

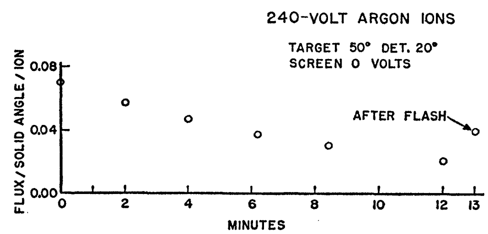

FIG. 8. Time dependence of detector signal for cold-filament condition.

established by a two-collision mechanism rather than by a single collision. The value 3.75 ev was arbitrarily used in the determination of thresholds although values of 3 ev and 4.5 ev could equally well be used without disturbing the conclusions.

Typical time dependence of cold filament ionization is indicated in Fig. 8. Potassium atoms were produced in sputtering by 240-v argon ions incident at an angle of 50° on the potassium surface. The detector assembly was located 20° from the surface normal. The platinum filament had not been heated since closure of the vacuum chamber. At the initial time the potassium surface was cleaned by slicing. No signal was observed by the detector for a period of 5 min, indicating that positive ions were not getting through the grid. The platinum filament was then flashed and the initial point of the curve was obtained. A decay of signal with time was observed until the filament was again flashed, yielding the last point of the curve. The difference in value between the initial and final points is attributed to deterioration of the potassium surface. Under other conditions, when the level of background pressure had

[20] O. C. Yonts and D. E. Harrison, Jr., J. Appl. Phys. **31**, 1583 (1960).
[21] J. R. Werning, University of California Radiation Laboratory Report UCRL-8455, 1958 (unpublished).
[22] S. Datz and E. H. Taylor, J. Chem. Phys. **25**, 389 (1956).

been reduced to 10^{-7} mm by a day's pumping, the deterioration of the potassium occurred much more slowly, and decay in detected signal attributable to coating of the platinum filament was virtually nonexistent. It may be concluded that the partial pressures of non-noble gases had been reduced to very low levels.

ACKNOWLEDGMENT

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Image of the Fermi Surface in Spin-Wave Spectra of Rare-Earth Metals*

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Calculations of spin-wave spectra in rare earth metals were carried out to find whether images of the electronic Fermi surface might be observable. In the space of spin-wave vectors **q** there should occur surfaces on which the frequencies have an infinite gradient with respect to **q**, the location of such abrupt changes, "kinks" in the dispersion curves, being determined by the shape of the Fermi surface. The spin-wave spectrum is found by assuming that the coupling between ionic spins takes place primarily through exchange scattering of conduction electrons, paralleling the calculation on the coupling of nuclear spins by Ruderman and Kittel. Spin-wave dispersion curves in two directions of high symmetry are computed numerically. The sought-for kinks in the dispersion curves are found to amount to about 2% of the maximum excitation frequency. The development is for ferromagnets, but extension to spiral antiferromagnets is taken up briefly.

I. INTRODUCTION

THE suggestion has been made by Kohn that, because of screening of the ions in a metal by conduction electrons, images of the electronic Fermi surface might show up in the phonon spectrum.[1] Kohn also pointed out that a similar effect might exist for spin waves in cases where the interaction between ionic spins originates primarily from exchange scattering with the conduction electrons.[2] In the space of wave vectors, for either of these excitations there will occur certain surfaces on which the frequencies will vary rather abruptly with **q**; the location of these surfaces will be determined by the shape of the Fermi surface. This raises the possibility of obtaining direct experimental information about the Fermi surface by examining these spectra, for example, with neutron scattering techniques.

For phonons special considerations, which will be discussed in a later publication, cause the effect to be so small as, probably, to be unobservable at present. A study of spin waves in the rare-earth metals, which will be the subject of the present paper, leads to more encouraging results.

The calculation of the spin-wave spectrum is similar to that carried out by Kasuya.[3] In view of the small radius of the $4f$ shell in the rare-earth metals the lattice spins are assumed to be coupled primarily through exchange scattering of conduction electrons.[3,4] A quasi-Hamiltonian describing the interaction of any two lattice spins is obtained, first, by choosing suitable interaction potentials between otherwise free conduction electrons and individual lattice spins. The change in total energy is then found by a perturbation calculation carried out to second order, paralleling the calculation on the coupling of nuclear spins by Ruderman and Kittel.[2] This leads to the familiar Heisenberg scalar-product interaction for the lattice spins. The spin-wave excitations are found in the usual way by expanding the Hamiltonian in powers of $1/S$.[3,5]

Since the rare-earth metals of interest have hexagonal close-packed (hcp) structure, that is, they have two atoms per unit cell, an additional transformation is required to diagonalize the Hamiltonian. Approximate numerical calculations of the spin wave frequencies as functions of the propagation vector **q** in two directions of high symmetry are carried out, so that the magnitudes of the expected discontinuities can be determined. Throughout the development the assumption is made that the ground state of the crystal is ferromagnetic. The extension of these results to the antiferromagnetic case is taken up briefly in the final discussion.

II. THE CALCULATION

In view of the small radius of the ions, the interaction between a conduction electron and the ionic

* Supported in part by the Office of Naval Research.
† National Science Foundation Predoctoral Fellow at the Carnegie Institute of Technology, Pittsburgh, Pennsylvania.
[1] W. Kohn, Phys. Rev. Letters **2**, 393 (1959).
[2] M. A. Ruderman and C. Kittel, Phys. Rev. **96**, 99 (1954).
[3] T. Kasuya, Progr. Theoret. Phys. (Kyoto) **16**, 58 (1956).
[4] P-G. de Gennes, Compt. rend. **247**, 1836 (1958).
[5] T. Holstein and H. Primakoff, Phys. Rev. **58**, 1098 (1940).