Dispersion of Gyromagnetic Ratios in Complex Spectra*

NORBERT ROSENZWEIG
Argonne National Laboratory, Argonne, Illinois

AND

C. E. PORTER
Brookhaven National Laboratory, Upton, New York

(Received March 20, 1961)

It is pointed out that the random-matrix hypothesis, which was previously used to explain the "repulsion" of energy levels in complex atomic and nuclear spectra, leads to a dispersion in the values of the gyromagnetic ratios which depends on the relative strengths of central and spin-dependent interactions and the number of states which interact with one another in the *LS* coupling scheme. The dispersion based on experimentally known atomic g factors is computed in three regions of the periodic table, and the empirical trends are found to be in qualitative agreement with the theoretical analysis.

WE have previously worked out some of the consequences of a statistical hypothesis for the Hamiltonian proposed by Wigner[1] for highly excited nuclear states. In particular, we observed and explained a varying degree of repulsion of atomic energy levels in terms of a suitable random matrix hypothesis which takes into account the relative strengths of spin-orbit and Coulomb interactions.[2,3] The random-matrix hypothesis also implies that the associated eigenfunctions have statistical properties which are similar to those of a random vector which is uniformly distributed on the surface of a unit sphere of high dimensionality. This property of the wave function was used in reference 2 for a derivation of the distribution of neutron widths.[4] We now find that the same ideas lead to some interesting statistical properties for the gyromagnetic ratios of complex quantum systems, and that these properties are easily discerned in the abundant experimental material of atomic spectroscopy.[5]

As an illustration, let us consider the hypothetical case of a complex atom in which a limited number of electron configurations interact strongly with one another but only very weakly (or not at all) with all other configurations. Suppose N energy levels of a definite parity and J value arise in these configurations. It is further assumed that these N states interact strongly in the SL coupling scheme (i.e., we want to consider the case of intermediate coupling). Let $\phi_j (j=1, \cdots, N)$ denote a complete orthonormal set of SLJ wave functions and g_j the well-known Landé factor. Let $\psi_i (i=1, \cdots, N)$ denote the correct wave function and G_i the associated gyromagnetic ratio. We have the well-known relations,

$$\psi_i = \sum_j U_{ij}\phi_j, \quad UU^T = 1, \tag{1}$$

and the g sum rule,

$$\langle G \rangle = \langle g \rangle, \tag{2}$$

where the average is to be performed, of course, over the N states in question. The dispersion of the correct gyromagnetic ratios about the mean $\langle G \rangle$ is given by

$$\langle G^2 \rangle - \langle G \rangle^2 = \sum_{j,k} g_j g_k \sum_i \frac{1}{N} U_{ij}^2 U_{ik}^2 - \langle g \rangle^2. \tag{3}$$

Strictly speaking, the sum $\sum_i U_{ij}^2 U_{ik}^2$ depends on the particular matrix U. However, if the system is sufficiently complex then one may expect that this dependence will be rather weak. In the spirit of our earlier work we introduce the hypothesis that the sum will be given approximately by its average value over the invariant distribution of orthogonal matrices in N dimensions.[6,7] This leads to our main result,

$$r^2 \equiv \frac{\langle G^2 \rangle - \langle G \rangle^2}{\langle g^2 \rangle - \langle g \rangle^2} = \frac{2}{N+2}, \tag{4}$$

which shows that the dispersion of the true gyromagnetic ratios may be expected to be much smaller than the dispersion of the Landé g factors in the LS coupling limit. This result (suitably modified to take into account the presence of both neutrons and protons and their intrinsic magnetic moments) suggests to us that the nuclear levels of a given parity and spin at sufficiently high excitation will have nearly the same magnetic moment. For example, this prediction applies to the resonance levels observed in the reaction $n+U^{238}$.

The extreme statistical model leading to Eq. (4) can not be expected to apply to atomic spectra without some

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] E. P. Wigner, *Proceedings of the Conference on Neutron Physics by Time-of-Flight*, Gatlinburg, Tennessee, 1957 [Oak Ridge National Laboratory Rept. ORNL-2309, (unpublished)], p. 59.

[2] C. E. Porter and N. Rosenzweig, Ann. Acad. Sci. Fennicae, No. 44 (1960).

[3] N. Rosenzweig and C. E. Porter, Phys. Rev. **120**, 1698 (1960).

[4] First obtained by R. G. Thomas and C. E. Porter, Phys. Rev. **104**, 483 (1956).

[5] *Atomic Energy Levels*, edited by C. E. Moore, National Bureau of Standards Circ. No. 467 (U. S. Government Printing Office, Washington, D. C., 1948).

[6] It should be noted that according to our hypothesis the dispersion of G is itself a random variable (although of small dispersion) and its "value" as given by Eq. (4) is its average value.

[7] As in our previous work, we neglect for simplicity the selection rules for spin-orbit coupling.

TABLE I. Average values and dispersions of measured G factors compared with the corresponding quantities g in the LS coupling limit for the three homologous elements Fe, Ru, and Os.

Spectrum	J	n_J	$\langle G\rangle$	$\langle g\rangle$	$\langle G^2\rangle-\langle G\rangle^2$	$\langle g^2\rangle-\langle g\rangle^2$	r^2
Fe I	1	35	1.33	1.33	0.648	0.755	0.856
	2	50	1.25	1.26	0.242	0.263	0.919
	3	53	1.21	1.21	0.109	0.116	0.939
	4	47	1.19	1.20	0.061	0.066	0.920
	5	35	1.17	1.18	0.033	0.035	0.934
	6	16	1.19	1.19	0.018	0.019	0.953
Ru I	1	21	1.18	1.17	0.301	0.532	0.566
	2	32	1.28	1.28	0.191	0.232	0.823
	3	27	1.26	1.26	0.070	0.092	0.761
	4	20	1.26	1.27	0.051	0.069	0.734
	5	17	1.20	1.20	0.035	0.040	0.860
	6	6	1.23	1.25	0.022	0.024	0.928
Os I	1	25	1.21	a	0.181	a	0.6[b]
	2	36	1.28	a	0.094	a	0.5[b]
	3	38	1.26	a	0.037	a	0.5[b]
	4	32	1.24	a	0.025	a	0.5[b]
	5	13	1.28	a	0.014	a	0.4[b]
	6	10	1.22	a	0.011	a	0.5[b]

[a] The number of levels for which LS assignments have been made are not sufficient to permit a direct estimate.
[b] Estimated by using the values of $\langle g^2\rangle-\langle g\rangle^2$ of Ru I.

modification because it is found[8] that even in intermediate coupling one particular LS component frequently dominates the wave function. However, a statistical treatment is still possible without serious inconsistencies. In analogy with (1), we now write

$$\psi_i=a_i\phi_i+b_i\sum_j U_{ij}\phi_j, \quad UU^T=1, \tag{5}$$

where ϕ_i is the dominant LS component. Applying the same statistical hypothesis to U, one now obtains

$$r^2=\langle(1-b^2)^2\rangle+\langle b^4\rangle[2/(N+2)]. \tag{6}$$

Pure LS coupling corresponds to $b\rightarrow 0$, in which case $r^2\rightarrow 1$. In the opposite limit, $b\rightarrow 1$ and r^2 is reduced to the small value given by Eq. (4). Thus one expects to find a systematic decrease in the value of r^2 as the departure from LS coupling increases.

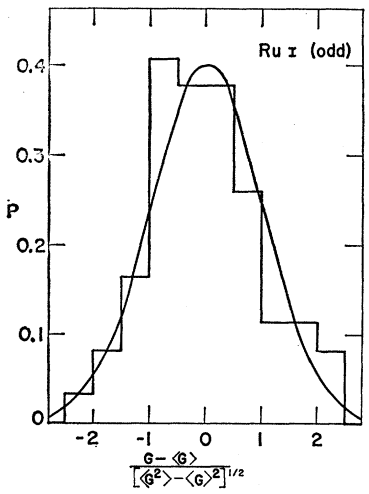

FIG. 1. Distribution of experimentally observed G values for the odd-parity levels of Ru I.

[8] For example, R. E. Trees, Phys. Rev. **49**, 838 (1959).

To a fairly good approximation, the spectra which are homologous with those of the iron group of elements may be regarded as arising from the same electron configurations. Thus, according to the above theory, the differences in the dispersion of the true G values should be determined mainly by the degree of departure from LS coupling. Generally speaking, this departure increases with increasing atomic number. Correspondingly, r^2 should decrease with increasing atomic number, and this is indeed found to be the case. As illustration, some results for the odd-parity levels of Fe I, Ru I, and Os I are shown in Table I, in which n_j denotes the number of levels with a given J value in the computation of the averages. In the case of Fe I and Ru I, only those levels were considered for which G has been measured *and* SL values have been assigned.[5] However, in the case of Os I the number of levels for which definite values of S and L have been assigned is so small that we have used all the levels for which G has been determined. The following points should be noted: (i) Although the states for which G has been measured do not form a complete set, the data are sufficiently representative for the sum rule (2) to hold in a statistical sense. In this connection, the comparison for Ru I, in which there is an appreciable departure from LS coupling, is more significant than the case of Fe I. (ii) For a fixed value of J the dispersion $\langle G^2\rangle-\langle G\rangle^2$, and also the ratio r^2, decreases with increasing atomic number. (iii) For a fixed element, $\langle G^2\rangle-\langle G\rangle^2$ decreases with increasing value of J. This can be understood in terms of the dependence of the Landé g factor on S, L, and J and the proportionality of the two dispersions which is exhibited by both (4) and (6).

It should be noted that in the case of spectra for which the SL composition of a sufficient number of individual states is known, the applicability of the above theory can be further scrutinized by evaluating both sides of

Eq. (6) (as well as certain terms which vanish in the statistical theory). An analysis to this end of existing theoretical eigenfunctions[8] is in progress.

In addition to the first and second moments, we have also studied the entire empirical distribution of G for many elements and found it to be fairly close to Gaussian in a majority of cases. The situation is illustrated in Fig. 1 for the 123 odd levels of Ru I. (The plot is actually a superposition of six histograms, one for each value of J.) On the theoretical side, we merely note that the discussion preceding Eq. (4) can evidently be extended to all higher moments, and this defines, at least formally, an entire distribution of G. The third moment is given by

$$\frac{\langle(G-\langle G\rangle)^3\rangle}{\langle(g-\langle g\rangle)^3\rangle}=\frac{8}{(N+2)(N+4)}, \tag{7}$$

which, like Eqs. (2) and (4), has the interesting form of a ratio of corresponding quantities in the LS and intermediate coupling schemes.

ACKNOWLEDGMENT

One of us (N.R.) is pleased to acknowledge a helpful discussion with A. Bohr.

PHYSICAL REVIEW VOLUME 123, NUMBER 3 AUGUST 1, 1961

"Breaks" in the Activation Curve of the $P^{31}(\gamma,n)P^{30}$ Reaction

D. SADEH*
Laboratoire de Synthèse Atomique, Centre National de la Recherche Scientifique, Ivry, France
(Received March 17, 1961)

Discontinuities in the slope of the activation curve of the $P^{31}(\gamma,n)P^{30}$ reaction were found while using two large NaI(Tl) crystals to detect the annihilation gammas from P^{30}. These breaks correspond well with resonances found recently in the $Si^{30}(p,n)P^{30}$ reaction. The correspondence between breaks and known resonances in some light nuclei is discussed. Such a correspondence was found to exist in the case of N^{14}, F^{19}, and P^{31} and is in doubt in the case of C^{12}. The detection system used allowed accurate measurements of the thresholds of the $P^{31}(\gamma,n)P^{30}$ reaction (12.23±0.04 Mev) and the $Cl^{35}(\gamma,n)Cl^{34}$ reaction (12.66±0.04 Mev).

I. INTRODUCTION

SINCE 1952 "breaks" in the activation curves of some light elements irradiated with gamma rays from a betatron have been found. These breaks were attributed to narrow resonances in the photon absorption cross section.[1–6]

The breaks of P^{31} were found only by Schull[2] in 1955 with a poor counting efficiency using Geiger counters. (One break in P^{31} was found by Geller *et al.*[7]) However, mass data and detecting systems recently made available permit us to calibrate the energy scale of the betatron with a better precision than that obtained by Schull. Our sensitive way of detecting the activity combined with the good energy stability of the betatron allowed us to determine these breaks to ±40 kev.

Recently the doubt about the origin of the breaks, in general was emphasized when Gove *et al.*[8] could not find any resonance in the $B^{11}(p,n)C^{12}$ reaction although such a resonance should have been observed[5] because of the breaks detected in the $C^{12}(\gamma,p)B^{11}$. This will be discussed in the light of our results.

II. EXPERIMENTAL PROCEDURE

A. Activity Measurements

Cylindrical samples of red phosphorus, 4.2 cm in diameter and 5 cm in height, were placed at a distance of 25 cm from the anticathode of the betatron. After irradiation, the sample was placed between two NaI(Tl) crystals of 5-in. diam and 4-in. high. A single-channel analyzer and a coincidence circuit ensured that only the two annihilation gammas from the 2.5-min β^+ of P^{30} were counted.

In practice, the crystals were placed in a lead castle 15-cm thick. A brick of lead provided with a cylindrical hole to contain the sample, was placed between the two crystals to prevent false coincidences between photons entering one crystal and being scattered to the other.

The identity of the samples was tested by irradiating each with an energy much higher than its threshold energy, and counting its activity.[1] The samples used were identical within ±0.2%.

The samples were irradiated and their activity was measured for a period of 8 min, pausing 30 sec between the end of the irradiation and the beginning of counting.

* Present address: Atomic Energy Commission of Israel, P.O.B. 7056, Hakirya, Tel-Aviv, Israel.
[1] L. Katz *et al.*, Phys. Rev. **95**, 464 (1954).
[2] C. G. Schull, Suppl. Nuovo cimento **4**, 1162 (1956).
[3] A. S. Penfold and B. M. Spicer, Phys. Rev. **100**, 1377 (1955).
[4] H. King and L. Katz, Can. J. Phys. **37**, 1387 (1959).
[5] I. M. Thorson and L. Katz, Proc. Roy. Soc. (London) (to be published).
[6] D. Sadeh, Compt. rend. **249**, 531 and 2313 (1959).
[7] K. N. Geller *et al.*, Phys. Rev. **118**, 1302 (1960).
[8] H. E. Gove *et al.*, Phys. Rev. Letters **3**, 177 (1959).