Specular Reflection in the Diffraction of Slow Electrons near Normal Incidence

J. P. HOBSON AND I. H. KHAN
Radio and Electrical Engineering Division, National Research Council, Ottawa, Canada
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During experiments on the diffraction of slow electrons (0–180 ev) at normal incidence, a diffracted beam was observed corresponding to the direction of specular reflection; i.e., straight back along the incident direction. This beam had properties indicating that it should not be considered as a limiting case of beams diffracted at other colatitude angles. Maxima in reflected intensity of this beam were observed at 4, 16, and 31 ev incident energy. The target used was a single crystal of tungsten with the (110) face exposed. Ultra-high-vacuum techniques were employed.

AN electron diffraction apparatus, similar in general design to that recently described by Scheibner *et al.*[1] has been constructed. A schematic diagram of the main diffraction chamber of the apparatus is shown in Fig. 1. Electrons from a low-voltage gun, with a coiled tungsten filament 1 mm in diameter, entered the chamber through the collimator *a*, and were finally focussed on the target *c* by the lens formed by the collimator and coaxial cylinder *b*. The target was a single crystal of tungsten with the (110) face exposed. It was about 4 mm in diameter and was electropolished until no deviations from an ideal surface were observable by standard electron diffraction and microscopic techniques. The reflected beam could be made visible on the fluorescent screen of willemite by application of 1000 v between this screen and the pair of nesting grids *d*. Electrode *e* was constructed with fins to give a low-reflection coefficient for electrons. The construction of the apparatus permitted visual observations to be made down to a colatitude angle of 2.5°. Helmholtz coils were used to cancel the earth's field. The apparatus was enclosed in a Pyrex envelope and was evacuated by ultrahigh-vacuum techniques to a total residual pressure less than 5×10^{-10} mm Hg.

Diffraction patterns similar to those reported by Scheibner *et al.*[1] were observed but in addition, because of slight misalignment of the target, a spot corresponding to the center of the diffraction pattern was observed on the screen. This spot evidently arose from direct specular reflection of electrons from the crystal. It was visible almost in the center of the optical pattern on the screen caused by reflection at the target of light from the gun filament. The spot was not photoelectric in origin since its presence depended upon the electron beam rather than the light beam. An interesting feature of this central spot was that it remained visible below incident energies of about 20 ev, when the main surface diffraction pattern had disappeared from the screen at right angles to the target surface. The central spot and the main diffraction pattern persisted for many days after the target had been outgassed at 2100°K for 2 hr. It appears likely that the target had at least a monolayer of gas on it by this time. In this condition maxima in the central spot as a function of incident energies less than 50 ev were observed by measuring the current on electrode *b*. Figure 2 shows the ratio of the current on electrode *b* (I_b) to that entering the measuring chamber $I_t=I_b+I_c+I_d+I_e$ for four settings of the chamber voltages. The flourescent screen was joined to *d* for these measurements. All voltages represent energies of the slowest electrons in the thermal energy spread from the gun, as established by retarding field plots. The estimated accuracy is $\pm\frac{1}{2}$ ev. The four curves are included to demonstrate that the positions of the main peaks at energies of about 4, 16, and 31 ev do not depend to first order upon focusing conditions in the measuring chamber which are a function of the chamber voltages. However, only qualitative significance may

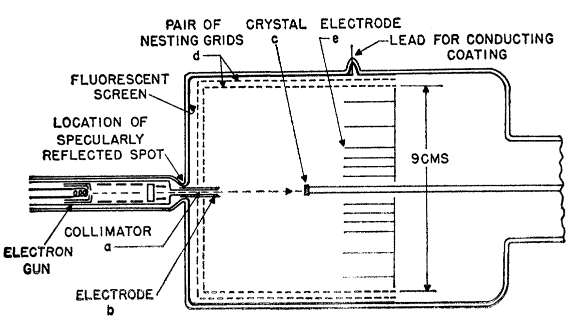

FIG. 1. Schematic diagram of apparatus.

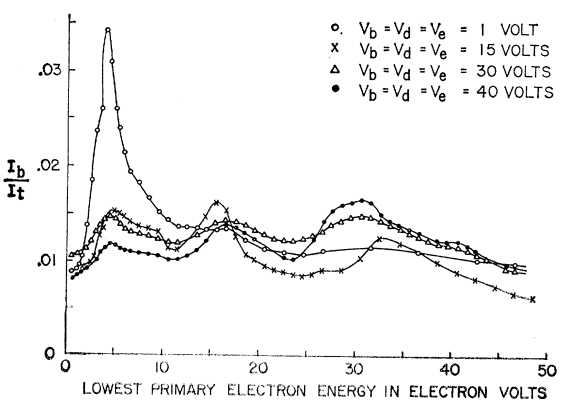

FIG. 2. The ratio of the current on electrode *b* (I_b) to the total current entering the measuring chamber (I_t) as a function of incident electron energy on the target.

[1] E. J. Scheibner, L. H. Germer, and C. D. Hartman, Rev. Sci. Instr. 31, 112 (1960).

be attributed to the relative amplitudes which are clearly dependent on focusing conditions. Extensive tests were performed to establish that only a minor portion of the incident beam missed the target and that the results shown are characteristic of the reflection from the target and have no other origin.

In the great majority of slow-electron diffraction units operating at normal incidence, the specularly reflected beam cannot be observed because it is lost in the structure of the gun. However, Sproull,[2] using a magnetic deflection method, has studied this beam from the (112) and (100) faces of tungsten. Sproull's analysis was within the framework of conventional diffraction theory and he viewed the specularly reflected beam as a special case (colatitude angle zero) in which the volume interference condition was satisfied. In this case a specularly reflected spot splits into two spots which diverge along a principal azimuth as the incident energy is raised. There appears no doubt that Sproull's observations on the (112) face fit this interpretation. However, in our experiments, even though we could visually observe the central spot almost continuously for incident energies from 0 to 180 ev, we observed no tendency for this spot to split. We suggest therefore that there exist cases in which the specularly reflected beam should not be considered as a limiting case of beams diffracted at other colatitude angles. The theory of MacColl[3] on the reflection of electrons by metallic crystals appears to be a good starting point for these cases. We have not yet attempted to apply this theory to the results of Fig. 2.

[2] W. T. Sproull, Phys. Rev. **43**, 516 (1933).

[3] L. A. MacColl, Bell System Tech. J. **30**, 888 (1951).

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Cyclotron Resonance and de Haas-van Alphen Oscillations of an Interacting Electron Gas*

WALTER KOHN
University of California at San Diego, La Jolla, California

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An electron gas with short-range interactions is considered in the presence of a uniform magnetic field. It is shown that (1) the cyclotron resonance frequency is independent of the interaction; (2) for a two-dimensional gas, the de Haas–van Alphen period is independent of the interaction. The low-lying excited states are briefly discussed.

THERE has been considerable interest in recent months in the effects of the electron-electron interaction on the cyclotron resonance frequency and de Haas–van Alphen oscillations of a gas of electrons. As some of the theoretical treatments of these problems use very sophisticated methods,[1] and others are based on incorrect qualitative reasoning, we wish here to present some simple considerations which we think shed some light on what has been a rather confusing situation.

In the present paper we restrict ourselves to the case of a short range electron-electron interaction, deferring specific effects of the long-range Coulomb force to a later account.

We write the Hamiltonian of our system, in a uniform magnetic field $\mathcal{H}$ in the z direction, as

$$H=\frac{1}{2m}\sum_{i=1}^{N}\mathbf{P}_i^2+U, \tag{1}$$

where

$$\mathbf{P}_i=[p_{i,x},\, p_{i,y}+(e\mathcal{H}/c)x_i,\, p_{i,z}], \tag{2}$$

and the interaction U is

$$U=\sum_{i,j} u(\mathbf{r}_i-\mathbf{r}_j). \tag{3}$$

CYCLOTRON RESONANCE

We verify directly that if we define the kinetic momentum of the whole system as

$$\mathbf{P}\equiv\textstyle\sum_i \mathbf{P}_i, \tag{4}$$

then

$$\frac{d\mathbf{P}}{dt}=\frac{i}{\hbar}[H,\mathbf{P}]=-\frac{e}{mc}\mathbf{P}\times\mathcal{H}, \tag{5}$$

which is the Lorentz equation for the whole system in operator form. We now define

$$P_\pm\equiv P_x\pm iP_y, \tag{6}$$

and the cyclotron frequency,

$$\omega_c=e\mathcal{H}/mc. \tag{7}$$

Then by (5) we find that

$$[H,P_\pm]=\pm\hbar\omega_c P_\pm. \tag{8}$$

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[1] J. M. Luttinger, Phys. Rev. **121**, 1251 (1961).